ANALYTICAL EVALUATION OF INSURANCE MARKET RISK

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Agenda

Mean and Variance of Prospective Assets

- General Formulas
- The Black-Scholes-Merton return model
- A special case

Market Risk Target Capital

- The cost-of-capital approach
- The coherent SST approach
- Life Insurance

References

Mean and Variance of Prospective Assets (1)

Multi-period discrete time stochastic model of insurance

A_t : *assets* at time t

L_t : *actuarial liabilities* at time t

P_{t-1} : *loaded premium* paid at time t-1 (fully invested)

X_t : *insurance costs* paid at time t (insurance benefits, expenses and bonus payments for period (t-1,t])

R_t : accumulated *rate of return* on investment for period (t-1,t]

Equation of dynamic evolution of the random assets over the time horizon [0,T]:

$$A_t = \left(A_{t-1} + P_{t-1}\right) \cdot R_t - X_t, \qquad t \in \{1, \ldots, T\}$$

In terms of the initial capital A(0) the random prospective assets at time T equals

$$A_T = A_0 \cdot \prod_{t=1}^{T} R_t + \sum_{t=1}^{T} \left\{P_{t-1} \cdot R_t - X_t\right\} \cdot \prod_{j=t+1}^{T} R_j$$

Mean and Variance of Prospective Assets (2)

Model Assumptions (Multivariate normal model of logarithmic returns)

(M1) The random premiums and insurance costs are independent from the returns and their means are denoted by $\mu_{P_t} = E[P_t]$ and $\mu_{X_t} = E[X_t]$

(M2) The random vector of *logarithmic returns* $Z = (Z_1,\ldots,Z_T) = (\ln R_1,\ldots,\ln R_T)$ has a multivariate normal distribution with characteristic function

$$\varphi(x) = E[\exp(ix'Z)] = \exp\left(ix'\mu - \tfrac{1}{2}x'\Sigma x\right),$$

$$x' = (x_1,\ldots,x_T), \quad \mu' = (\mu_1,\ldots,\mu_T), \quad \Sigma = (\rho_{st}\sigma_s\sigma_t; 1 \le s \le t \le T),$$

Notation: $e(T-t), t \in \{1,\ldots,T\},$ column vector containing zeros in the first t entries followed by $T-t$ entries with ones

Theorem 2.1. Under the model assumptions (M1) and (M2), the mean and variance of the random assets of an insurance portfolio are given by

$$E[A_T] = (A_0 + P_0)\cdot\exp\left(e(T)'\mu + \tfrac{1}{2}e(T)'\Sigma e(T)\right)$$

$$+\sum_{t=1}^{T-1}\left(\mu_{P_t} - \mu_{X_t}\right)\cdot\exp\left(e(T-t)'\mu + \tfrac{1}{2}e(T-t)'\Sigma e(T-t)\right) - \mu_{X_T}$$

Mean and Variance of Prospective Assets (3)

$$
\begin{aligned}
Var[A_T] &= (A_0+P_0)^2\cdot\exp\left(2e(T)'\mu+e(T)'\Sigma e(T)\right)\cdot\left(\exp\left(e(T)'\Sigma e(T)\right)-1\right)\\
&+2\left(A_0+P_0\right)\cdot\sum_{t=1}^{T-1}\left\{\begin{array}{l}(\mu_{P_t}-\mu_{X_t})\cdot\\ \exp\left(e(T)'\mu+e(T-t)'\mu+\tfrac{1}{2}e(T)'\Sigma e(T)+\tfrac{1}{2}e(T-t)'\Sigma e(T-t)\right)\cdot\\ \left(\exp\left(e(T-t)'\Sigma e(T-t)\right)-1\right)\end{array}\right\}\\
&+\sum_{t=1}^{T-1}\exp\left(2e(T-t)'\mu+e(T-t)'\Sigma e(T-t)\right)\cdot\left\{\begin{array}{l}(\mu_{P_t}-\mu_{X_t})^2\cdot\left(\exp\left(e(T-t)'\Sigma e(T-t)\right)-1\right)\\ +Var[P_t-X_t]\cdot\exp\left(e(T-t)'\Sigma e(T-t)\right)\end{array}\right\}\\
&+2\cdot\sum_{1\le s<t\le T-1}\left(\begin{array}{l}\exp\left(e(T-s)'\mu+e(T-t)'\mu+\tfrac{1}{2}e(T-s)'\Sigma e(T-s)+\tfrac{1}{2}e(T-t)'\Sigma e(T-t)\right)\cdot\\ \left\{\begin{array}{l}\left(\mu_{P_s}-\mu_{X_s}\right)\left(\mu_{P_t}-\mu_{X_t}\right)\cdot\left(\exp\left(e(T-t)'\Sigma e(T-t)\right)-1\right)\\ +Cov[P_s-X_s,P_t-X_t]\cdot\exp\left(e(T-t)'\Sigma e(T-t)\right)\end{array}\right\}\end{array}\right)\\
&-2\left(A_0+P_0\right)\cdot\sum_{t=1}^{T-1}Cov[P_t-X_t,X_T]\cdot\exp\left(e(T-t)'\mu+\tfrac{1}{2}e(T-t)'\Sigma e(T-t)\right)+Var[X_T]
\end{aligned}
$$

Mean and Variance of Prospective Assets (4)

The Black-Scholes-Merton return model

(M1) The random premiums and insurance costs are independent from the returns

(M2)=(BSM) The random accumulation factors are independent and identically log-normally distributed such that $Z_t = \ln\{R_t\}, t \in \{1,\ldots,T\}$ is normally distributed with mean μ and standard deviation σ. The quantity $r = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$ is the one-period *expected accumulation factor* over the time horizon $(t-1,t]$, $t \in \{1,\ldots,T\}$.

Corollary 2.1. Under the model assumptions (M1) and (BSM), the mean and variance of the random assets of an insurance portfolio are given by

$$E[A_T] = r^T \cdot \left\{ A_0 + P_0 + \sum_{t=1}^{T-1} r^{-t} \cdot \left(\mu_{P_t} - \mu_{X_t}\right) \right\} - \mu_{X_T}$$

$$Var[A_T] = r^{2T} \cdot \begin{Bmatrix} (A_0 + P_0)^2 \cdot \left(e^{T\sigma^2} - 1\right) \\ + 2(A_0 + P_0) \cdot \sum_{t=1}^{T-1} r^{-t} \left(\mu_{P_t} - \mu_{X_t}\right)\left(e^{(T-t)\sigma^2} - 1\right) \\ + \sum_{t=1}^{T-1} r^{-2t} \cdot \left\{ \left(\mu_{P_t} - \mu_{X_t}\right)^2 \left(e^{(T-t)\sigma^2} - 1\right) + Var[P_t - X_t] \cdot e^{(T-t)\sigma^2} \right\} \\ + 2 \cdot \sum_{1 \le s < t \le T-1} r^{-(s+t)} \cdot \begin{Bmatrix} \left(\mu_{P_s} - \mu_{X_s}\right)\left(\mu_{P_t} - \mu_{X_t}\right)\left(e^{(T-t)\sigma^2} - 1\right) \\ + Cov[P_s - X_s, P_t - X_t] \cdot e^{(T-t)\sigma^2} \end{Bmatrix} \end{Bmatrix}$$

$$- 2(A_0 + P_0) \cdot r^T \cdot \sum_{t=1}^{T-1} r^{-t} Cov[P_t - X_t, X_T] + Var[X_T]$$

Mean and Variance of Prospective Assets (5)

Black-Scholes-Merton return model: a special case

Model assumptions:

(A1) Insurance cash-flows are valued with constant discount rate $v_0 = r_0^{-1}$

The quantities $CF_{t-1} = v_0 \cdot X_t - P_{t-1}, \quad t = 1,\ldots,T,$ represent the future (net) *insurance cash-flows* at time t -1 over the period *(t-1,t]*.

(A2) Insurance cash-flows and premiums are deterministic quantities (i.e. focus on market risk)

(A2) => insurance costs are deterministic, i.e. $X_t = E[X_t]$

Negative of insurance cash-flows = premium loadings:

$$\Theta_{t-1} = -CF_{t-1} = P_{t-1} - v_0 \cdot E[X_t], \quad t = 1,\ldots,T,$$

Vector notations:

$$P = (P_0,\ldots,P_{T-1}), \Theta = (\Theta_0,\ldots,\Theta_{T-1}), \mu_P = (0, P_1,\ldots,P_{T-1}), \mu_X = r_0 \cdot (0, P_0 - \Theta_0,\ldots,P_{T-2} - \Theta_{T-2})$$

Present value functions:

$$PV_T(a,r) = \sum_{t=0}^{T-1} r^{-t} \cdot a_t \qquad PV_T(a, r_a; b, r_b) = \sum_{0 \le s < t \le T-1} r_a^{-s} r_b^{-t} a_s b_t$$

$$a = (a_0,\ldots,a_{T-1}) \qquad a = (a_0,\ldots,a_{T-1}) \quad b = (b_0,\ldots,b_{T-1})$$

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Mean and Variance of Prospective Assets (6)

Mean of discounted assets:

$$E\left[r^{-T}A_T\right]=A_0+PV_T\left(P-\tfrac{r_0}{r}(P-\Theta),r\right),$$

Fair value assumption: $r_0=r$

$$E\left[r^{-T}A_T\right]=A_0+PV_T(\Theta,r)$$

(initial capital + PV of all premium loadings)

Variance of discounted assets:

$$Var\left[r^{-T}A_T\right]=(A_0+P_0)^2\cdot\left(e^{T\sigma^2}-1\right)+S_1+S_2+S_3,$$

$$S_1=2(A_0+P_0)\cdot\begin{pmatrix} e^{(T-1)\sigma^2}\cdot PV_T\left(e^{\sigma^2}P-\tfrac{r_0}{r}(P-\Theta),re^{\sigma^2}\right) \\ -PV_T\left(P-\tfrac{r_0}{r}(P-\Theta),r\right)-P_0\cdot\left(e^{T\sigma^2}-1\right)\end{pmatrix}$$

$$S_2=e^{T\sigma^2}\cdot PV_T\left((\mu_P-\mu_X)^2,r^2e^{\sigma^2}\right)-PV_T\left((\mu_P-\mu_X)^2,r^2\right)$$

$$S_3=2\cdot\left(e^{T\sigma^2}\cdot PV_T\left(\mu_P-\mu_X,r;\mu_P-\mu_X,re^{\sigma^2}\right)-PV_T\left(\mu_P-\mu_X,r;\mu_P-\mu_X,r\right)\right)$$

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Market Risk Target Capital (1)

The Cost-of-Capital approach

$C_t = A_t - L_t$: *risk-bearing capital* (RBC) at time t

$SC_t = -C_t$: *shortfall risk-bearing capital* at time t (negative of RBC)

- Discounted shortfall RBC and its time period change

$$SC_t^d = v^t \cdot SC_t = v^t \cdot (L_t - A_t), \quad t = 1,2,\ldots,T$$

$$\Delta SC_t^d = SC_t^d - SC_{t-1}^d = v^t \cdot (L_t - rL_{t-1}) - v^t \cdot (A_t - rA_{t-1}), \quad t = 1,2,\ldots,T$$

- Economic Capital (EC) to risk measure R(·), e.g. VaR / CVaR

$$EC := R\left[\Delta SC_1^d\right]$$

- Risk Margin (RM) / Market value Margin (MvM)

$$RM := i_{CoC} \cdot \sum_{t=2}^{T} R\left[\Delta SC_t^d\right]$$

- Target Capital (TC) : TC = EC + RM

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Market Risk Target Capital (2)

VaR Target Capital

$$TC^{CoC/VaR} = C_0 + VaR_\alpha[v \cdot (L_1 - A_1)] + i_{CoC} \cdot \sum_{t=2}^{T} VaR_\alpha[v^t \cdot (L_t - rL_{t-1}) - v^t \cdot (A_t - rA_{t-1})]$$

CVaR Target Capital = SST Target Capital (FOPI(2004/2006))

$$TC^{CoC/CVaR} = C_0 + CVaR_\alpha[v \cdot (L_1 - A_1)] + i_{CoC} \cdot \sum_{t=2}^{T} CVaR_\alpha[v^t \cdot (L_t - rL_{t-1}) - v^t \cdot (A_t - rA_{t-1})]$$

Goal: Analytical evaluation for deterministic liabilities

Justification for simplifying assumption: *best- and worst-case insurance scenarios* s.th.

$$L_t^b \le L_t \le L_t^w, \quad t = 1,2,\ldots,T \quad \Rightarrow \quad L_t^b - rL_{t-1}^w \le L_t - rL_{t-1} \le L_t^w - rL_{t-1}^b, \quad t = 2,3,\ldots,T$$

$$\Rightarrow \quad TC^b \le TC \le TC^w, \quad t = 1,2,\ldots,T.$$

Market Risk Target Capital (3)

Economic Capital (= Solvency II 1st year *SCR* for the market risk)

$$EC = C_0 - E[vC_1] + R[E[vA_1] - vA_1]$$ with $$L_1^A = E[vA_1] - vA_1$$ (1st year asset loss)

1st year Solvency II SCR (VaR measure, lognormal approx.)

$$VaR_\alpha[L_1^A] = \rho_\alpha^{VaR}(\sigma A_1)\cdot(A_0 + \Theta_0), \quad \rho_\alpha^{VaR}(x) = 1 - \frac{\exp\left\{\Phi^{-1}(1-\alpha)\cdot\sqrt{\ln(1+x^2)}\right\}}{\sqrt{1+x^2}},$$

$$\sigma A_1 = \left(\frac{A_0 + P_0}{A_0 + \Theta_0}\right)\cdot\sqrt{e^{\sigma^2} - 1}$$

(up to sign change & exchange of confidence level/loss probability = SCR Non-Life)

SST market risk EC (CVaR measure, lognormal approx.)

$$CVaR_\alpha[L_1^A] = \rho_\alpha^{CVaR}(\sigma A_1)\cdot(A_0 + \Theta_0), \quad \rho_\alpha^{CVaR}(x) = 1 - \frac{\Phi\left(\Phi^{-1}(1-\alpha) - \sqrt{\ln(1+x^2)}\right)}{1-\alpha},$$

(up to sign change & exchange of confidence level/loss probability = SST Non-Life)

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Market Risk Target Capital (4)

Comparison of solvency capital ratios $\rho_\alpha^\bullet(\sigma A_1)/\sigma A_1$

	VaR Method			CVaR Method		
confidence level	0.99	**0.995**	0.99612	**0.98720**	0.99	0.995
percentile	-2.326	**-2.576**	-2.662	**-2.232**	-2.326	-2.576
return volatility σ						
5.0%	2.216	**2.436**	2.512	**2.438**	2.512	2.709
5.5%	2.205	**2.422**	2.497	**2.424**	2.497	2.691
6.0%	2.194	**2.409**	2.482	**2.410**	2.482	2.674
6.5%	2.183	**2.395**	2.468	**2.396**	2.467	2.656
7.0%	2.172	**2.381**	2.453	**2.382**	2.453	2.639
7.5%	2.162	**2.368**	2.438	**2.368**	2.438	2.621
8.0%	2.151	**2.354**	2.424	**2.355**	2.423	2.604
8.5%	2.140	**2.341**	2.410	**2.341**	2.409	2.587
9.0%	2.129	**2.328**	2.395	**2.328**	2.394	2.570
9.5%	2.118	**2.314**	2.381	**2.314**	2.380	2.553
10.0%	2.108	**2.301**	2.367	**2.301**	2.365	2.536

Market Risk Target Capital (5)

Risk Margin

$$R\left[\Delta SC_t^d\right] = E\left[v^t(rC_{t-1} - C_t)\right] + R\left[L_t^A\right], \quad t = 2,\ldots,T$$

$$L_t^A = E\left[v^t(A_t - rA_{t-1})\right] - v^t(A_t - rA_{t-1}), \quad t = 2,\ldots,T$$

(increase of the *t*-th year discounted change in assets with respect to the mean)

Solvency II Risk Margin (VaR measure, lognormal approx.)

$$VaR_\alpha\left[L_t^A\right] = \rho_\alpha^{VaR}(\sigma A_t)\cdot E\left[v^t(A_t - rA_{t-1})\right], \qquad \sigma A_t = \frac{\sqrt{Var[A_t] - r^2 Var[A_{t-1}]}}{E[A_t - rA_{t-1}]}, \quad t = 2,\ldots,T$$

SST Risk Margin (CVaR measure, lognormal approx.)

$$CVaR_\alpha\left[L_t^A\right] = \rho_\alpha^{CVaR}(\sigma A_t)\cdot E\left[v^t(A_t - rA_{t-1})\right],$$

Remark. $A_t,\ A_t - rA_{t-1}$ only approximately lognormal

Can consider improved comonotone approx. and log-elliptical extensions

Market Risk Target Capital (6)

SST target capital via SST risk measure

$$TC^{SST} = C_0 + R_\alpha^{SST}\left[SC^d\right]$$

with the *SST risk measure* $R_\alpha^{SST}\left[SC^d\right] := CVaR_\alpha\left[SC_1^d\right] + i_{CoC} \cdot \sum_{t=2}^{T} CVaR_\alpha\left[SC_t^d - SC_{t-1}^d\right]$

not a coherent multi-period risk measure (**Filipovic and Vogelpoth(2008)**):

$X \geq Y$ with probability one does not imply $R_\alpha^{SST}[X] \geq R_\alpha^{SST}[Y]$

Coherent SST target capital via coherent SST risk measure

$TC^{SST,c} = C_0 + R_\alpha^{SST,c}\left[SC^d\right]$ with the *coherent SST risk measure*

$R_\alpha^{SST,c}\left[SC^d\right] := (1 - i_{CoC}) \cdot CVaR_\alpha\left[SC_1^d\right] + i_{CoC} \cdot CVaR_\alpha\left[SC_T^d\right]$ with the lognormal approx.:

$$TC^{SST,c} = C_0 - E\left[vC_1\right] + i_{CoC} \cdot E\left[vC_1 - v^T C_T\right]$$

$$+ (1 - i_{CoC}) \cdot \rho_\alpha^{CVaR}(\sigma A_1) \cdot (A_0 + \Theta_0) + i_{CoC} \cdot \rho_\alpha^{CVaR}(\sigma A_T) \cdot (A_0 + PV_T(\Theta, r)),$$

$$\sigma A_1 = \left(\frac{A_0 + P_0}{A_0 + \Theta_0}\right) \cdot \sqrt{e^{\sigma^2} - 1}, \quad \sigma A_T = \frac{\sqrt{(A_0 + P_0)^2 \cdot \left(e^{T\sigma^2} - 1\right) + S_1 + S_2 + S_3}}{A_0 + PV_T(\Theta, r)}.$$

coeff.'s of variation

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Market Risk Target Capital (7)

Example: portfolio of identical life insurance policies

$\pi_0 = \pi$: pure level premium

$\theta_t = \theta, t = 0,\ldots,T-1$: constant premium loading factor

$P_{t-1} = {}_{t-1}p_x \cdot (1+\theta)\pi, t = 1,\ldots,T$: loaded premiums

${}_t p_x$: survival probabilities

One has $\Theta_t = {}_t p_x \cdot \theta\pi$, $\mu_{P_t} - \mu_{X_t} = ((1+\theta)_t p_x - r \cdot {}_{t-1} p_x) \cdot \pi, t = 1,\ldots,T-1$

$$TC^{SST,c} = C_0 - E[vC_1] + i_{CoC} \cdot E[vC_1 - v^T C_T] + R_\alpha^{SST,c}(\sigma, T, A_0, \pi, \theta, {}_t p_x, r, i_{CoC}),$$

$$R_\alpha^{SST,c}(\sigma, T, A_0, \pi, \theta, {}_t p_x, r, i_{CoC})$$

$$= (1 - i_{CoC}) \cdot \rho_\alpha^{CVaR}(\sigma A_1) \cdot (A_0 + \Theta_0) + i_{CoC} \cdot \rho_\alpha^{CVaR}(\sigma A_T) \cdot (A_0 + PV_T(\Theta, r))$$

Comparison of coherent SST risk measure ratio $R_\alpha^{SST,c} / A_0$ with approximate ratio obtained from approximation

$$\sigma A_T^* = \left(\frac{A_0 + P_0}{A_0 + PV_T(\Theta, r)} \right) \cdot \sqrt{e^{T\sigma^2} - 1}$$

Market Risk Target Capital (8)

Example 1: $\alpha = 0.99, \sigma = 7.5\%, \pi = 100, \theta = 10\%, A_0 = 1000, r = 1.025, i_{CoC} = 6\%$

α	0.99	coherent SST risk measure ratio by varying time horizon														
T	${}_{T-1}p_x$	$a=\mu_P-\mu_x$	PV(Θ,v)	PV(a,v)	$PV(a,v_\sigma)$	S1	S2	S3	σA_1	σA_T	σA^*_T	$\rho_\alpha(\sigma A_1)$	$\rho_\alpha(\sigma A_T)$	$\rho_\alpha(\sigma A^*_T)$	R_c/A_0	R^*_c/A_0
1	1.00000	7.402	10.0	0.0	0.0	0	0.0	0.0	8.3%	8.3%	8.3%	20.0%	20.0%	20.0%	20.0%	20.0%
2	0.99911	7.387	19.7	7.2	7.2	90	0.3	0.0	8.3%	11.6%	11.6%	20.0%	27.0%	26.9%	20.4%	20.4%
3	0.99815	7.371	29.2	14.3	14.1	269	0.9	0.6	8.3%	14.2%	14.1%	20.0%	31.9%	31.8%	20.7%	20.7%
4	0.99710	7.353	38.5	21.1	20.9	535	1.7	1.7	8.3%	16.3%	16.1%	20.0%	35.8%	35.5%	21.0%	21.0%
5	0.99596	7.334	47.5	27.8	27.4	886	2.8	3.3	8.3%	18.1%	17.9%	20.0%	39.0%	38.6%	21.2%	21.2%
6	0.99473	7.313	56.3	34.2	33.7	1320	4.1	5.5	8.3%	19.8%	19.5%	20.0%	41.7%	41.2%	21.4%	21.4%
7	0.99338	7.290	64.9	40.5	39.8	1835	5.7	8.1	8.3%	21.3%	20.9%	20.0%	44.1%	43.5%	21.6%	21.5%
8	0.99193	7.266	73.2	46.7	45.7	2430	7.5	11.1	8.3%	22.7%	22.2%	20.0%	46.2%	45.5%	21.7%	21.7%
9	0.99035	7.239	81.4	52.6	51.4	3103	9.5	14.6	8.3%	24.0%	23.4%	20.0%	48.1%	47.2%	21.9%	21.8%
10	0.98863	7.210	89.3	58.4	56.9	3852	11.7	18.5	8.3%	25.2%	24.5%	20.0%	49.8%	48.8%	22.0%	22.0%
11	0.98677	7.179	97.0	64.1	62.2	4676	14.1	22.8	8.3%	26.3%	25.6%	20.0%	51.3%	50.3%	22.1%	22.1%
12	0.98475	7.145	104.5	69.5	67.4	5573	16.7	27.5	8.3%	27.4%	26.6%	20.0%	52.8%	51.6%	22.3%	22.2%
13	0.98256	7.108	111.8	74.9	72.3	6542	19.5	32.5	8.3%	28.5%	27.5%	20.0%	54.1%	52.9%	22.4%	22.3%
14	0.98019	7.068	118.9	80.0	77.1	7581	22.4	37.8	8.3%	29.5%	28.4%	20.0%	55.4%	54.0%	22.5%	22.4%
15	0.97761	7.025	125.8	85.0	81.7	8688	25.4	43.5	8.3%	30.4%	29.3%	20.0%	56.5%	55.1%	22.6%	22.5%
16	0.97482	6.978	132.6	89.9	86.2	9863	28.7	49.5	8.3%	31.3%	30.1%	20.0%	57.6%	56.1%	22.7%	22.6%
17	0.97179	6.928	139.1	94.6	90.5	11103	32.0	55.8	8.3%	32.2%	30.9%	20.0%	58.6%	57.1%	22.8%	22.7%
18	0.96851	6.873	145.5	99.1	94.6	12406	35.5	62.3	8.3%	33.1%	31.6%	20.0%	59.6%	58.0%	22.9%	22.8%
19	0.96496	6.814	151.7	103.5	98.6	13773	39.2	69.1	8.3%	33.9%	32.4%	20.0%	60.5%	58.8%	23.0%	22.8%
20	0.96111	6.750	157.7	107.8	102.4	15200	42.9	76.2	8.3%	34.8%	33.1%	20.0%	61.4%	59.6%	23.0%	22.9%

Market Risk Target Capital (9)

Example 2: $\alpha = 0.99,\ \sigma = 5\%,\ \pi = 100,\ \theta = 10\%,\ A_0 = 1000,\ r = 1.025,\ i_{CoC} = 6\%$

α	0.99	coherent SST risk measure ratio by varying time horizon														
T	${}_{T-1}p_x$	$a=\mu_P-\mu_X$	PV(Θ,v)	PV(a,v)	PV(a,v_σ)	S1	S2	S3	σA_1	σA_T	σA^*_T	$\rho_\alpha(\sigma A_1)$	$\rho_\alpha(\sigma A_T)$	$\rho_\alpha(\sigma A^*_T)$	R_c/A_0	R^*_c/A_0
1	1.00000	7.402	10.0	0.0	0.0	0	0.0	0.0	5.5%	5.5%	5.5%	13.7%	13.7%	13.7%	**13.7%**	**13.7%**
2	0.99911	7.387	19.7	7.2	7.2	40	0.1	0.0	5.5%	7.7%	7.7%	13.7%	18.8%	18.8%	**14.1%**	**14.1%**
3	0.99815	7.371	29.2	14.3	14.2	119	0.4	0.3	5.5%	9.4%	9.4%	13.7%	22.5%	22.3%	**14.3%**	**14.3%**
4	0.99710	7.353	38.5	21.1	21.0	237	0.8	0.7	5.5%	10.8%	10.7%	13.7%	25.4%	25.2%	**14.5%**	**14.5%**
5	0.99596	7.334	47.5	27.8	27.6	392	1.2	1.5	5.5%	12.0%	11.9%	13.7%	27.8%	27.5%	**14.7%**	**14.6%**
6	0.99473	7.313	56.3	34.2	34.0	583	1.8	2.4	5.5%	13.1%	12.9%	13.7%	29.9%	29.6%	**14.8%**	**14.8%**
7	0.99338	7.290	64.9	40.5	40.2	810	2.5	3.6	5.5%	14.1%	13.8%	13.7%	31.8%	31.3%	**14.9%**	**14.9%**
8	0.99193	7.266	73.2	46.7	46.2	1071	3.3	4.9	5.5%	15.0%	14.7%	13.7%	33.5%	32.9%	**15.1%**	**15.0%**
9	0.99035	7.239	81.4	52.6	52.1	1366	4.2	6.4	5.5%	15.9%	15.5%	13.7%	35.0%	34.4%	**15.2%**	**15.1%**
10	0.98863	7.210	89.3	58.4	57.7	1695	5.2	8.2	5.5%	16.7%	16.2%	13.7%	36.4%	35.7%	**15.3%**	**15.2%**
11	0.98677	7.179	97.0	64.1	63.2	2055	6.2	10.0	5.5%	17.4%	16.9%	13.7%	37.7%	36.9%	**15.4%**	**15.3%**
12	0.98475	7.145	104.5	69.5	68.6	2447	7.3	12.1	5.5%	18.1%	17.5%	13.7%	38.9%	38.0%	**15.5%**	**15.4%**
13	0.98256	7.108	111.8	74.9	73.7	2869	8.5	14.3	5.5%	18.8%	18.1%	13.7%	40.1%	39.0%	**15.6%**	**15.5%**
14	0.98019	7.068	118.9	80.0	78.7	3320	9.8	16.6	5.5%	19.4%	18.7%	13.7%	41.1%	40.0%	**15.7%**	**15.6%**
15	0.97761	7.025	125.8	85.0	83.5	3801	11.1	19.1	5.5%	20.0%	19.3%	13.7%	42.1%	40.9%	**15.8%**	**15.7%**
16	0.97482	6.978	132.6	89.9	88.2	4310	12.5	21.6	5.5%	20.6%	19.8%	13.7%	43.1%	41.7%	**15.8%**	**15.8%**
17	0.97179	6.928	139.1	94.6	92.7	4846	14.0	24.4	5.5%	21.2%	20.3%	13.7%	44.0%	42.5%	**15.9%**	**15.8%**
18	0.96851	6.873	145.5	99.1	97.1	5409	15.5	27.2	5.5%	21.8%	20.8%	13.7%	44.8%	43.3%	**16.0%**	**15.9%**
19	0.96496	6.814	151.7	103.5	101.3	5998	17.0	30.1	5.5%	22.3%	21.3%	13.7%	45.6%	44.0%	**16.1%**	**16.0%**
20	0.96111	6.750	157.7	107.8	105.4	6612	18.7	33.2	5.5%	22.8%	21.7%	13.7%	46.4%	44.7%	**16.1%**	**16.0%**