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Erlangen-Nürnberg

The Impact of Natural Hedging on a Life Insurer's Risk Situation

AFIR 2011 Colloquium
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Introduction
Motivation

- Demographic risk can significantly impact a life insurer’s solvency level
  - ➢ Increase in life expectancy poses serious problems to life insurers selling annuities
  - ➢ However, risk of unexpected high mortality (e.g. due to pandemics) has increased as well; problem for term life
- But: Hedging instruments are still scarce
  - ➢ “Natural Hedge” between term life insurance (death benefit) and annuities (lifelong survival benefits) is effective alternative
  - ➢ Use opposed reaction of term life insurance and annuities towards shocks to mortality
  - ➢ Hedge shocks to mortality internally through portfolio composition

Introduction
Aim of paper

- Previous literature:
  - Cox/Lin (2007), Bayraktar/Young (2007), Gründl/Post/Schulze (2006), Wang et al. (2010), Wetzel/Zwiesler (2008)

- Aim of this paper:

1. Quantify impact of natural hedging on a life insurance company's insolvency risk
   - Holistic model, take into account dynamic interaction between assets and liabilities for a two-product life insurer
2. Simultaneously *immunize* an insurer's solvency situation against changes in mortality and *fix the absolute level of risk*
   - Use investment strategy

Model framework
Modeling and forecasting mortality

- Extension of the Lee-Carter (1992) model by Brouhns/Denuit/Vermunt (2002):

$$D_{x,t} \sim Poisson\left(E_{x,t} \cdot \mu_x(t)\right) \;\; \mu_x(t) = \exp\left(a_x + b_x \cdot k_t\right) \;\; q_x(t) = 1 - \exp\left(-\mu_x(t)\right)$$

  - $D_{x,t}$ Poisson-distributed number of deaths, $E_{x,t}$ exposure at risk
  - a_x and b_x indicating the general shape of mortality over age
  - k_t indicating the general level of mortality in the population (with negative drift)
  - Forecasting of k_t (and $\mu_x(t)$) by ARIMA process for estimated time series of k_t

Model framework
Modeling systematic mortality risk

- Analyze systematic mortality risk in two ways:

1. Shock to (decreasing) mortality time trend: e^*k_t
   - ➢ Leads to an unexpected change in the level and future development of mortality
   - ➢ Shocks e > 1: mortality rates decrease (longevity scenario)
   - ➢ Shocks e < 1: mortality rates increase (pandemic scenario)
   - ➢ How to compose a portfolio of term life and annuities in order to immunize the portfolio against shocks to mortality?

2. Use empirically observed changes in mortality
   - ➢ Analyze usefulness of natural hedging under realized changes in mortality
   - ➢ Similar results

Model framework

Model of a life insurance company

- Simplified balance sheet:

Assets	Liabilities
$A(t)$	$E(t)$
	$B_A(t)$ } $L(t)$
	$B_L(t)$ } $L(t)$

  - $A(t)$: market value of assets at time t
  - $B_A(t)$: book value of liabilities for annuities at time t
  - $B_L(t)$: book value of liabilities for term life insurance at time t
  - $E(t)$: equity at time t

➢ Default of the insurance company, if $L(t) = B_L(t) + B_A(t) > A(t)$

Model framework

Liabilities – Premium and benefit calculation

- Premiums and benefits: use actuarial equivalence principle
  - Term life insurance

$$\sum_{t=0}^{T-1} P \cdot {}_t p_x \cdot (1+r)^{-t} = \sum_{t=0}^{T-1} DB \cdot {}_t p_x \cdot q_{x+t} \cdot (1+r)^{-(t+1)}$$

  - Life-long immediate annuity

$$SP = \sum_{t=0}^{T-1} a \cdot {}_t p_x \cdot (1+r)^{-(t+1)}$$

- Improve comparability and isolate effect of natural hedging:
  - Calibrate input parameters such that volume of both contract types is identical at inception
  - Fix the number of contracts sold

Model framework
Liabilities – Book value of liabilities

- Use actuarial reserve to determine book value of liabilities
- Value of one term life insurance contract:

$$B_L(t) = \sum_{s=0}^{T-t-1} \left[DB \cdot {}_s\, p_{x+t}(e) \cdot q_{s+x+t}(e) \cdot (1+i)^{-(s+1)} - P \cdot {}_s\, p_{x+t}(e) \cdot (1+i)^{-s} \right]$$

- Value of one annuity:

$$B_A(t) = \sum_{s=0}^{T-t-1} a \cdot {}_s\, p_{x+t}(e) \cdot (1+i)^{-(s+1)}$$

➢ Mortality rates are subject to shock e

➢ Value of liabilities *L*(*t*):

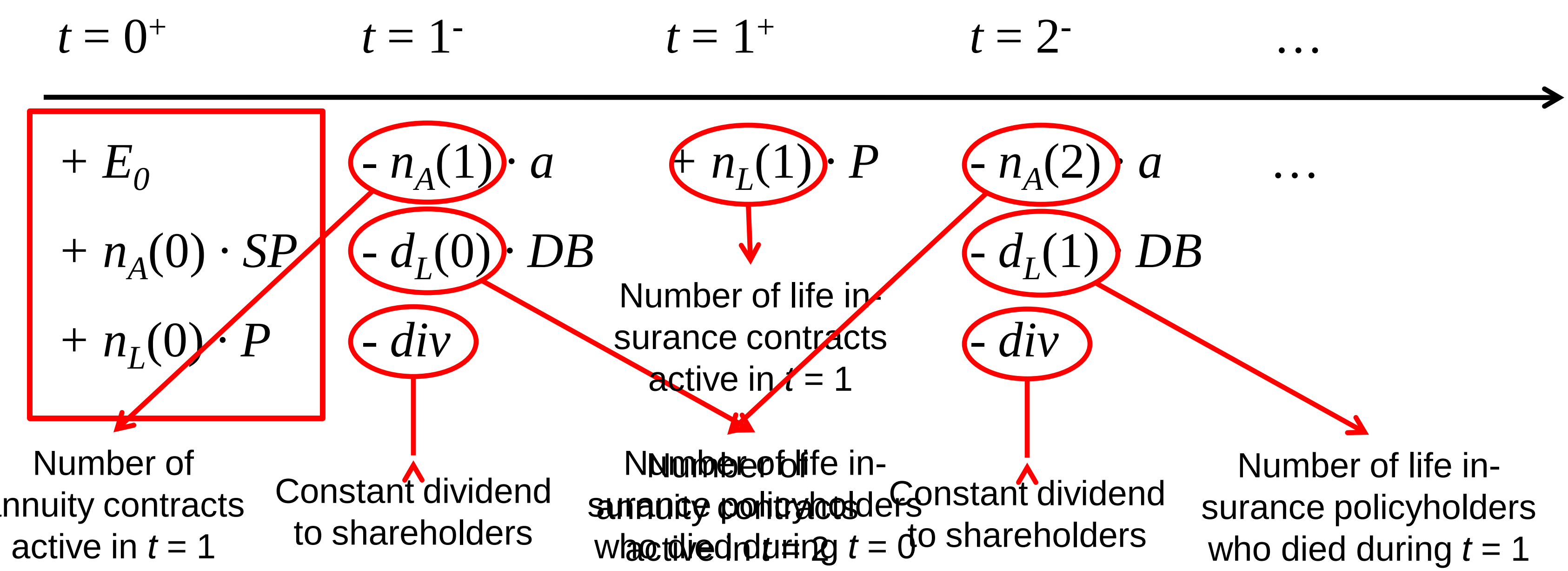
$$L(t) = n_A(t) \cdot B_A(t) + n_L(t) \cdot B_L(t)$$

Model framework
Assets

- Assets follow a geometric Brownian motion:

$$dA(t) = \mu \cdot A(t) \cdot dt + \sigma \cdot A(t) \cdot dW^P(t)$$

- Development of asset base depends on cash-flows of insurance portfolio



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Model framework
Risk measurement

- Probability of default (PD): $PD = P\left(T_d \leq T\right)$

  with $T_d = \left(T+1\right) \vee \inf\left\{t : A(t) < L(t)\right\}, t = 1,\ldots,T.$

- Mean Loss (ML): $ML = E\left(\max\left(\left(L(T_d) - A(T_d)\right)\cdot(1+r)^{-T_d}, 0\right)\cdot 1\left\{T_d \leq T\right\}\right)$

- Expected Shortfall (ES) $ES = \frac{ML}{PD}$

- Contractual Payment Obligations (CP)

$$CP = n_L(0)\cdot\sum_{t=0}^{T-1} DB\cdot {}_t p_x(e)\cdot q_{x+t}(e)\cdot(1+r)^{-(t+1)} + n_A(0)\cdot\sum_{t=0}^{T-1} a\cdot {}_t p_x(e)\cdot(1+r)^{-(t+1)}$$

  - Only liability side
  - Linear in portfolio composition

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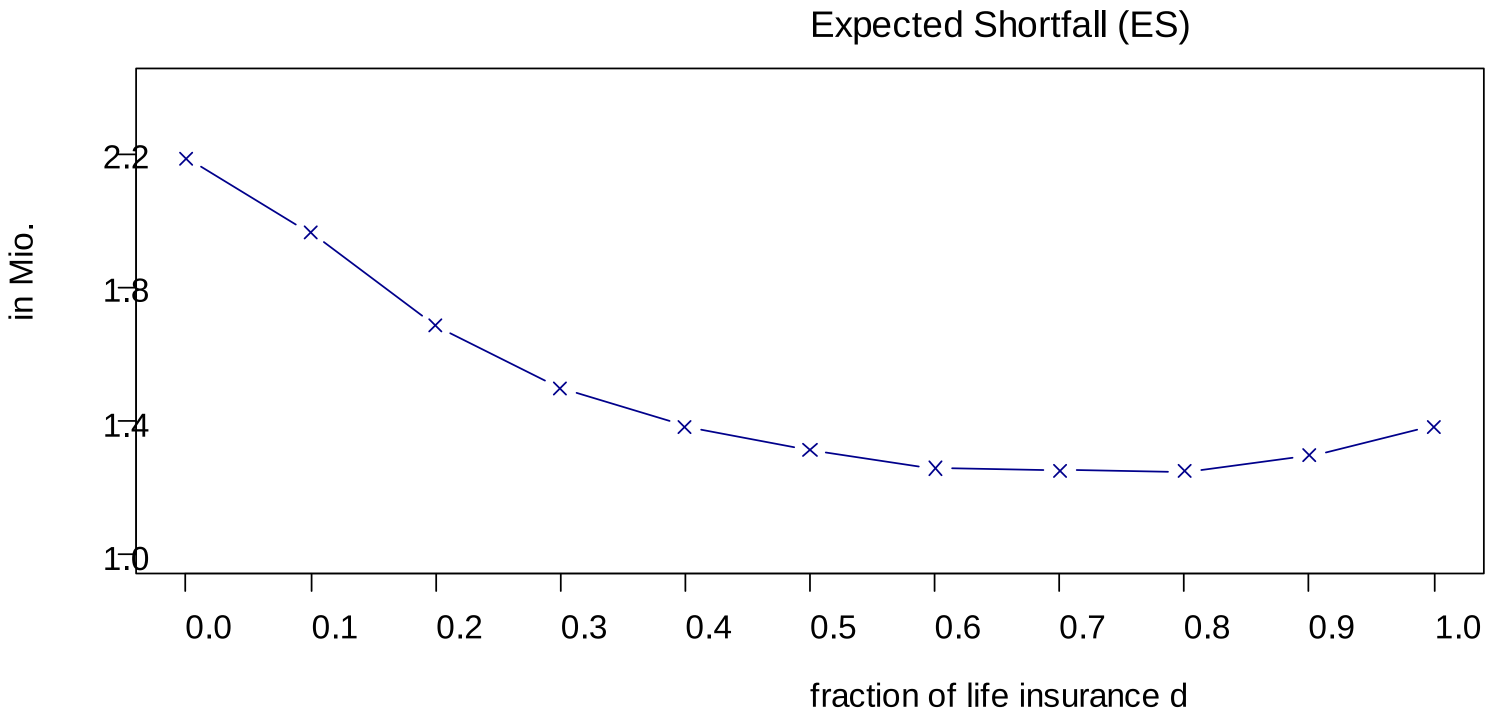
Numerical results
Input parameters

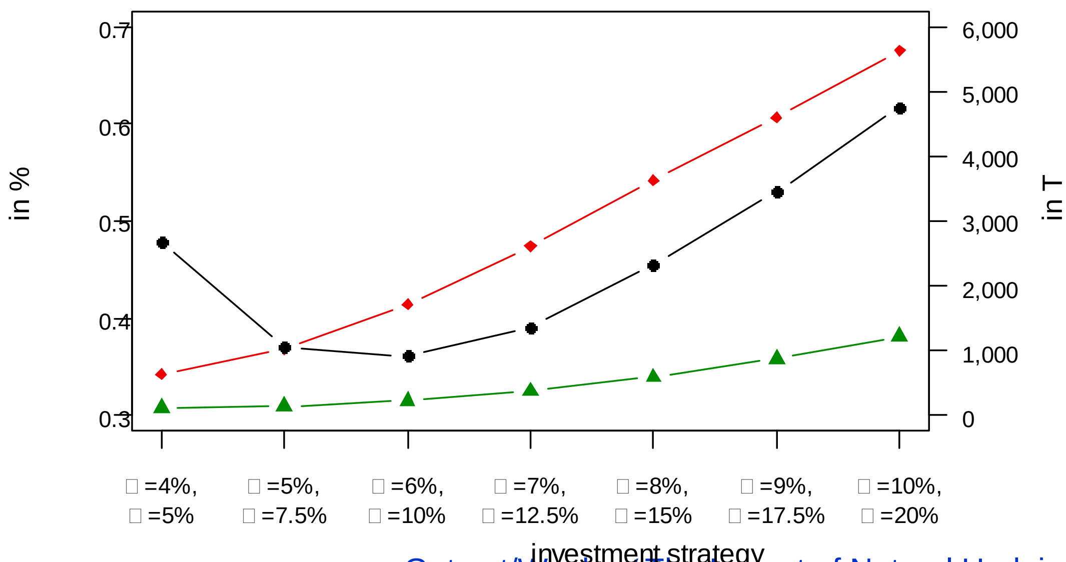
- Liabilities

Age at inception of term life	30
Max. duration of term life	35
Age at inception of annuity	65
Premium for life insurance (*P*)	417
Single premium for annuity (*SP*)	10,000
Yearly annuity (*a*)	725
Death benefit (*DB*)	88,724
Total number of contracts sold	10,000

- Assets

Drift of assets (μ)	6%
Volatility of assets (σ)	10%
Risk-free interest rate (r)	3%


Expected Shortfall (ES)
in Mio.
2.2
1.8
1.4
1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
fraction of life insurance d


in %
0.7
0.6
0.5
0.4
0.3
in T
6,000
5,000
4,000
3,000
2,000
1,000
0
□ =4%, □ =5%
□ =5%, □ =7.5%
□ =6%, □ =10%
□ =7%, □ =12.5%
□ =8%, □ =15%
□ =9%, □ =17.5%
□ =10%, □ =20%
investment strategy

Summary

- Results show: Natural hedging can considerably reduce absolute risk level of an insurer and immunize it against shocks to mortality
  - Optimal portfolio composition depends on risk measure
  - Holistic consideration of mortality risk with respect to insurer’s overall risk level is vital (focus on liability side only underestimates risk)
- Investment strategy can have substantial impact on the effectiveness of natural hedging
  - Use investment strategy to simultaneously *fix a risk level* and *immunize* the portfolio against shocks to mortality
  - Changing the investment strategy requires adjustment of portfolio mix to immunize portfolio against changes in mortality

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Thank you very much for your attention!

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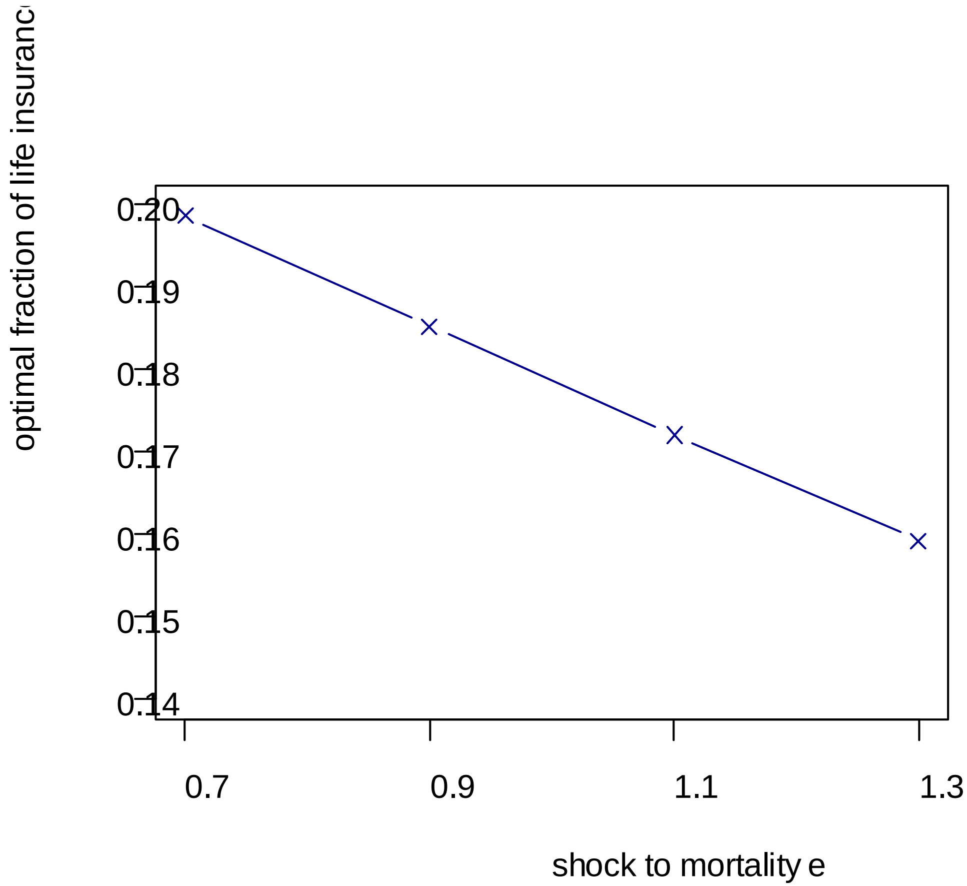
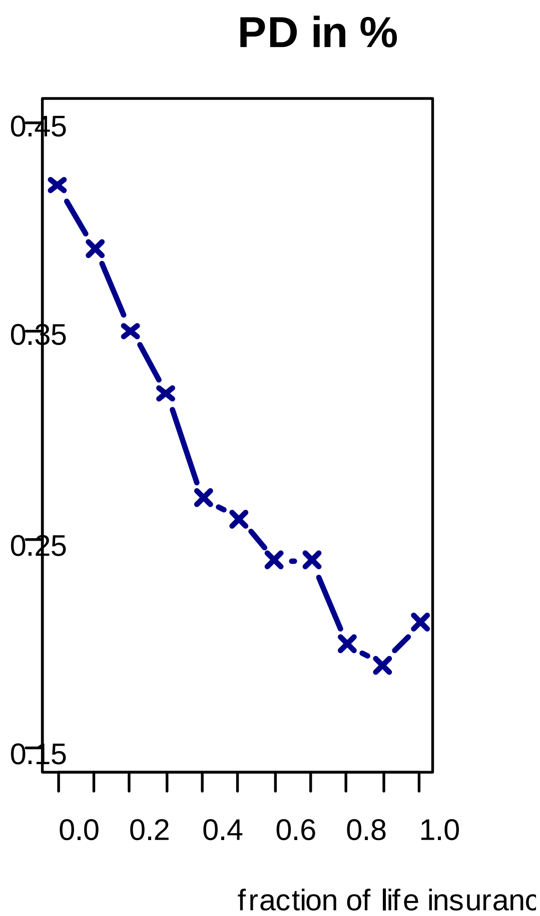

optimal fraction of life insuranc
0.20
0.19
0.18
0.17
0.16
0.15
0.14
0.7
0.9
1.1
1.3
shock to mortality e


PD in %
0.45
0.35
0.25
0.15
0.0
0.2
0.4
0.6
0.8
1.0
fraction of life insuranc

Model framework

Risk measurement – liability side

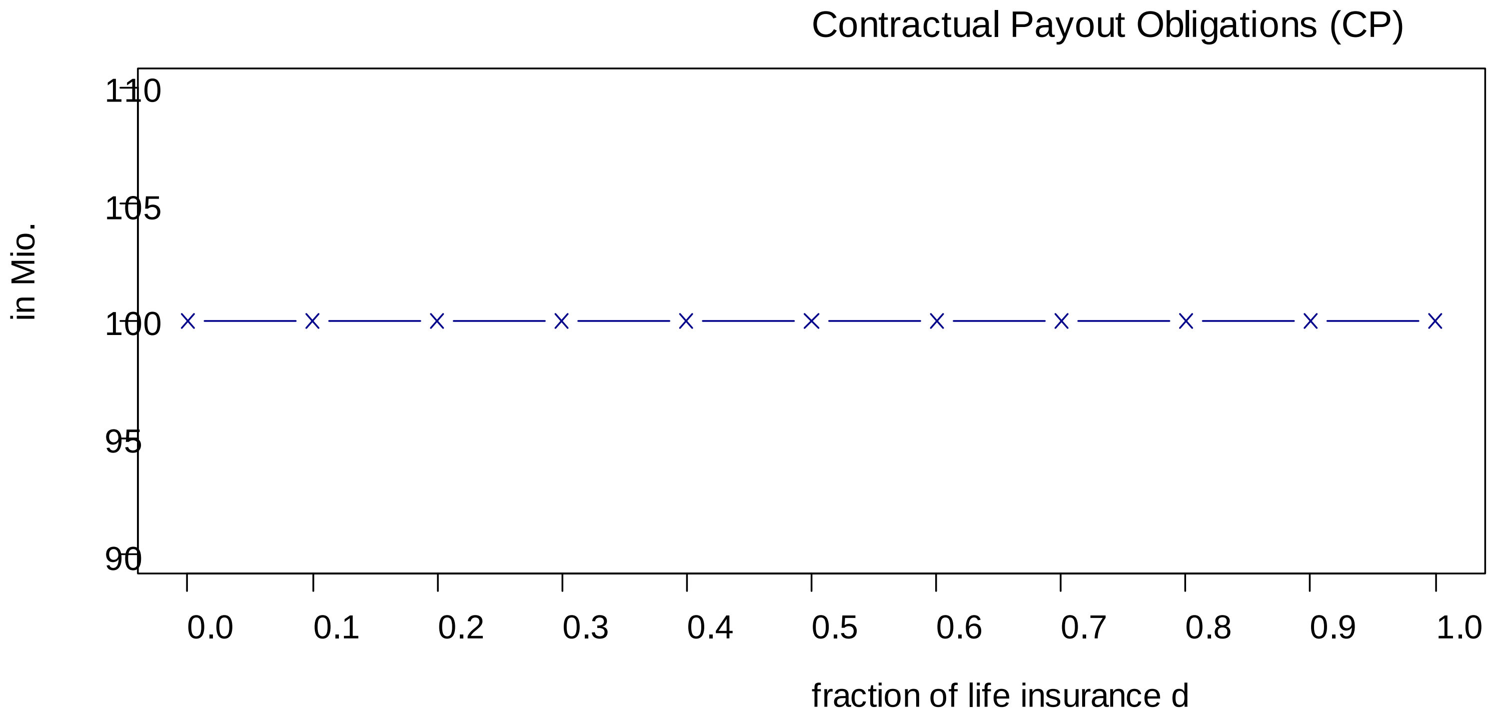
- Contractual Payout Obligations (CP)

$$CP = n_L(0)\cdot\sum_{t=0}^{T-1} DB \cdot {}_t p_x(e)\cdot q_{x+t}(e)\cdot(1+r)^{-(t+1)}$$

$$+n_A(0)\cdot\sum_{t=0}^{T-1} a \cdot {}_t p_x(e)\cdot(1+r)^{-(t+1)}$$

- take into account only liabilities – focus of previous literature
- linear in portfolio composition

- Here, additional consideration of default risk measures



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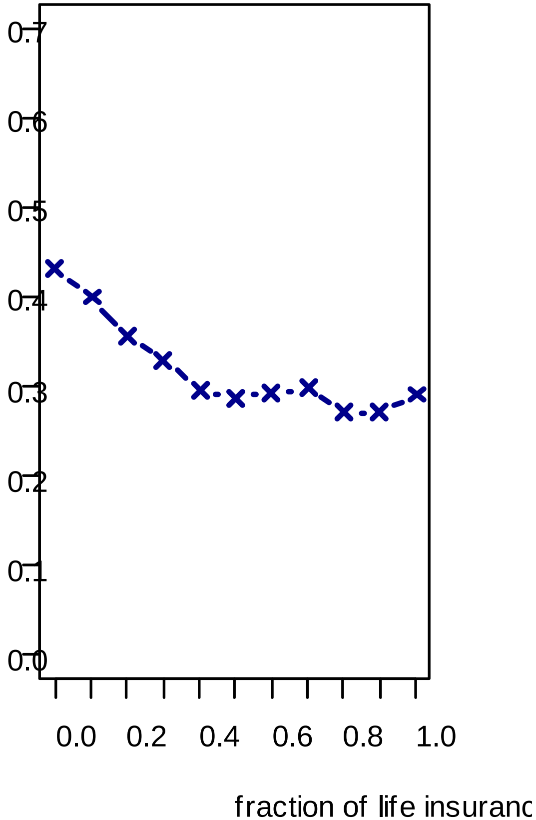
PD in %


0.7
0.6
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0.4
0.3
0.2
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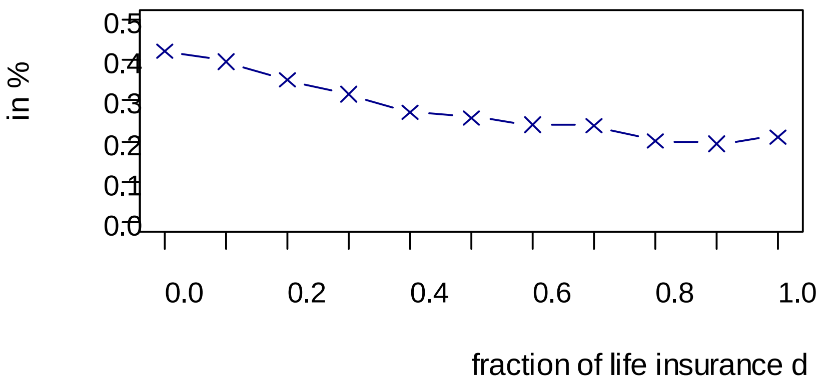
0.0
0.0
0.2
0.4
0.6
0.8
1.0
fraction of life insuranc

Probability of Default (PD)



Model framework

Risk measurement – liability side

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