Friedrich-Alexander-Universität Erlangen-Nürnberg

Mortality Risk and its Effect on Shortfall and Risk Management in Life Insurance

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Introduction
Motivation

- Recently there has been growing interest in mortality risk and its management, especially due to the demographic development

- Therefore, several alternative instruments for managing demographic risk have been proposed and discussed, e.g.
  - transferring mortality or longevity risk to the capital market or
  - natural hedging

- To analyze the effectiveness of these alternative risk management strategies comprehensively, mortality risk can be divided into three components

Introduction
Motivation

- Mortality risk components
  - Unsystematic mortality risk: individual time of death is a random variable with a certain probability distribution
  - Systematic mortality risk: probability distribution of the time of death is subject to sudden unexpected change
  - Adverse selection: which here refers to the fact that the mortality rate differs for different groups of insured, i.e. the mortality rate for annuitants is lower than for the population as a whole

- All of these mortality components might have considerable impact on the risk situation and risk management of a life insurance company

Introduction

Aim of paper

1. Study the interactions between different types of mortality risk with respect to the risk situation of an insurance company
   - explicitly modeling unsystematic mortality risk, systematic mortality risk and adverse selection

2. Analyze the impact of mortality risk components on the effectiveness of different risk management tools, namely
   - purchasing Mortality Contingent Bonds (MCB)
   - natural hedging, i.e. hedging systematic mortality risk through portfolio composition

Model framework
Modeling and forecasting unsystematic mortality

- Extension of the Lee-Carter (1992) model by Brouhns, Denuit and Vermunt (BDV) (2002):

$$D_{x,t} \sim Poisson\ \ E_{x,t} \cdot \mu_x\ \ t \qquad \text{with} \quad \mu_x\ \ t\ = e^{a_x + b_x \cdot k_t}$$

with

- $D_{x,t}$ poisson-distributed number of deaths,
- $E_{x,t}$ exposure at risk
- a_x and b_x indicating the general shape of mortality over age
- k_t indicating the general level of mortality in the population

- Forecasting of k_t, respectively $\mu_x(t)$

⇨ ARIMA process for estimated time series of k_t

Model framework

Modeling mortality basis risk and systematic mortality risk

- Adverse selection:
  - extension of the brass-type relational model by Brouhns, Denuit and Vermunt (2002)

$$\ln\ \mu_{x,t}^{annu} = \alpha + \beta_1 \cdot \ln\ \mu_{x,t}^{pop} + \beta_2 \cdot\ \ln\ \mu_{x,t}^{pop} \cdot t + \varepsilon_{x,t}.$$

→Implies a different level and trend of annuitant mortality

- Systematic mortality risk:
  - modeled through a change in the drift of the time trend of k_t

→Leads to an unexpected change in the level and in the future development of mortality

Model framework
Model of a life insurance company

- Simplified balance sheet of a two-product life insurance company:

	Assets $A(t)$	Liabilities $L(t)$	
$A(t)$	$A_{high}(t)$	$M_A(t)$	$L(t)$
	$A_{low}(t)$	$M_L(t)$	
	$M_{bond}(t)$	$E(t)$	

- $A_i(t)$: market value of assets at time t for i = high risk, low risk
- $M_{bond}(t)$: value of mortality contingent bond (MCB) at time t
- $M_i(t)$: value of liabilities at time t for i = annuities, life insurance
- $E(t)$: equity in time t

- Default of insurance company, if $L(t) > A(t)$

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Model framework
Liabilities

- Premiums and benefits are calculated using the actuarial equivalence principle and risk-neutral valuation
- Based on this, the value of liabilities for both insurance products is defined as the net of future payment obligations for the insurance company
- The mortality rates used in the calculation
  - are those forecasted stochastically using the BDV model and
  - differ for life insurance policyholder and annuitants

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Model framework
Assets

- Value of assets in $t = 0$

  $$A(0) = E(0) + n_A(0) \cdot SP_A + n_L(0) \cdot P_L - \Pi_{x,d}$$

  with $\Pi_{x,d}$ premium of MCB

- $A(t)$ can the be calculated via

  $$A(t) = A_{high}(t) + A_{low}(t) + n_L(t) \cdot P_L - n_A(t) \cdot a - d_L(t) \cdot DB + X(t) - div(t)$$

  where

  - $A_{low}(t) = \alpha \cdot A(t)$, i.e. a constant fraction α is invested in low risk assets
  - *X*(*t*) is the coupon payment of the MCB in time *t* and
  - *div*(*t*) is the dividend paid to shareholders in return for their investment

Model framework
Mortality Contingent Bond (MCB)

- Proposed by Blake and Burrows (2001) under the name "survivor bond"

- In return for a premium paid in advance, the insurance company receives a variable coupon payment $X(t)$ at the end of year t
  - The coupon payment $X(t)$ depends on the number of survivors in the reference population $n_{ref}(t)$

$$X(t) = \frac{n_{ref}(t)}{n_{ref}(0)} \cdot C$$

- Mortality in the reference population is thereby equal to population mortality, which differs from annuitant mortality (= adverse selection) → Basis risk

Model framework

Risk measurement – default risk measures

- Probability of default (PD)

$$PD = P\left(T_d \leq T\right)$$

with $T_d = \inf\left\{t : A(t) < L(t)\right\}, t = 1,\ldots,T.$

- Mean Loss (ML)

$$ML = E\left[\left(L(T_d) - A(T_d)\right) \cdot (1+r)^{-T_d} \cdot 1\left(T_d \leq T\right)\right]$$

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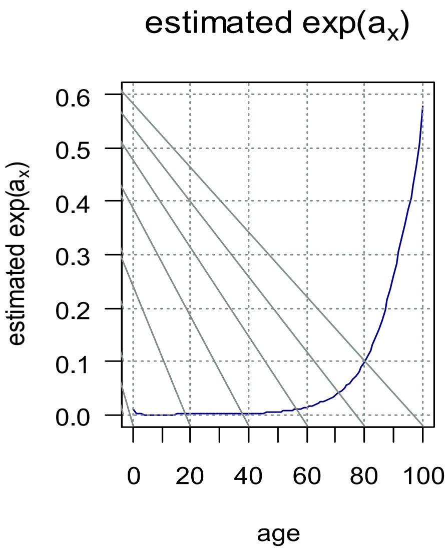
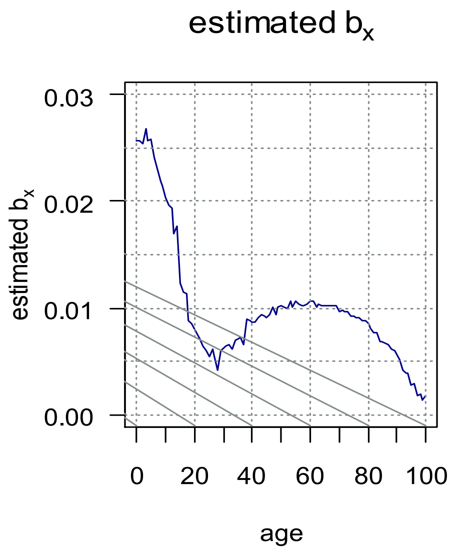
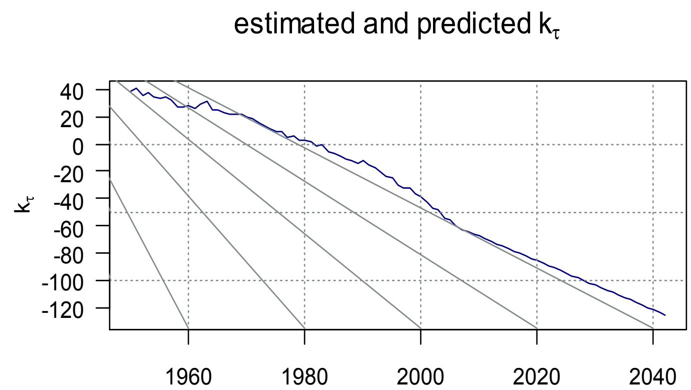
Numerical results
Estimation of mortality risk

- Estimation of mortality of the population (Switzerland)







ARIMA (1,1,0) process

- Estimation of adverse selection

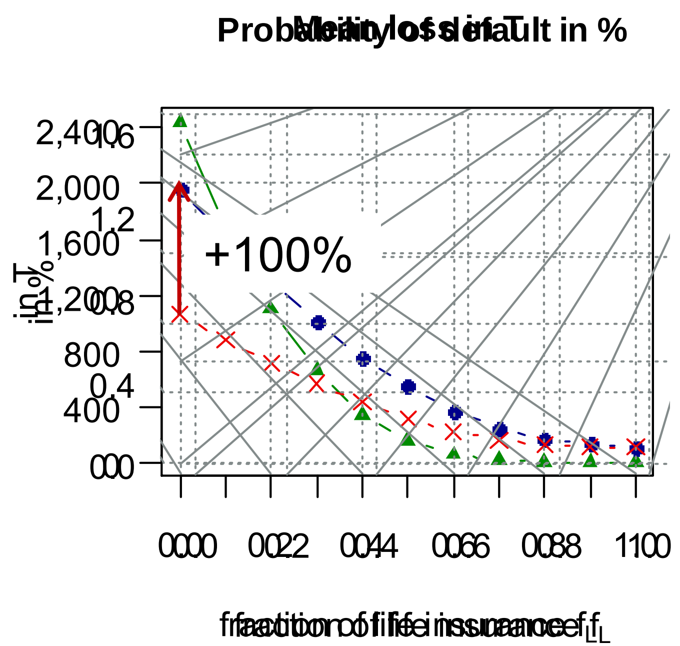
$$\ln \mu_{x,t}^{annu} = -0.3197 + 1.0747 \cdot \ln \mu_{x,t}^{pop} - 0.0004 \cdot \ln \mu_{x,t}^{pop} \cdot t$$

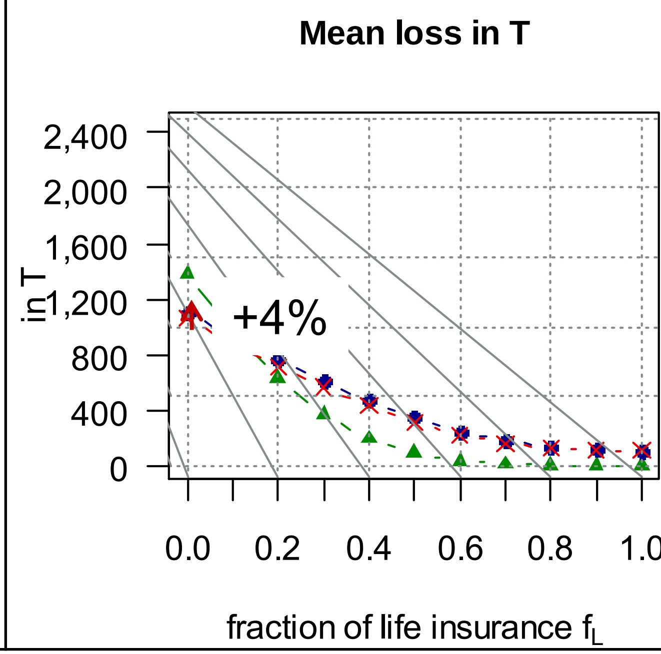
Numerical results

Impact of adverse selection on the risk situation

- Two assumptions concerning adverse selection

information about the generally lower mortality of annuitants is partly hidden (asymmetric information) ➡ adverse selection mispriced	insurance company is able to forecast adverse selection completely (e.g. through experience rating) ➡ adverse selection perfectly priced





unsystematic risk | unsystematic risk+ adverse selection | unsystematic risk+ adverse selection+ systematic risk

Numerical results
Effectiveness of MCBs

Effectiveness of MCBs for reducing the impact of systematic mortality risk

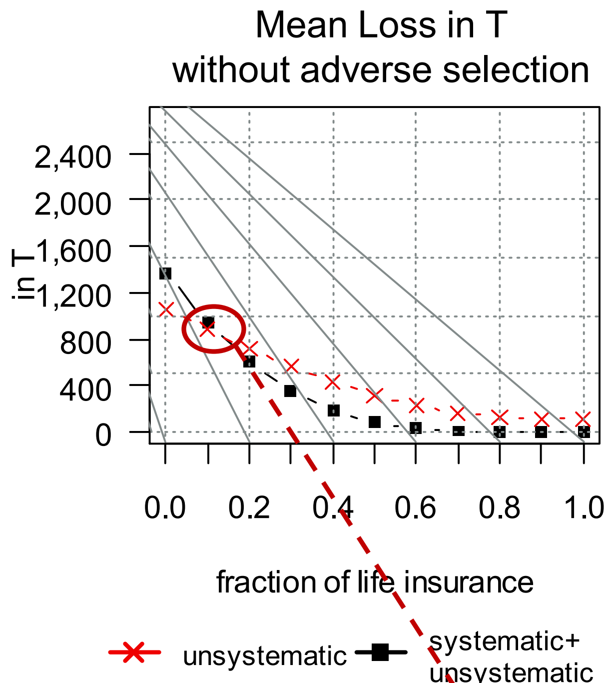
→ here: measured through the change in the riskiness of an insurance company in response to a change in mortality

For a portfolio with only annuities $f_L = 0$	Without adverse selection (no basis risk)	With adverse selection (in the presence of basis risk)	
		misestimated	perfectly estimated
	Mean Loss	Mean Loss	Mean Loss
Without MCB	1,369 T	2,442 T	1,386 T
With MCB	749 T	1,631 T	776 T
Relative reduction through MCB	45.3%	33.2%	44.0%

The relative reduction is defined as $\frac{ML_{without\ MCB} - ML_{with\ MCB}}{ML_{without\ MCB}}$.

Numerical results

Natural hedging under adverse selection



fraction of life insurance needed to eliminate the impact of un-expected low mortality decreases under adverse selection, despite greater implied change in life expectancy of annuitants

	Without adverse selection	With adverse selection	
		misestimated	Perfectly estimated
	ML	ML	ML
Fraction of life insurance	13.1%	12.4%	11.8%

Summary

- Our results show an increase in the risk of an insurance company through adverse selection for all portfolios, even if it can be perfectly forecasted
  - This effect is stronger when considering mixed portfolios as compared to a portfolio consisting only of annuities
- That adverse selection does not impair the effectiveness of natural hedging, however it does affect the immunizing portfolio composition
- In terms of hedging against systematic mortality risk, the effectiveness of MCBs is decreased slightly, given that adverse selection can be properly forecasted and is taken into account in pricing

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Thank you very much for your attention!

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