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ADJUSTED FORWARD RATES WITHIN TWO THEORY OF INTEREST RATES

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AIM of the Paper

✓Graduation methods are **commonly** used in actuarial science for smoothing mortality rates. However, a graduation could be applied to the course of stock or commodity prices and successful speculation be based on the slope of the curve. It has been developed procedures to obtain those non risk spot rates from Treasury bonds like bootstrapping procedure. But the rates calculated with this procedure are the **observed ones** and could reflect –and reflect- any anomaly from the market. We can use graduation methods to smooth those anomalies.

✓This is the principal aim of this paper: to **establish a simple procedure to estimate those future rates according to two theories of interest rates**.

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Financial Background

- Interest rates are used to value pension fund (A-L) and a lot of financial products. The flows are valued at **market rates** obtained from day to day values.
- But perhaps the now a day observed interest rates will **not** be the future real interest rates.
- In this sense, the procedure used to calculate the term structure of interest rates has some similarities with the procedure used to estimate mortality rates. So, it is developed the Whittaker-Henderson graduation for looking an **adjusted future forward** rate which reflects those expectations of the investors, smoothed and fit within two theories that explains the curve of future interest rates: rational expectations theory and preferred habitat theory

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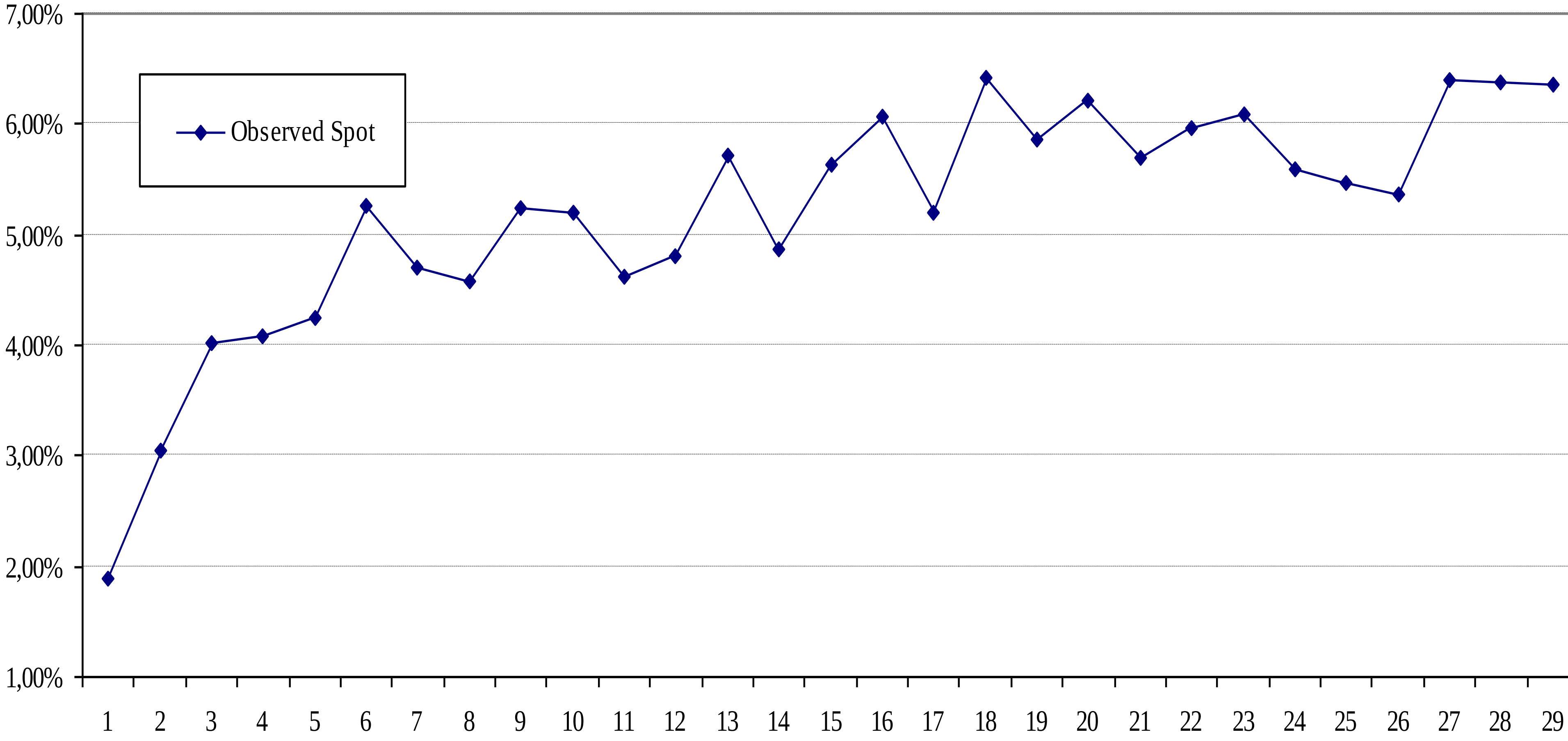
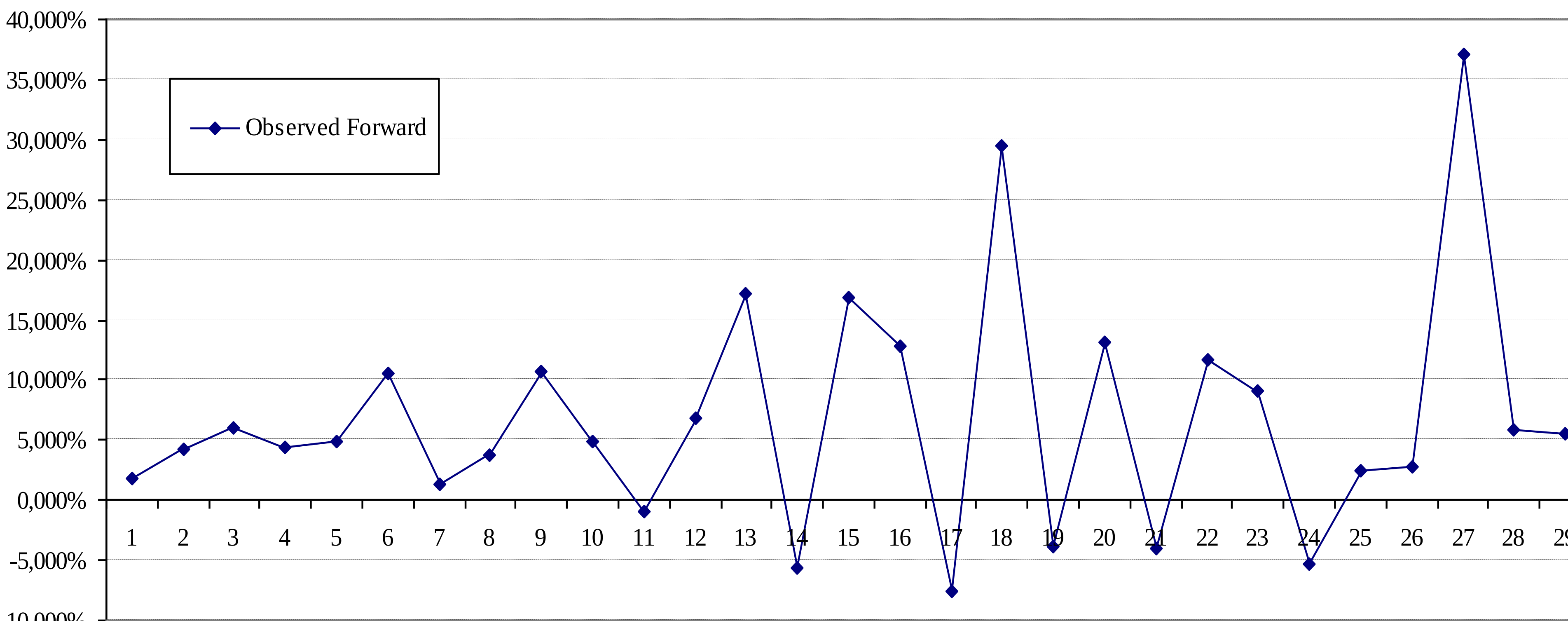
Observed Spot rate curve at 2011, April 12

Spot rate depends directly on the period in which it is paid [0, t]



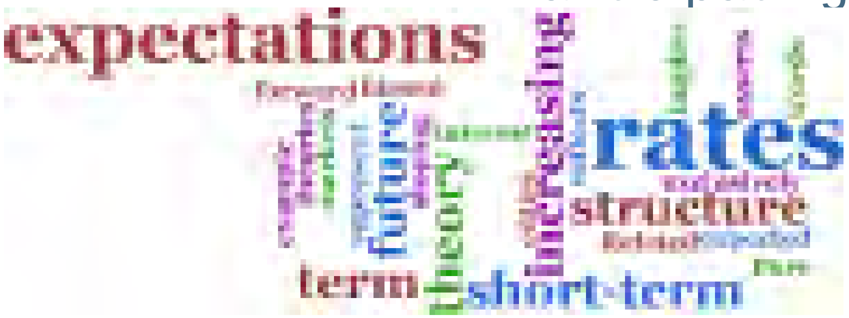
Observed Forward rate curve at 2011, April 12

The forward rate is the rate of interest, implicit in currently spot rate that could be applicable from one point of time in the future to another point of time in the future. It is an interest rate **expected to be** in a fixed lapse of time in the future.



Useful information for :

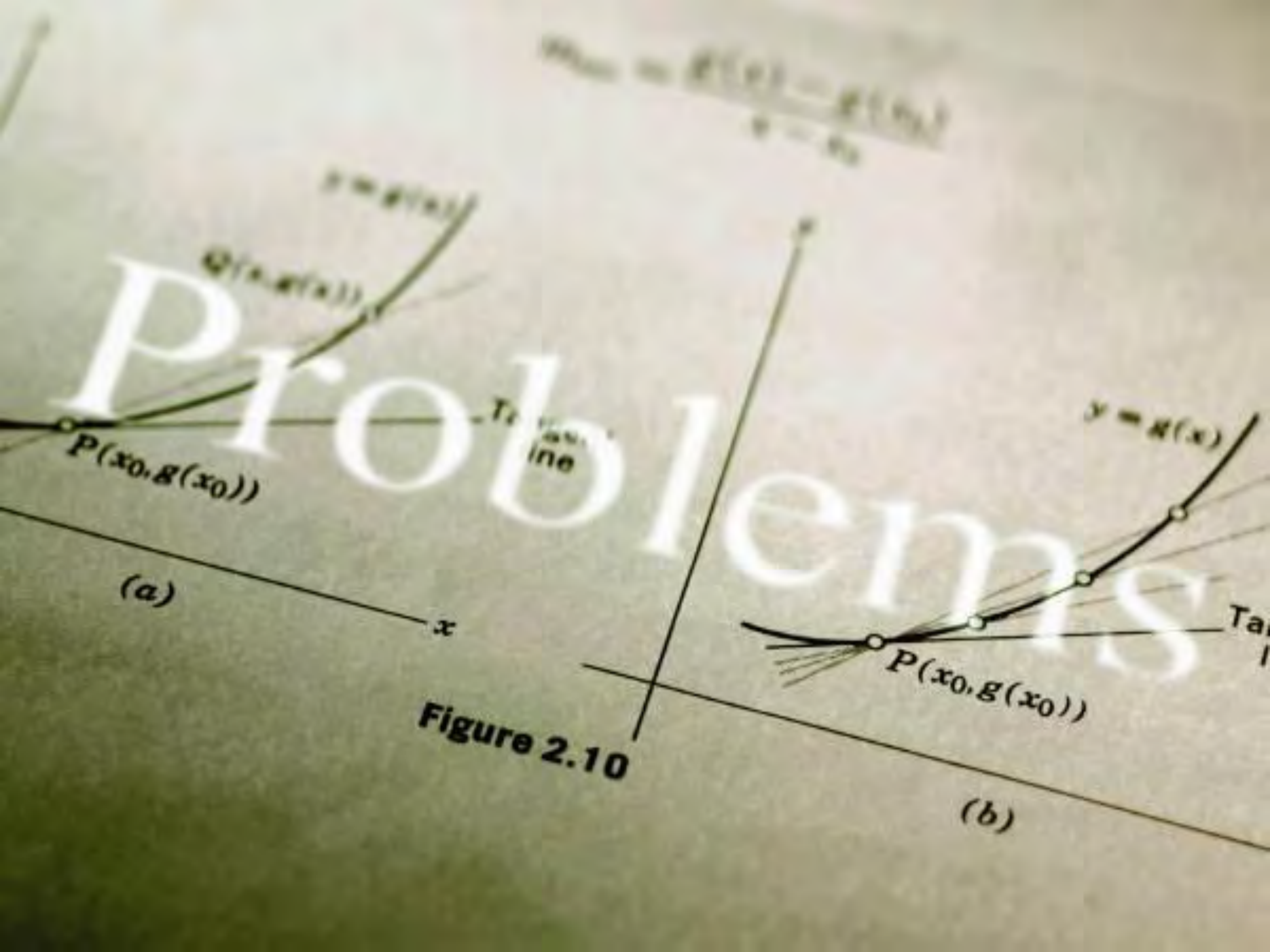
-anticipating expectations in changing markets,
-estimating future inflation,
-changes into interest rates,
-economical growing,
-monetary political matters.



->**Theory of the pure expectations**: the form of the curve is determined by the expectations that market investors have on the future interest rates.

-> **Preferred habitat theory**: investors will invest outside of their maturity preference if the benefit that could obtain compensates the assumed risk.

Problems



- The current *t* period forward rate is calculated using current information, however, the actual forward rate will likely differ from what was calculated, as we get closer to *t*: The **tops and the bottoms of the curve will be softer**.

- There are only a finite number of bonds and their prices define a finite number of points. With those values it is possible to construct the interest rate curve although with several **problems**.

i) Spot rates are **not observed**.

ii) In Spain, bonds pay **periodical coupons** at fixed dates.

iii) Observed rates include **other risks** or other effects as liquidity risk, taxes, etc.

iv) Observed interest rates obtained at a specific moment are according to the **expectations** that the market has **at that moment**.

v) The **crisis** and its effect into several firms could give us different values at different periods.

vi) The necessity of money (**liquidity**) in a specific moment could do the value of a bond became higher from one moment to another, but it is due to a punctual anomaly.

vii) **Political factors** affect interest rates at different periods

viii) It could happen that the investor could be at a market **without the n-year spot rate** or n-year forward rate.

For solving some of these problems we can use a graduation method

Whittaker-Henderson Graduation

The aim of the graduation technique is to obtain the **best kind of adjusted values** that represents and fits the observed values.

The graduated values will be **close** to the observed values because those are **the best estimate of the true values**.

Parametric and **non parametric** methods.

The problem of graduation is to find the **best fitting values**, which satisfy mathematical and actuarial constrains.

(Whittaker 1923 and Henderson, 1924)

It requires minimizing a function, which is the sum of a **fitness** measure and a **smoothness** measure.

The method assumes that there is a simple trade-off between a measure inversely related to fidelity (F) and a measure inversely related to smoothness (S).

$$M = F + h \cdot S$$

$$S = \sum_{j=1}^{n \cdot z} \left(\Delta^z f_j \right)^2$$

$$F = \sum_{j=1}^{n} w_j \cdot \left(f_j - \overline{f_j} \right)^2$$

fidelity

$$F = \sum_{j=1}^{n} w_j \cdot \left(f_j - \overline{f_j}\right)^2$$

This sum explains the **level of fitness** between the observed values and the adjusted values. It is the sum of the squares of all the *n* graduated values.

PET: Observed values are forward rates determined by the expectations that market investors have on the future interest rates.

PHT: w_j are the weights corresponding to each value. The weight of the squares is different from one period to another because the investors invest in different kind of bonds (outside of their maturity preference).

$$S = \sum_{j=1}^{n-z} \left(\Delta^z f_j\right)^2$$

The second sum of the function is the **measure of smoothness** of the estimations. The parameter *h* (real positive number) it is a control element that gives the relative equilibrium between smoothness and fitness.

So,

$$M = \sum_{j=1}^{n} w_j \cdot \left(f_j - \overline{f_j}\right)^2 - h \cdot \sum_{j=1}^{n-z} \left(\Delta^z f_j\right)^2$$

Application

The following applications take the value of **spots rates at 2011, April 12** at the Spanish bond market. All bonds come from the Treasury, so they are free of credit risk, and with different maturities (the top is 29 years).

For the graduation we choose

Z=3 and h=3, so…

Graduated Values at 2011, April 12

t	Observed spot	Observed forward	weights	Graduated forward weight	Graduated Spot weight	Graduated Forward	Graduated spot
1	1,883832%	1,883832%	150,69	1,883832%	1,883832%	1,883832%	1,883832%
2	3,040778%	4,210862%	75,06	4,507341%	3,187249%	3,834069%	2,854328%
3	4,012339%	5,983031%	174,92	5,718326%	4,024135%	5,067756%	3,586908%
4	4,092189%	4,332105%	57,00	4,527674%	4,149792%	5,705007%	4,112420%
5	4,240222%	4,834462%	213,00	5,246230%	4,368162%	5,980606%	4,483404%
6	5,266067%	10,548730%	60,00	8,444059%	5,036681%	5,933552%	4,723709%
7	4,702038%	1,380766%	3,00	5,717739%	5,133706%	5,554384%	4,841976%
8	4,577631%	3,710910%	886,00	3,835210%	4,970510%	5,314647%	4,900943%
9	5,234052%	10,635941%	27,98	7,154626%	5,210974%	5,158134%	4,929489%
10	5,193038%	4,824634%	799,20	4,692388%	5,159000%	4,883364%	4,924876%
11	4,626857%	-0,870023%	276,18	0,000000%	4,679203%	5,060815%	4,937226%
12	4,810340%	6,850013%	249,78	7,546336%	4,915183%	6,124287%	5,035639%
13	5,714814%	17,196993%	271,18	14,792071%	5,643796%	7,481821%	5,221814%
14	4,866711%	-5,558905%	290,33	0,000000%	5,230312%	8,603798%	5,459852%
15	5,630557%	16,927451%	40,00	8,975704%	5,475950%	9,869145%	5,748219%
16	6,061670%	12,743590%	385,20	12,345859%	5,892737%	10,173252%	6,019501%
17	6,022954%	5,405405%	375,81	6,496635%	5,928165%	9,256927%	6,207255%
18	6,429852%	13,591011%	109,00	8,189562%	6,052549%	7,682694%	6,288690%
19	5,859467%	-3,900280%	381,56	0,000000%	5,725049%	5,954410%	6,271071%
20	6,212721%	13,153063%	381,78	10,736441%	5,970145%	4,920761%	6,203144%
21	5,701363%	-4,024222%	349,89	0,000000%	5,677934%	4,209487%	6,107349%
22	5,965321%	11,663268%	208,15	9,831278%	5,863269%	3,946484%	6,008160%
23	6,097850%	9,055794%	237,87	8,912400%	5,994047%	3,730941%	5,908119%
24	5,596372%	-5,305549%	202,90	0,000000%	5,737267%	4,355016%	5,842947%
25	5,472154%	2,534379%	101,75	0,156207%	5,508165%	6,951390%	5,887064%
26	5,366373%	2,756043%	101,49	9,727976%	5,667424%	10,365966%	6,055920%
27	6,397193%	37,044545%	192,21	31,460203%	6,525652%	11,750722%	6,261570%
28	6,379537%	5,903955%	218,98	9,588712%	6,633559%	8,629145%	6,345231%
29	6,349142%	5,501580%	125,00	3,150989%	6,511535%	3,498328%	6,245770%

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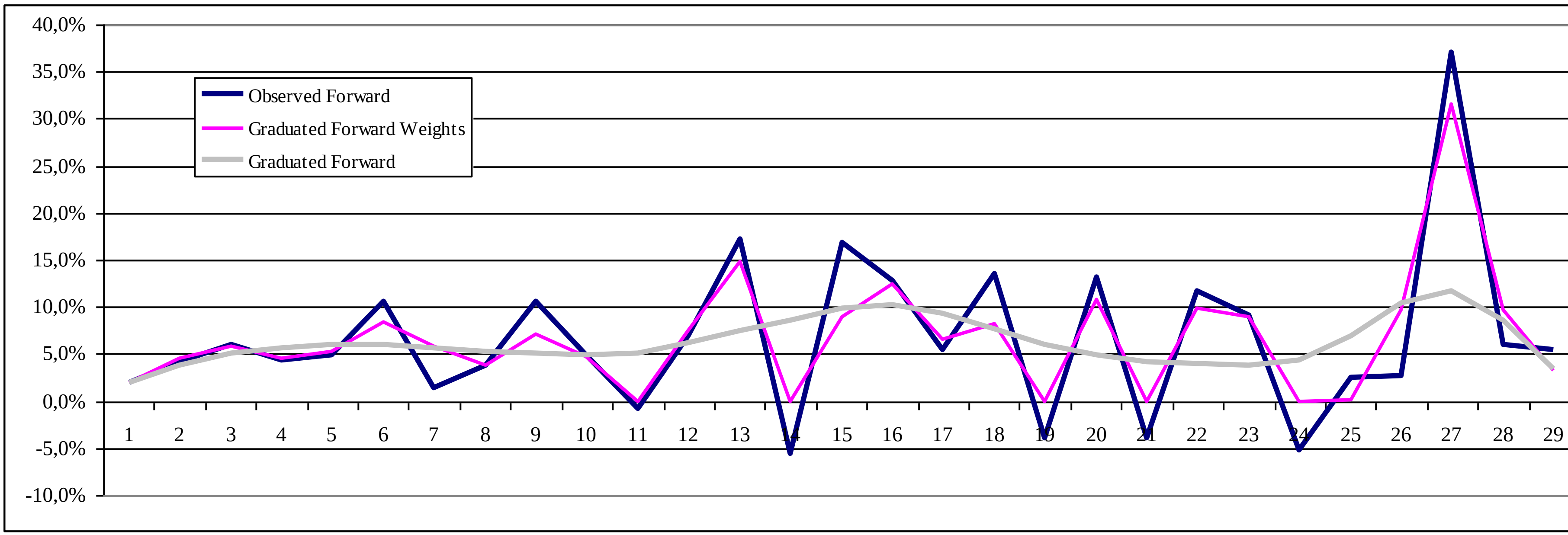
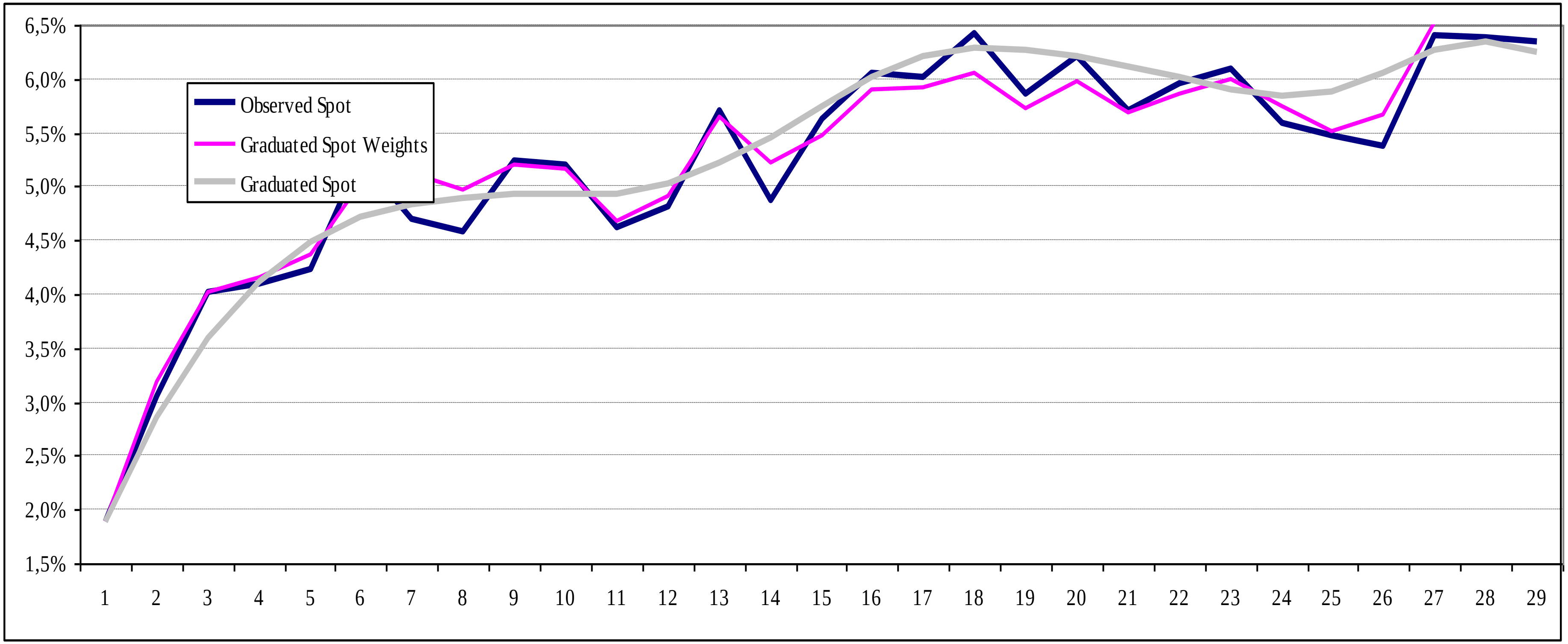
Observed and Graduated Forward rate curve at 2011, April 12

They represent annual interest rates used, for instance, for the reinvestment of pension annual plan contributions and other assets. **Using weights** there is a better fit than without it, but in this case, **persists the problem** of existing top and bottoms into the future rates.



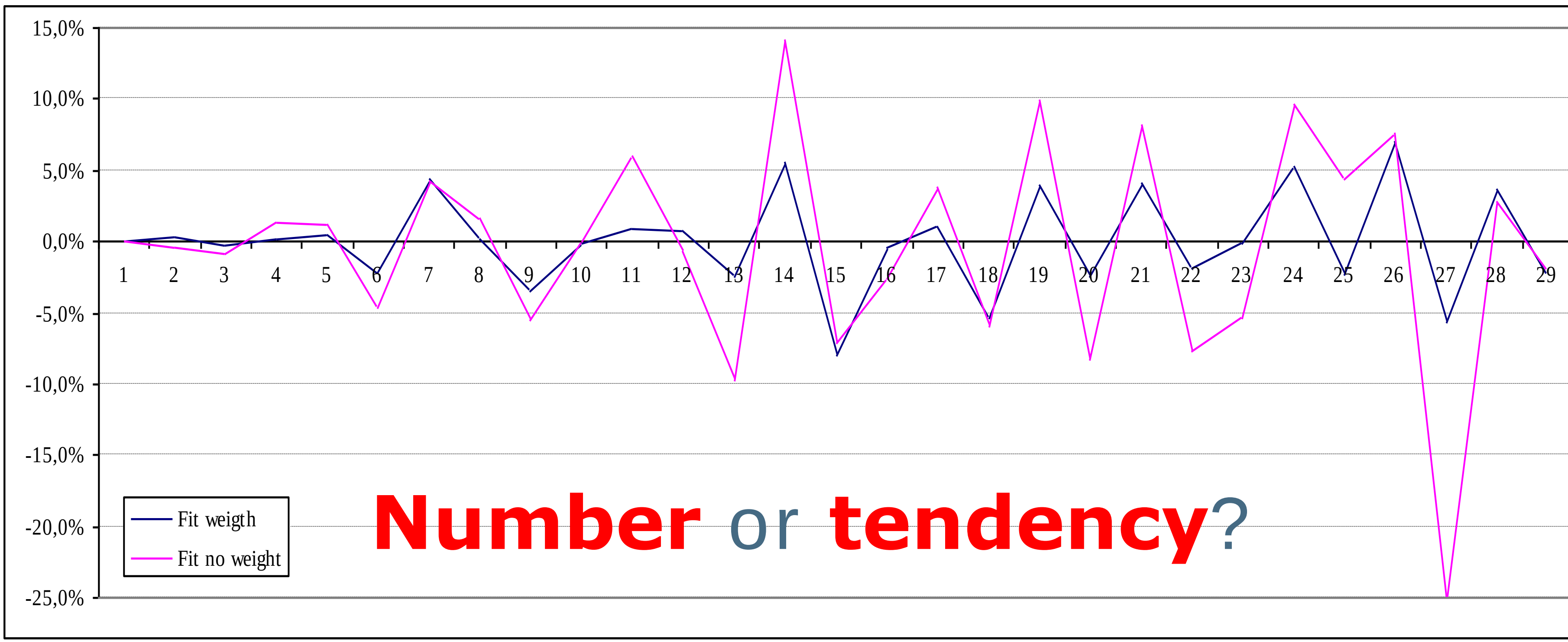
Observed and Estimated Term structure of interest rate at 2011, April 12

This curve could be used to valuate liability flows or to value the assets whom fund the liability into an immunization program.



Fitness of the Graduation at 2011, April 12

When this difference is zero fitness is the best. In this case the best graduation is done using the Whittaker-Henderson Method within two theories of interest rates, but what are we looking for?



Number or **tendency**?

Several comments

a) The information that results from the adjusted forward rates is useful **to anticipate** market expectations into the interest rates, to anticipate inflation effects, economical growth but in all the cases according to market (investors) expectations.

b) The adjusted interest rates include the expectations of the market but **smoothed** throughout different years.

c) The Whittaker – Henderson graduation method is **fast and easy** to apply.

d) All the observations are used to obtain the graduation.

e) This graduation method allows making estimations and future valuations according to the expectations that, **in a certain moment**, exist on the interest rate.

f) We can use the **inefficiencies** observed in the market to anticipate to them before they arrive.

g) To determine the inefficiencies of the market over the expectations, which could affect the valuations over the adjusted interest rates, **is necessary a periodic follow-up**.

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