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# The Impact of Policyholder Behavior on Pricing, Hedging, and Hedge Efficiency of Withdrawal Benefit Guarantees in Variable Annuities

Alexander Kling, Frederik Ruez and Jochen Ruß

Frederik Ruez
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# Research Purpose

- Variable Annuities are fund-linked annuities
  - the policyholder typically pays a single premium, which is invested in one or several mutual funds
  - several guarantee riders available on top of this
- ***“Guaranteed Lifetime Withdrawal Benefits”*** (GLWB)
  - the policyholder is guaranteed lifelong minimum withdrawals
  - the invested capital is not annuitized
    
    ➔ fund assets remain accessible to the policyholder
  - withdrawals are deducted from the policyholder’s account value as long as it has not been depleted
    - afterwards, the insurer has to compensate for the guaranteed withdrawals until the insured’s death
  - in return for this guarantee, the insurer receives guarantee fees deducted from the policyholder’s fund assets

➔ combination of policyholder behavior, longevity and market risk that is difficult to hedge

# Research Purpose – Previous Paper

- Kling, Ruez and Ruß (2011):
  "The Impact of Stochastic Volatility on Pricing, Hedging, and Hedge Efficiency of Withdrawal Benefit Guarantees in Variable Annuities"
  - analysis of the impact of stochastic equity volatility on pricing and hedging of GLWB riders
    - similar framework used as in Bauer et al. (2008)
  - comparison of different hedging strategies and their efficiency
    - no hedge, delta hedge, and delta-vega hedge
  - focus lies on model risk (financial market)
    - hedging model vs. data-generating model
  - policyholder behavior modeled deterministically

    ➔ **starting point of current paper**

# Research Questions

– The focus of this paper lies on behavioral risk in the context of GLWB riders. Its key question is:

  What is the impact of policyholder behavior on pricing and hedging of GLWB riders?

  - What is the magnitude of potential losses if assumptions about future policyholder behavior prove to be wrong?
  - What effect does the product design of the GLWB rider have on the results?

## Main findings

– Policyholder behavior that differs from pricing / hedging assumptions may lead to significant losses to the insurer.
  - assuming optimal (financially rational) behavior, however, may lead to products that are not competitive

➔ Use product design to reduce sensitivity to policyholder behavior.
  - insurer may then assume optimal behavior without losing (too much) competitiveness
  - but: such product designs may have other disadvantages
    – harder to hedge and higher sensitivity to changes in volatility

– In order to assess their behavioral assumptions, insurers should know what ‘optimal’ policyholder behavior would be.
  - modeled behavior in more elaborate models may still be far from ‘optimal’

## Agenda

- Product designs
- Market models
- Models of the policyholder behavior
- Pricing results
- Hedging results
- Outlook

# Product designs of the GLWB option

- All considered designs guarantee an annual minimum withdrawal amount for the lifetime of the insured.
  - surrender benefit = account value less surrender fees
  - death benefit = account value

- Depending on the product design, the guaranteed withdrawal amount increases if the fund performs well.

➔ Three different ratchet mechanisms considered:

**1) No Ratchet**
- the guaranteed withdrawal amount remains constant

**2) Lookback Ratchet**
- the guaranteed withdrawal amount is calculated as a percentage of the highest account value at all past policy anniversaries

**3) Remaining Withdrawal Benefit Base (WBB) Ratchet**
- if the account value exceeds a certain reference value, the surplus is used to increase the guaranteed withdrawal amount for all following payments

# Market models used for pricing, hedging and simulation

- constant interest rates
- no spreads / no transaction costs
- The dynamics of the contract's underlying fund is given by the following SDEs:
  - **Black-Scholes (1973)**

$$dS(t) = \mu S(t)dt + \sigma_{BS} S(t)dW(t), \quad S(0) \geq 0$$

  - **Heston (1993)**

$$dS(t) = \mu S(t)dt + \sqrt{V(t)}S(t)dW_1(t), \quad S(0) \geq 0$$

$$dV(t) = \kappa\big(\theta - V(t)\big)dt + \sigma_v \sqrt{V(t)}dW_2(t), \quad V(0) \geq 0$$

  - with
    - $\mu$ - drift
      $\sigma_{BS}$ - Black-Scholes volatility
    - $V$(t) - local variance at time t
      $\kappa$ - speed of mean reversion
      $\theta$ **-** long-term average variance
      $\sigma_v$ - "volatility of volatility"
      $W_{1/2}$ - Wiener processes
      $\rho$ - correlation between $W_1$ and $W_2$

# Policyholder Behavior

- Only two options considered:
  - policyholder withdraws guaranteed amount or
  - policyholder withdraws all of the remaining fund assets
    - ➔ full surrender
- Behavior within a pool of policies modeled as annual percentage indicating the portion of policyholders who surrender their contract each year

## Policyholder Behavior – Considered Models

1) **Deterministic behavior**
   - each year, a deterministic but time-dependent percentage of the policyholders perform full surrender
2) **Optimal (financially rational) behavior**
   - approximated via Least-Squares-MC approach (LSMC)
3) **Moneyness approach**
   - practitioner's approach
   - use deterministic behavior as base
   - determine factor between 1/3 and 5 depending on the 'moneyness' of the guarantee
   - we use the ratio between surrender value and the NPV of an immediate annuity as 'moneyness'
4) **(➔ paper)**

# Selected Pricing Results

– Potential loss from mispricing: assumed vs. actual
  - assumed: Black-Scholes, $\sigma_{BS}$ = 22%, deterministic behavior

➔ actual option value at inception (in % of single premium):

| | No Ratchet | Lookback | Rem. WBB |
|---|---|---|---|
| **correct** | 0.0 | 0.0 | 0.0 |
| **Moneyness** | -2.4 | -1.3 | -1.2 |
| **Optimal** | -4.6 | -2.4 | -2.3 |
| **Vol 25 %** | -1.1 | -2.1 | -2.1 |

# Selected Hedging Results

- homogeneous pool of policies
- Black-Scholes delta hedge
  - monthly rebalancing of hedge portfolio

- What if actual behavior differs from pricing and hedging assumptions?
  - data-generating model Black-Scholes
  - values represent (relative changes in) the risk measure
    - CTE90 of final P&L in % of single premium

| Assumed / Actual | **No Ratchet** | **Lookback** | **Rem. WBB** |
|---|---|---|---|
| **Det. / Det.** | **1.2** | **2.6** | **2.4** |
| **Det. / Moneyness** | **8.2** (+583%) | **3.9** (+50%) | **3.6** (+50%) |
| **Det. / Optimal** | **12.5** (+942%) | **6.2** (+138%) | **5.4** (+125%) |

# Selected Hedging Results

- data-generating model: Black-Scholes ➔ Heston
  - constant equity volatility ➔ stochastic equity volatility
  - relative change in risk (range given for different behavioral models):
  - No Ratchet: + 20% to + 27%
  - Lookback: + 43% to + 45%
  - Rem. WBB: + 44% to + 48%

# Outlook / Future Research

- interest rate risk
- include more options for the policyholder (e.g. withdrawing nothing)
- heterogeneous pool of policies
- use LSMC approach for hedging / Greeks calculation
- …

# Thank you for your attention!

Alexander Kling
a.kling@ifa-ulm.de

Frederik Ruez
frederik.ruez@uni-ulm.de

Jochen Ruß
j.russ@ifa-ulm.de