Mortality and Smoking Prevalence

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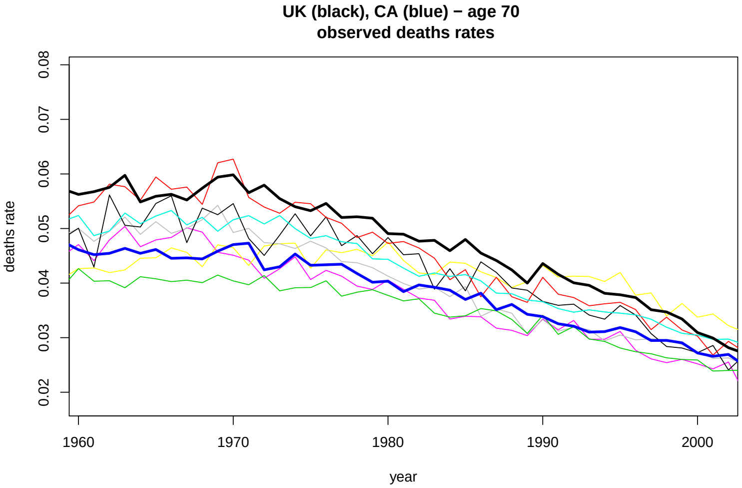
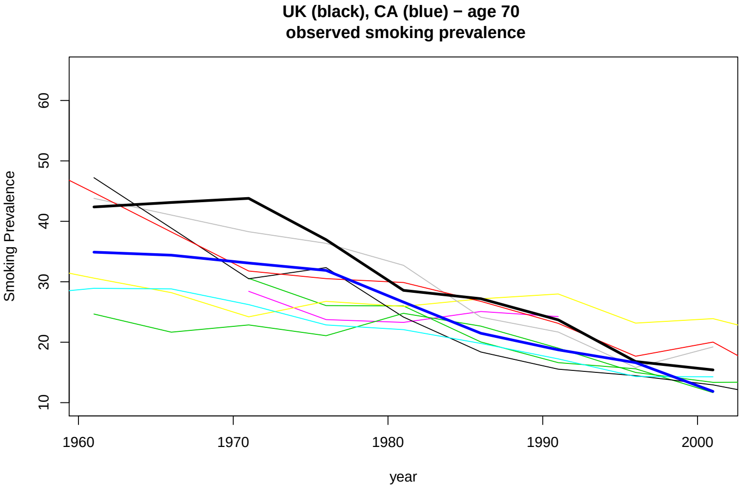
AFIR 2011 - MADRID

Mortality


UK (black), CA (blue) − age 70
observed deaths rates
deaths rate
0.02
0.03
0.04
0.05
0.06
0.07
0.08
1960
1970
1980
1990
2000
year

Smoking



UK (black), CA (blue) − age 70
observed smoking prevalence
Smoking Prevalence
10
20
30
40
50
60
1960
1970
1980
1990
2000
year

Mortality Models for Multiple Populations

Consider k different populations (countries).
For each country i, time t and age x we observe

$D_i(t,x)$: Number of deaths,

$E_i(t,x)$: Exposure-to-risk

$m_i(t,x)$: $= D_i(t,x)/E_i(t,x)$, deaths rate

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Core Hypothesis, Li & Lee (2005), Cairns et al. (2011): For all ages x and all i and j:

$$m_i(t,x)/m_j(t,x) \not\to \infty \text{ for } t \to \infty$$

Covariates

Covariates influencing individual life expectancy and disability-free life expectancy:

- life style (obesity, smoking, alcohol consumption, physical exercise, ...)
- socio-economic variables (income, wealth, Housing tenure, education, ...)
- genetic factors?

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Example: Smoking prevalence

Available data

What we observe:

- deaths rates (www.mortality.org), "1×1-table"
- smoking prevalence (International Smoking Statistics, P N Lee Statistics and Computing Ltd)
  there are different definitions (<u>total cigarettes</u>, manufactured cigarettes, any tobacco products)
  based on surveys

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What we do not observe:

- deaths rates for smokers and non-smokers, separately
- Cessation data
- “1×1-table”, in general, prevalence data are only available for age groups

Smoking and Mortality - British Doctors

R. Doll, R. Peto, J. Boreham and I. Sutherland: "Mortality in relation to smoking: 50 years' observations on male British Doctors"

- 34,439 British doctors,
- data about smoking habits was first obtained in 1951 and then periodically thereafter
- mortality was monitored for 50 years

Main results:

- substantial decrease in the mortality rates of non-smokers
- survival rates from age 35 for smokers are the same for cohorts born between 1900 to 1930, for non-smokers these survival rates have increased substantially

Smoking and Mortality

For each country i, time t and age x we define

$D_i(t,x)$: Number of deaths,
$D_i^N(t,x)$, $D_i^S(t,x)$ for non-smokers, smokers (not observed)
$D_i(t,x) = D_i^N(t,x) + D_i^S(t,x)$

$E_i(t,x)$: Exposure-to-risk

$m_i(t,x)$: $= D_i(t,x)/E_i(t,x)$,
$m_i^N(t,x)$, $m_i^S(t,x)$, deaths rates

$s_i(t,x)$: Smoking prevalence, in $[0,1]$,
the number of smokers is $s_i(t,x)E_i(t,x)$

We do not distinguish between life-long non-smokers and non-smokers who used to smoke.

Smoking and Mortality

$$\begin{aligned} D_i(t,x) &= D_i^N(t,x) + D_i^S(t,x) \\ &= m_i^N(t,x)\left[1 - s_i(t,x)\right]E_i(t,x) + m_i^S(t,x)s_i(t,x)E_i(t,x) \end{aligned}$$

where

$$\begin{aligned} m_i^N(t,x) &= \frac{D_i^N(t,x)}{\left[1 - s_i(t,x)\right]E_i(t,x)} \\ m_i^S(t,x) &= \frac{D_i^S(t,x)}{s_i(t,x)E_i(t,x)} \end{aligned}$$

We obtain

$$m_i(t,x) = \frac{D_i(t,x)}{E_i(t,x)} = m_i^N(t,x) + \left[m_i^S(t,x) - m_i^N(t,x)\right]s_i(t,x)$$

Smoking and Mortality

Modelling assumptions:

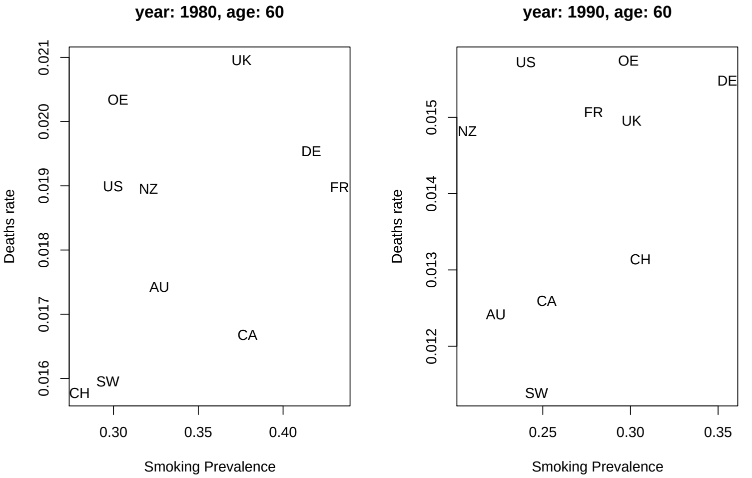
- Smoking has the same effect on mortality rates in all observed countries.
- Non-smokers' mortality in country i is the sum of general non-smokers' mortality and a "country effect"

$$m_i(t,x) \quad = \quad m^N(t,x) + \left[m^S(t,x) - m^N(t,x)\right] s_i(t,x) + C_i(t,x)$$

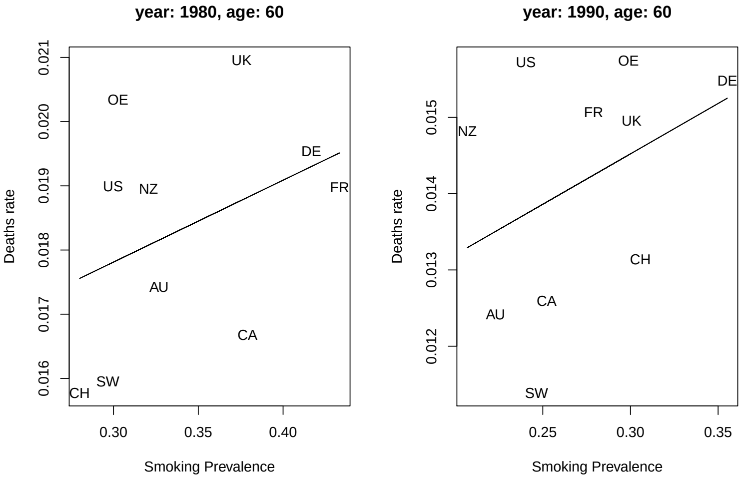
where $C_i(t,x)$ is a country specific effect.

First aim: Estimate $m^N(t,x)$ and $m^S(t,x)$.

Smoking and Mortality



Smoking and Mortality



Simplifying Assumptions

Motivated by the findings for British doctors, we assume that there is:

no improvement in smokers' mortality rates

$$m^S(t,x) = m^S(x)$$

$$m_i(t,x) \quad = \quad m^N(t,x) + \left[m^S(x) - m^N(t,x)\right] s_i(t,x) + C_i(t,x)$$

Constant smoker's mortality over time

Least-Square Estimation for a fixed age x:

$$\text{MSE}_x(m^S, m^N) = \sum_t \sum_i \left(C_i(t,x)\right)^2$$

$$= \sum_t \sum_i \Big(m_i(t,x) - m^N(t,x) - \big[m^S(x) - m^N(t,x)\big]\, s_i(t,x)\Big)^2$$

Note: $m^N = (m^N(1,x), \ldots, m^N(T,x))$
Choose m^S, m^N such that

$$\text{MSE}_x(m^S, m^N) \longrightarrow min$$

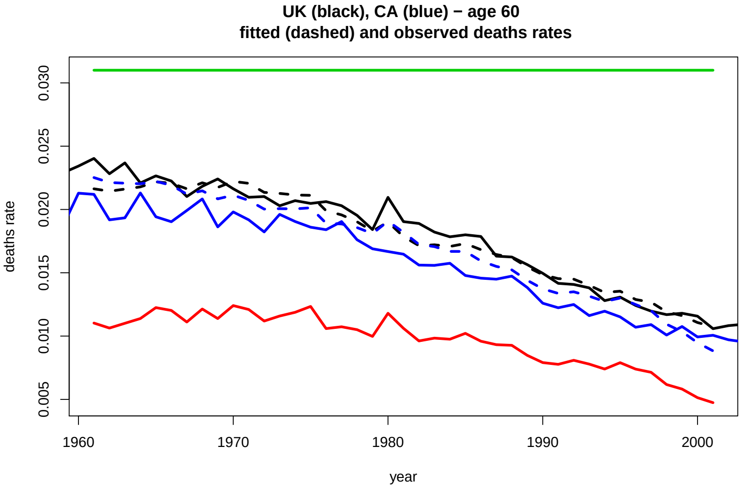
Constant smokers' mortality over time

Explicit solution for fixed age x is the solution of the following linear system of equations:

$$m^S = \frac{1}{\sum_t \sum_i s_i^2(t)} \sum_t \sum_i s_i(t) \underbrace{\left[m_i(t) - m^N(t)(1 - s_i(t)) \right]}_{m^S s_i(t) + C_i(t)}$$

$$m^N(t) = \frac{\sum_i (1 - s_i(t)) m_i(t)}{\sum_i (1 - s_i(t))^2} - m^S \frac{\sum_i (1 - s_i(t)) s_i(t)}{\sum_i (1 - s_i(t))^2}$$

Constant smokers' mortality over time



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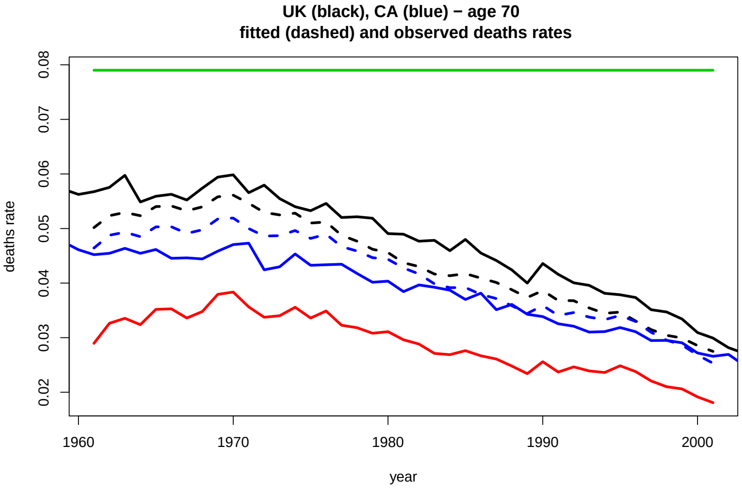
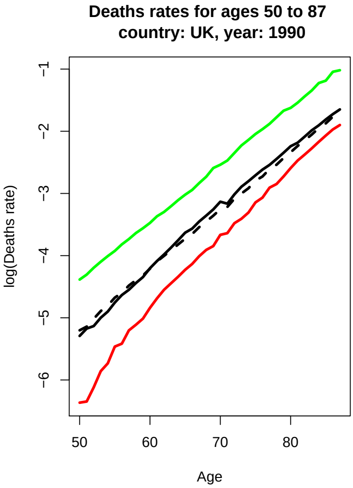
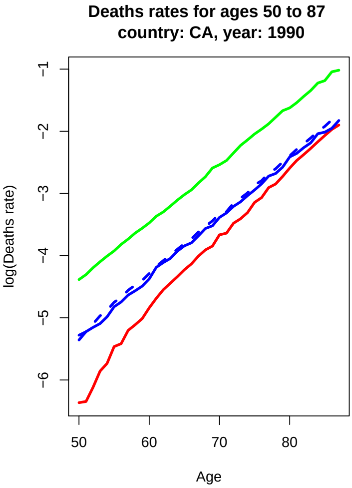
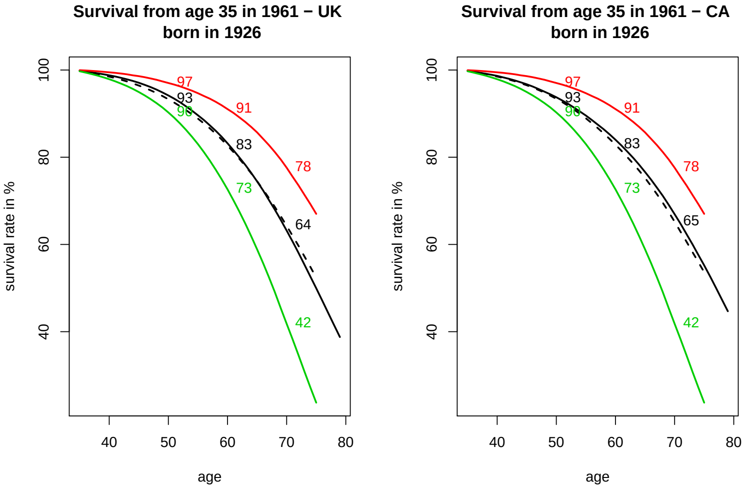


Constant smokers' mortality over time





Comparison with British Doctors



Modelling the Country effect

Model for $m_i(t,x)$:

$$m_i(t,x) \quad = \quad m^N(t,x) + \left[m^S(x) - m^N(t,x)\right] s_i(t,x) + C_i(t,x)$$

"Core Hypothesis"

$$m_i(t,x)/m_j(t,x) \not\to \infty \text{ for } t \to \infty$$

Since $m^S(x)$ is constant over time, the core hypothesis can only be fulfilled if $m^N(t,x) \to K(x) > 0$.

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Since we consider a covariate (smoking) we change the core hypothesis to:
For any $i \neq j$ and any fixed age x holds:

$$s_i(t,x) = s_j(t,x) \, \forall \, t \qquad \Rightarrow \qquad m_i(t,x)/m_j(t,x) \not\to \infty$$

for $t \to \infty$

Scenarios

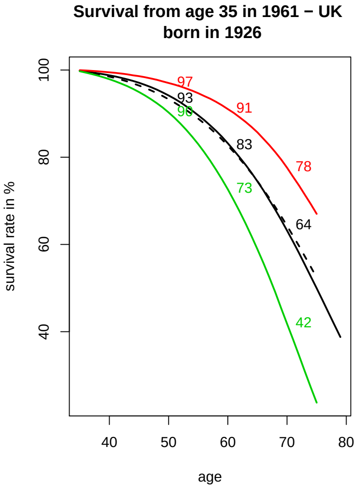
We can now investigate the effect of smoking on survival rates. With the estimates obtained earlier we consider

$$m_i(t,x) \quad = \quad m^N(t,x) + \mathbf{0.75}\left[m^S(x) - m^N(t,x)\right] s_i(t,x)$$

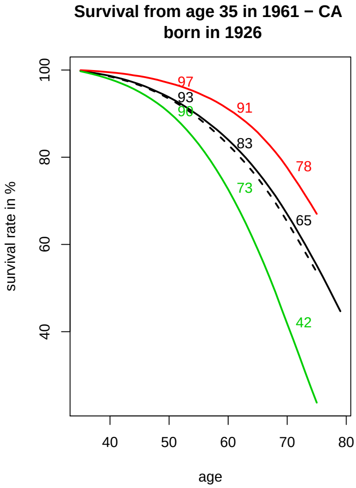
for the cohort aged 35 in 1961.
Rate of survival to age $x > 35$ for the cohort aged 35 in 1961:

$$\mathcal{S}(x, 1961, 35) = \prod_{j=1}^{x-35} \Big(1 - m_i(1961 + j, 35 + j)\Big)$$

Scenarios

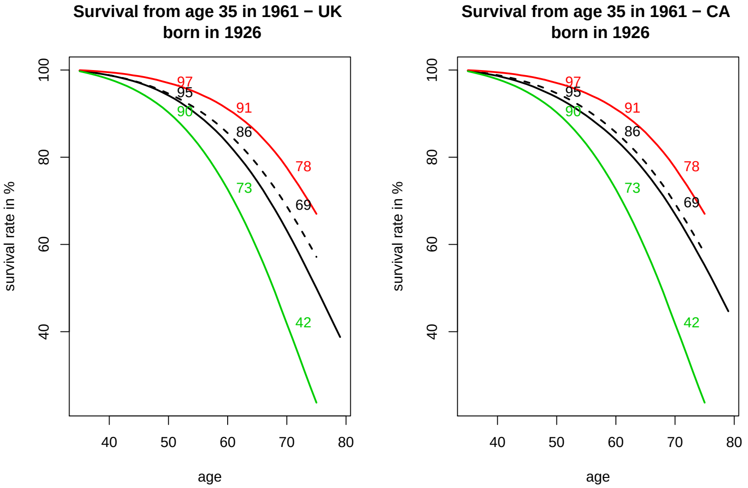


Survival from age 35 in 1961 − UK
born in 1926
survival rate in %
age
100
80
60
40
40
50
60
70
80
97
93
90
91
83
78
73
64
42



Survival from age 35 in 1961 − CA
born in 1926
survival rate in %
age
100
80
60
40
40
50
60
70
80
97
93
90
91
83
78
73
65
42

Scenarios - Smoking Prevalence reduced by 25%



Concluding remarks

- there is empirical evidence that smoking prevalence can be used to model deaths rates for entire countries and explain differences in country-specific mortality rates
- there is still a country-specific effect that has an impact on mortality
- there is only one "trend" component (non-smokers' mortality) in our model
- we require an assumption about the relationship between mortality rates of smokers and non-smokers when no cessation data are available
- the assumption of constant smokers' mortality rates is very strong, and other assumptions should be investigated
- these are all preliminary results

Concluding remarks

"... the easy application of a principle and its apparent adequacy give no very certain proof of its soundness ..."

Immanuel Kant (1785): "Groundwork for the Metaphysics of Morals", translated by Thomas Kingsmill Abbott