Pricing S-forwards via the Risk Margin under Solvency II

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Goal of the paper

- We need to estimate the market price of longevity risk (premium that a life insurer or pension plan might be willing to pay to release such a risk)
- In an incomplete market, such as the longevity-linked securities market, it is not possible to estimate a unique market price of risk
- Goal of the paper: investigating the possibility to derive the market price of longevity risk through the risk margin implicit in the evaluation of the technical provisions as required by Solvency II
- Technical provisions calculated as the sum of their best estimate plus a risk margin (the market value of the uncertainty on insurance obligations)
- Some authors suggested to link the pricing to the amount that the insurer should hold to cover unexpected losses. E.g.:
  - Börger (2010) : risk margin considered as the maximum price a life insurer would be willing to pay to transfer LR via securitization

Mortality model

- Mortality data coming from Italian male population : period 1974-2007, age 60-90
- Mortality model chosen between the ones compared in Cairns et al. 2008

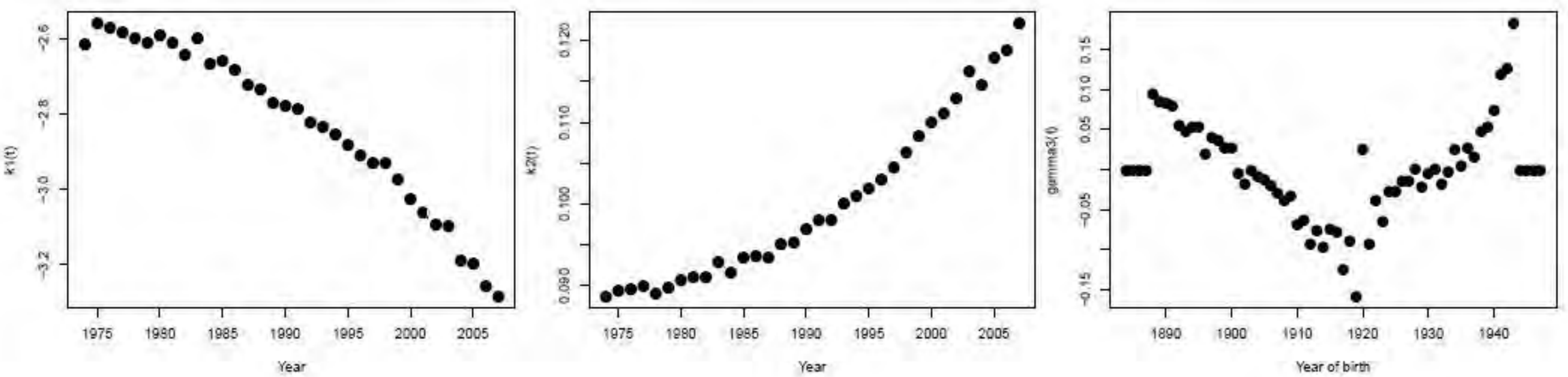
Model	MLL	BIC	Rank(MLL)	Rank(BIC)
LC	-9,823	-10,149	7	7
RH	-6,105	-6,754	1	4
Currie APC	-7,049	-7,486	5	5
CBD	-9,691	-9,927	6	6
CBD-1	-6,230	**-6,681**	3	**1**
CBD-2	-6,122	-6,688	2	2
CBD-3	-6,250	-6,709	4	3

Mortality model: CBD-1

- Death probabilities described by

$$logit(q_{x,t}) = ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = k_t^{(1)} + k_t^{(2)}(x-\bar{x}) + \gamma_c^{(3)}$$

- Estimated parameters



- Forecast of parameters via a multivariate ARIMA model

$$K_{s+1} = K_s + \phi(K_{s-2} - K_{s-1}) + \mu + CZ_{s+1}$$ with K_s vector of parameters $k_t^{(1)}$ $k_t^{(2)}$ and $\gamma_c^{(3)}$

Results: parameters estimation

Parameter	ARIMA	σ^2	μ	ϕ
$k^{(1)}$	(0,1,0)	0.000759	-0.020394	0
$k^{(2)}$	(0,1,0)	0.000001	0.001015	0
$\gamma^{(3)}$	(1,1,0)	0.000414	0.011075	-0.569975

Table 2: Fitted parameters of the ARIMA models

$$C = \begin{pmatrix} 0.027548 & 0 & 0 \\ 0.000469 & 0.001077 & 0 \\ 0 & 0 & 0.020358 \end{pmatrix}$$

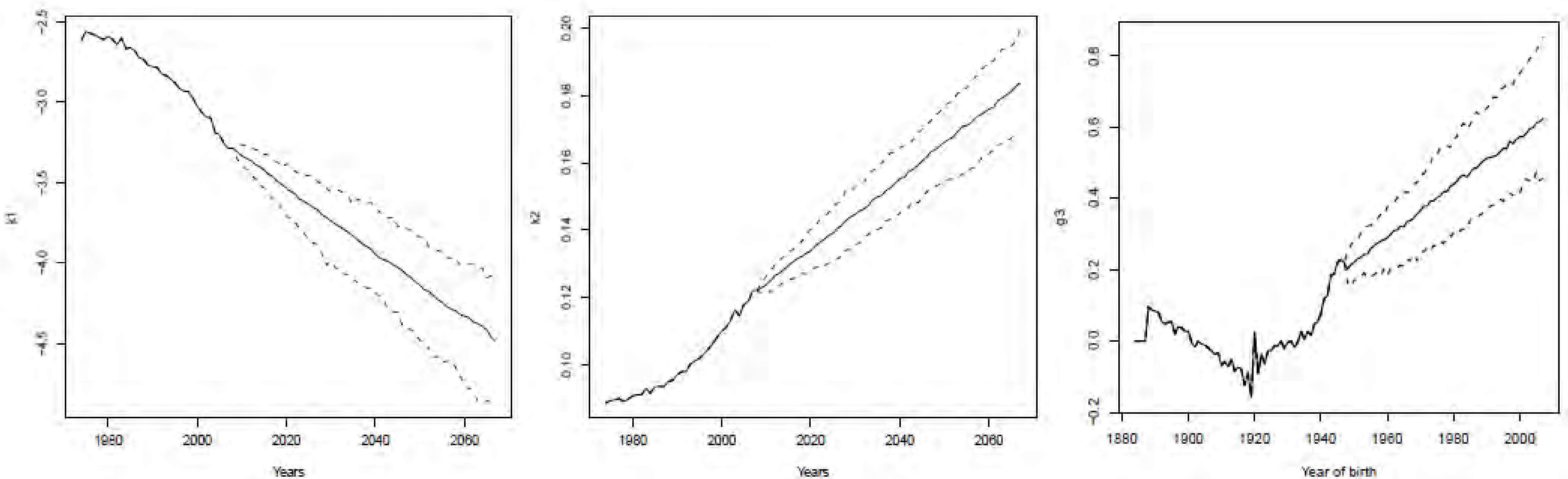


Figure 3: Parameters estimation with corresponding 5% and 95% prediction intervals, years 1974-2067.

Mortality model: CBD-1 under a risk-neutral measure

- The dynamics under a risk-neutral measure Q equivalent to the current real-world measure P becomes

$$K_{s+1} = K_s + \phi(K_{s-1} - K_s) + \mu + C(Z^Q_{s+1} - \lambda)$$

- whit $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ market prices of longevity risk associated with $Z^{(1)}$, $Z^{(2)}$, $Z^{(3)}$ under Q

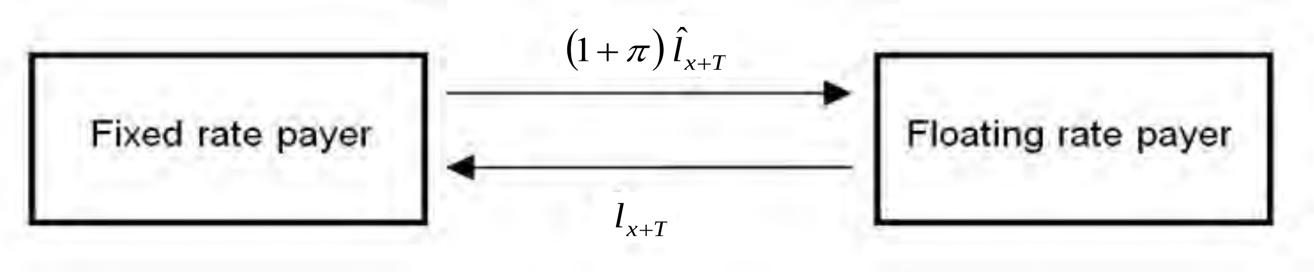
Portfolio of pure endowments

- Insurer with a portfolio of pure endowments paying a lump sum (1€) at maturity T to a cohort of l_x individuals all aged *x* at initial time
  - $\hat{l}_{x+T}$: expected number of survivors to age $x+T$ at time T
  - l_{x+T} : realized number of survivors to age $x+T$ at time T
- Exposure to risk of systematic deviations between l_{x+T} and $\hat{l}_{x+T}$
- $l_{x+T} - \hat{l}_{x+T}$: losses experienced by the insurer at time T

S-forward

- S-forward: agreement between two counterparties to exchange at maturity T an amount equal to the realized survival rate of a given population cohort (floating rate payment), in return for a fixed survival rate agreed at the inception of the contract (fixed rate payment), is a zero-coupon fixed-for-floating survival swap
  - π ($\pi \gtrless 0$): fixed proportional S-forward premium, set in a way that the swap value is zero at inception → market value of fixed leg = market value of floating leg

Figure: S-forward cash flows



- Compared with other mortality securities – e.g. longevity bonds – S-forwards:
  - Involve lower transactions cost
  - More flexible and tailor-made to meet different needs
  - Do not require the existence of a liquid market

S-forward (zc survival swap) pricing

- The basic survivor swap value at time zero to the fixed-rate payer is:

$$\text{Swap value} = V\left[l_{x+T}\right] - V\left[(1+\pi)\hat{l}_{x+T}\right]$$

market price of the floating leg in t=0

market price of the fixed leg in t=0

- Assuming the independence between interest rate and mortality:

$$V\left[(1+\pi)\hat{l}_{x+T}\right] = (1+\pi)\hat{l}_{x+T}\, d(0,T)$$

expected present value of the fixed leg under the real-world probability measure

$$V\left[l_{x+T}\right] = E^*\left(l_{x+T}\right) d(0,T)$$

expected present value of the floating leg under a risk-adjusted probability measure

$d(0,T)$: risk-free discount factor

Survivor swap premium ⇒

$$\pi = \frac{E^*\left(l_{x+T}\right)}{\hat{l}_{x+T}} - 1 = \frac{{}_T p_x^*}{{}_T \hat{p}_x} - 1$$

Estimation of the risk-adjusted probabilities

- Question: how to estimate the risk-adjusted survival probabilities?
- We propose an approach using the risk margin implicit in the evaluation of the technical provision under Solvency 2

Risk margin

- Under Solvency 2:
  - market value of liabilities = best estimate + Risk Margin
- The Risk Margin (RM) is used to provide a risk adjustment of the best estimate liabilities → sort of "market prices of liabilities"
- The cost-of-capital (CoC) approach
  - RM = the cost of providing an amount of capital equal to the Solvency Capital Requirements necessary to support the insurance obligations over their lifetime
  - Cost of capital rate set to 6% (QIS 5 technical specifications):

$$RM_t = 6\% \sum_{i=t}^{T-1} SCR_i \cdot d(t,i)$$

Capital requirements under Solvency 2

- Solvency Capital Requirement (SCR) under Solvency II
  - SCR defined as the amount of economic capital that an insurance company needs to hold to absorb unexpected losses over a 1-year at a 99.5% percentile level
  - Parameters of the standard formula determined to reflect a 99.5% Value-at-Risk (VaR) on the unexpected loss on a 1 year time horizon

- Solvency II Standard formula
  - SCR for longevity risk = change in the net value of assets minus liabilities due to a longevity shock

$$SCR_t = (\Delta NAV_t \,|\, longevity\ shock) = V_t^{'} - \hat{V}_t = l_{x,0}\left({}_T p'_{x,0} - {}_T \hat{p}_{x,0}\right) d(t,T)$$

Technical provision with shock ($V_t^{'}$) — Best estimate of technical provision ($\hat{V}_t$)

  - Longevity shock = permanent 20% reduction of mortality rates at each age
  - One level of possible shock → deterministic setting

Risk margin and market price of longevity risk

- Assumption: the insurance company is only exposed to longevity risk
- An insurer completely hedged against longevity risk : no solvency capital for longevity risk to provide → SCR=0 → RM=0
- Assumption: the insurer is interested in securitizing its LR if the transaction price is lower or equal to the PV of the future CoC required in presence of LR
- Risk Margin → maximum price the insurance company would be willing to pay for longevity risk securitization (see Börger (2010))
- Drawbacks:
  - The company's expected cost of capital µ ay be lower than the Risk Margin
  - A company might accept a higher market price of risk for strategic reasons (e.g. difficulties in raising capital and risk of increasing cost of capital in the future).
  - Different companies might accept different longevity risk prices
- The Risk Margin → a starting point for the pricing of longevity derivatives

Estimation of the risk-adjusted probabilities from the risk margin

- Solvency capital requirements (SCR) in t=0 for a portfolio of pure endowment

$$\mathrm{SCR}_0 = \underbrace{V'_0 - \hat{V}_0}_{} = l'_{x+T,T} \cdot d(0,T) - \hat{l}_{x+T,T} \cdot d(0,T) = l_{x,0}\underbrace{\left({}_T p'_{x,0} - {}_T \hat{p}_{x,0}\right)}_{} \cdot d(0,T)$$

Difference between technical provision with longevity shock and the best estimate

Difference between survival probabilities with longevity shock and the best estimate

- SCR in t=0 after a S-forward with maturity T (portfolio maturity)

$$\mathrm{SCR}_0^S = \left(l_{x,0} - l_{x,0}^S\right) \cdot \left({}_T p'_{x,0} - {}_T \hat{p}_{x,0}\right) \cdot d(0,T)$$

If $l_{x,0} = l_{x,0}^S$

then: $\mathrm{SCR}_0^S = 0$

- SCR requirements in t after a a S-forward with maturity T (portfolio maturity)

$$\mathrm{SCR}_t^S = \left(l_{x,0} - l_{x,0}^S\right) \cdot \left({}_T p'_{x,0} - {}_T \hat{p}_{x,0}\right) \cdot d(t,T)$$

If $l_{x,0} = l_{x,0}^S$

then: $\mathrm{SCR}_t^S = 0$

Assumptions: no credit risk and no basis risk

Estimation of the risk-adjusted probabilities from the risk margin

- Risk margin in t=0

[1] $$\mathrm{RM}_0 = 6\% \cdot \sum_{i=0}^{T-1} \mathrm{SCR}_{\mathrm{i}} \cdot d(0,i) \; = \; 6\% \cdot l_{x,0} \sum_{i=0}^{T-1} \left({}_T p'_{x,0} - {}_h \hat{p}_{x,0} \right) \cdot d(0,T) = 6\% \cdot l_{x,0} \cdot T \cdot \left({}_T p'_{x,0} - {}_h \hat{p}_{x,0} \right) \cdot d(0,T)$$

- Risk margin in t=0 after a S-forward with maturity T

[2] $$\mathrm{RM}_0^S = 6\% \cdot \sum_{i=0}^{T-1} \mathrm{SCR}_{\mathrm{i}}^S \cdot d(0,i) \; = \; 6\% \cdot T \cdot \left(l_{x,0} - l_{x,0}^S \right) \cdot \left({}_T p'_{x,0} - {}_T \hat{p}_{x,0} \right) \cdot d(0,T)$$

- Measure of the amount of RM saved by the insurer entering in a S-forward :

$$\mathrm{RM}_0 - \mathrm{RM}_0^S = 6\% \cdot T \cdot l_{x,0}^S \left({}_T p'_{x,0} - {}_T \hat{p}_{x,0} \right) d(0,T) \quad -- > \pi \cdot \hat{l}_{x+T,T}^S \cdot d(0,T)$$

$$\pi^{\max} = e^{\delta^{\max} T} - 1 = \frac{6\% \cdot T \cdot \left({}_T p'_{x,0} - {}_T \hat{p}_{x,0} \right)}{{}_T \hat{p}_{x,0}}$$

- RM saving = maximum premium to be paid by the insurer for hedging LR

Market price of longevity risk

- Assuming that $\pi = \pi^{\max}$:
- We find the "maximum" risk-neutral probabilities

$$E_Q[{}_T p_{x,0}] = {}_T \hat{p}_{x,0} + 6\% \cdot T \cdot \left({}_T p'_{x,0} - {}_T \hat{p}_{x,0} \right)$$

$$E_Q[{}_T p_{x,T}] = E_Q\left[\prod_{s=0}^{T-1} p_{x+s,t+s} \right] = E_Q\left[\prod_{s=0}^{T-1} \frac{1}{1+\exp\left(k_{t+s}^{(1)} + k_{t+s}^{(2)}(x-\bar{x}) + \gamma_{t-x}^{(3)}\right)} \right]$$

- And then the market price of longevity risk implicit in the risk-neutral probability measure:

$$\lambda = \arg\min_{\lambda} \sum_{i=1}^{m} \left[\delta_i(\lambda) - \delta_i^{\max} \right]^2$$

where: $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ and:

m = number of S-forwards the model is calibrated to

and: $$\delta_i^{\max} = \ln\left[\frac{{}_{T_i} \hat{p}_{x_i,0} + 6\% \cdot T_i \left({}_{T_i} p'_{x_i,0} - {}_{T_i} \hat{p}_{x_i,0} \right)}{{}_{T_i} \hat{p}_{x_i,0}} \right] \frac{1}{T_i}$$

$$\delta_i(\lambda) = \ln\left\{ \frac{E_{Q(\lambda)}\left[{}_{T_i} p_{x_i,0} \right]}{{}_{T_i} \hat{p}_{x_i,0}} \right\} \frac{1}{T_i}$$

Numerical application

- Portfolio of pure endowments
- Initial cohort of l_x=10,000 policyholders aged x in year 2007 (t=0)
- Death counts for age 60-90 and years 1974-2007
- Risk-free interest rate term structure taken from CEIOPS : year 2007
- Assumptions: no credit risk and no basis risk

Results: S-forward maximum price

Males

Year of birth:	1957	1952	1947	1942	1937
T			π^{max}		
5	0.000815	0.001465	0.002643	0.004608	0.007709
10	0.004140	0.007576	0.013922	0.024792	0.042504
15	0.012329	0.023027	0.043318	0.079302	0.140696
20	0.030432	0.058281	0.113004	0.215028	0.401905
25	0.069816	0.138029	0.279069	0.563423	-
30	0.157589	0.325354	0.701456	-	-

Table 3: Values of the S-forward "maximum price" π^{max} for different maturities, T

Results: S-forward maximum spread

Males

Year of birth:	1957	1952	1947	1942	1937
T			δ^{max}		
5	0.000163	0.000293	0.000528	0.000920	0.001536
10	0.000413	0.000755	0.001383	0.002449	0.004163
15	0.000817	0.001518	0.002827	0.005088	0.008776
20	0.001499	0.002832	0.005353	0.009738	0.016892
25	0.002699	0.005172	0.009845	0.017875	-
30	0.004878	0.009389	0.017716	-	-

Table 4: Values of the S-forward "maximum spread" δ^{max} for different maturities, T.

Results: market price of longevity risk

- The vector $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ calibrated to five S-forwards price with:
  - same maturity: T=10
  - different ages: 50, 55, 60, 65, 70 years
  - evaluated in the year 2007
- λ = (0.924316, 0.066557, 0.024869)

Further research

- Compare the results with other pricing methodology
- Effect of different approximations on SCR_t evaluation in the standard formula
- Adoption of an internal model instead of the standard formula
- More complex insurance portfolio (life annuities) and more complex hedging instruments (Plain vanilla survivor swaps)
- …

Thank you for your attention

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