DECOMPOSING HEDGE EFFECTIVENESS IN LONGEVITY HEDGES

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Plan

- Aim
- Preliminaries
- Case Study: model $+$ data
- Decomposing correlation

Motivation

- Pension plan wishes to hedge its exposure to longevity risk
- Options:
  - Customised hedge $\Rightarrow\sim$ 100% hedge effectiveness
  - Index-based hedge $\Rightarrow$ basis risk
- Aim: to understand contributors to:

  basis risk, hedge effectiveness and correlation

Key quantities

- T = future liability valuation date
- $a_k(T, x) =$
  - annuity value at T
  - life annuity of 1 per annum
  - for an individual aged x at T, in population k
  - allowing for future mortality improvements

Simple example

- Liability value $L(T) = a_2(T, 65)$
- Hedging instrument: deferred longevity swap

$$H(T) = a_k(T, x) - \hat{a}_k^{\text{fxd}}(0, T, x)$$

$$\hat{a}_k^{\text{fxd}}(0, T, x) = \text{value at } T \text{ of swap fixed leg}$$

- $k = 2 \Rightarrow$ CUSTOMISED hedge
- $k = 1 \Rightarrow$ INDEX hedge

Dynamic Models: Two stages

1. Simulation from 0 to T using model $M1(0)$

2. Valuation at T:

   - Why? Valuation regulations; Accounting standards

   - Requirement: a valuation model at T: $M2(T)$

What is model $M2(T)$?

- $M2(T) \neq M1(0)$
  - (Re-)calibration using data up to $T \Rightarrow$ realistic!
  - Valuers just observe historical mortality plus

    one future sample path of mortality from 0 to T

    $\Rightarrow$ they do not know the "true" $M1(0)$
  - Using $M1(0) \Rightarrow$ too optimistic (??) c.f. Black-Scholes

Hedge Effectiveness: basic idea

- $L =$ liability value
- $H =$ value of hedging instrument
- Hedge effectiveness depends on: $\rho = cor(L, H)$

(if “risk” $=$ S.D. or Variance; or if $(L, H) \sim$ multivariate normal)

Case Study

- Population 1: England and Wales males
- Population 2: UK CMI assured lives, males
- 1961–2005;   ages 50-89
- Here: 2-population model (Cairns et al., 2010)
- Model here: just one example

  (simple model: but both period and cohort effects)

Age-Period-Cohort model (APC) (M3-2 pops)

$m_k(t,x) =$ population k death rate

$$\log m_k(t,x) = \beta^{(k)}(x) + \kappa^{(k)}(t) + \gamma^{(k)}(t-x)$$

$\beta^{(1)}(x),\ \ \beta^{(2)}(x)$ population 1 and 2 age effects

$\kappa^{(1)}(t),\ \ \kappa^{(2)}(t)$ period effects

$\gamma^{(1)}(c),\ \ \gamma^{(2)}(c)$ cohort effects

$M1(0)$: simulation model – key features

- For each age x, $m_1(t,x)/m_2(t,x)$ does not diverge over time
- Bayesian approach $+$ MCMC

  $\Rightarrow$ full posterior for process params $+$ latent state variables

  $\Rightarrow$ easy to incorporate parameter uncertainty
- Simulation up to T:
  - with/without parameter uncertainty
  - with/without Poisson risk in death counts

$M2(T)$: valuation model – key features

- Simple deterministic approximation

fast & accurate ($+$ reality!)

- Consistent population 1 and 2 projections

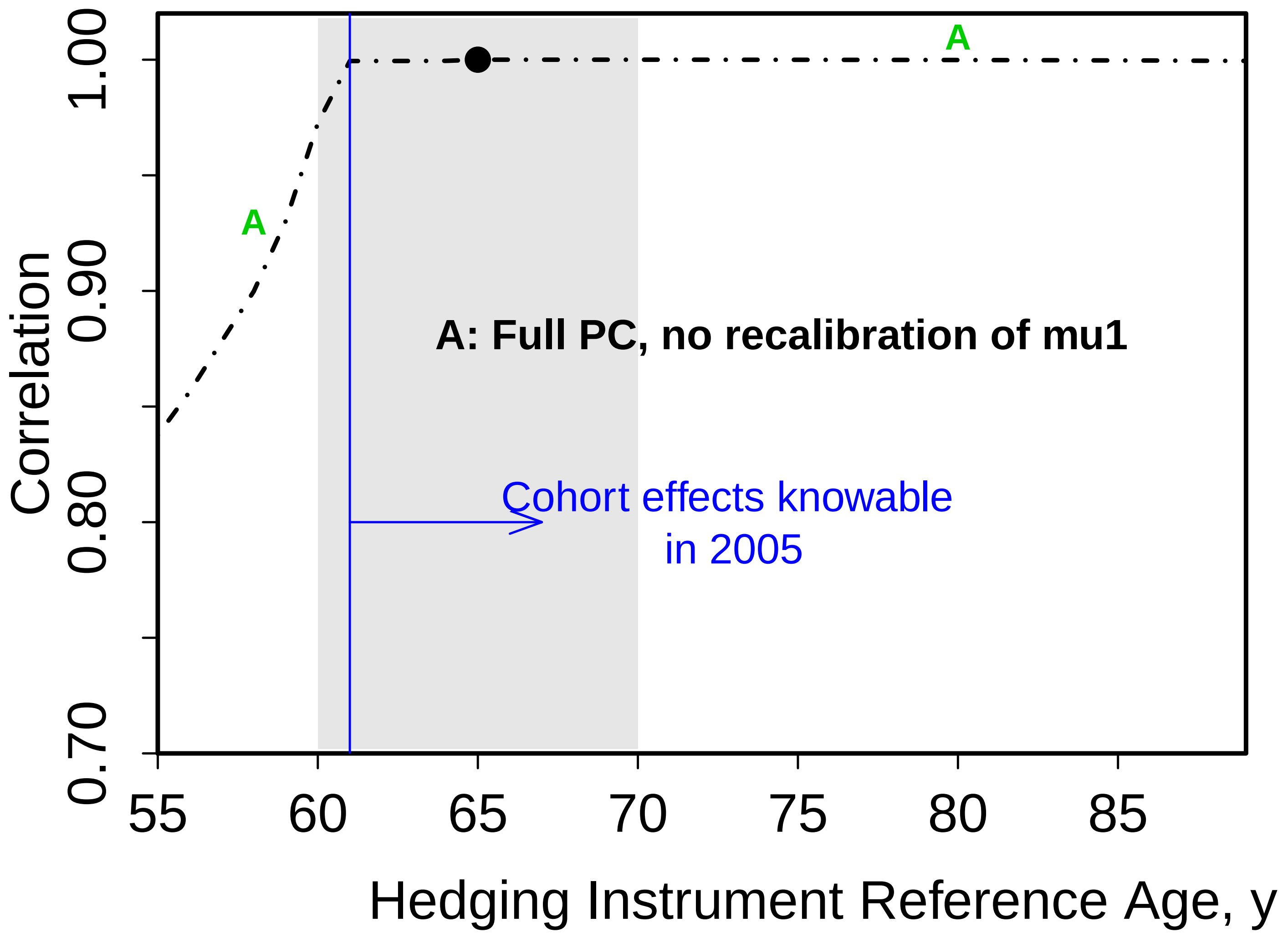
$$
\begin{aligned}
\hat{\kappa}^{(1)}(T+s) &= \kappa^{(1)}(T) + \mu s \\
\hat{\kappa}^{(2)}(T+s) &= \kappa^{(2)}(T) + \mu s
\end{aligned}
$$

i.e. median of a random walk with common drift μ

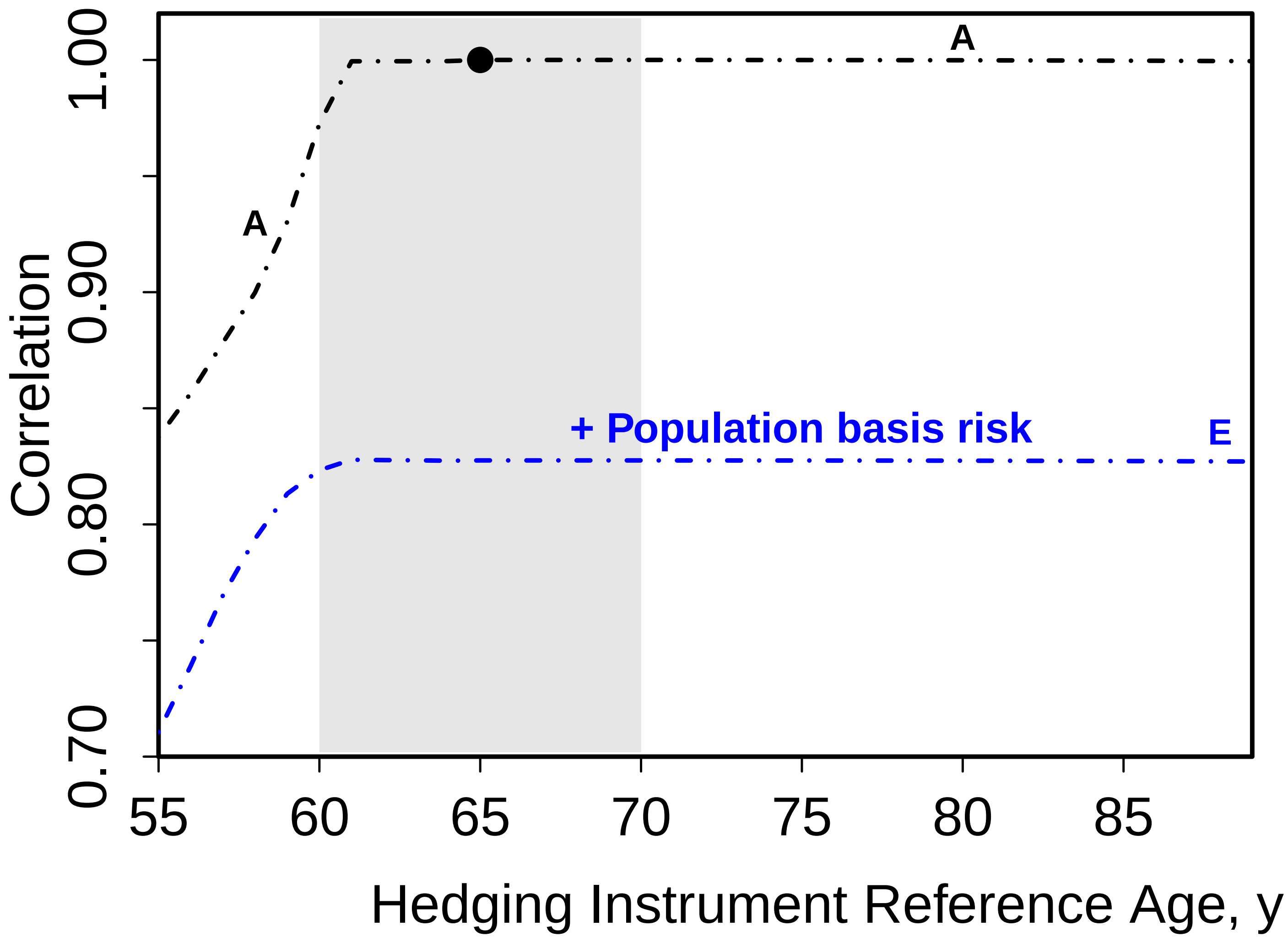
Variants

- Hedging with population 1 or population 2
- Full parameter uncertainty (PU) ($\longrightarrow M1(0)$)
  - $M2(T)$ recalibrated in 2015 using latest data
- Full parameter certainty (PC):
  - *PC version of* $M1(0)$ *for simulation*
  - $M2(T)$ calibration fixed in 2005
- Partial PC:
  - *PC version of* $M1(0)$
  - $M2(T)$ recalibrated in 2015 using latest data
- With and without Poisson Risk

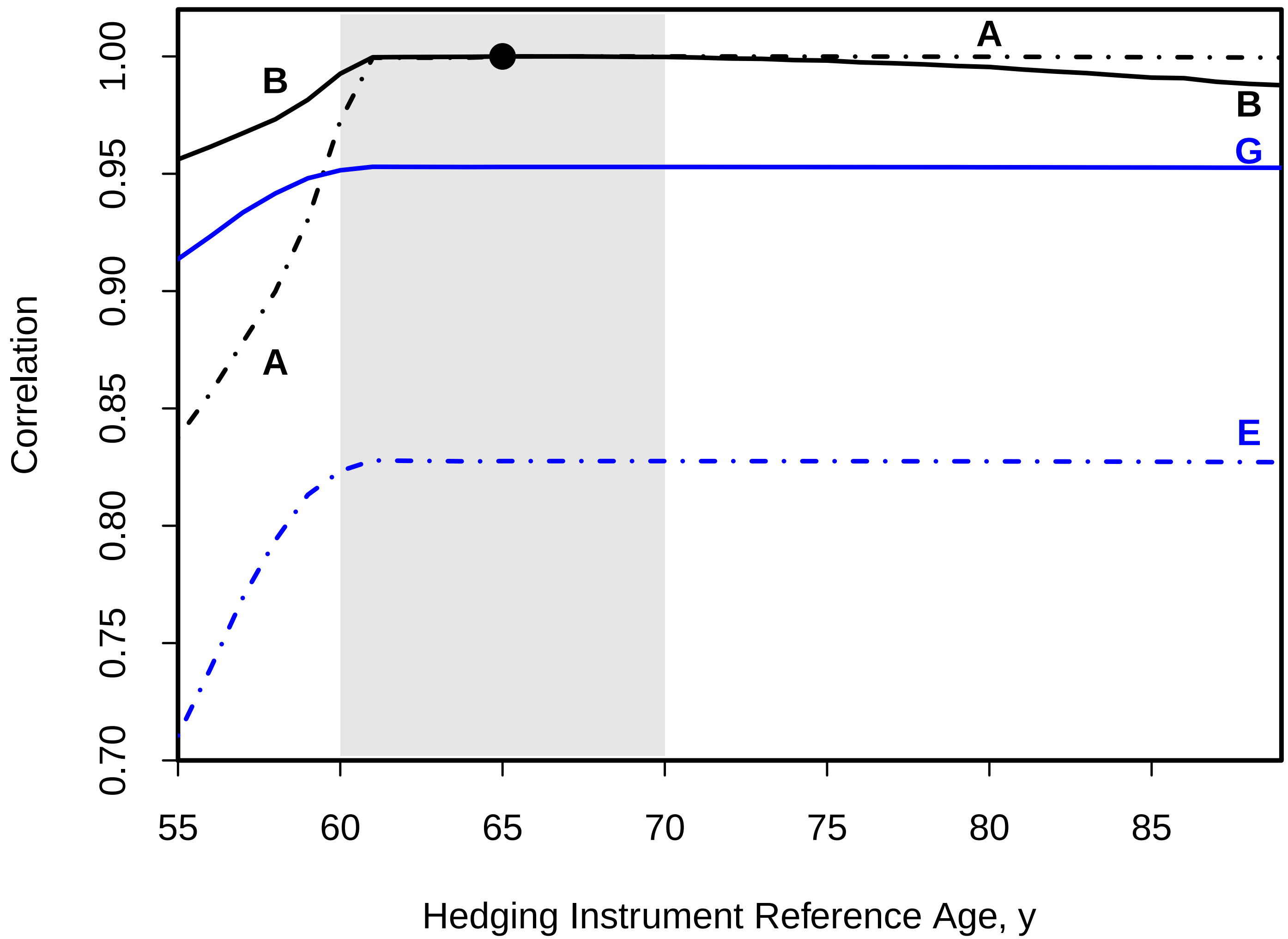
Customised hedge; PC; No Poisson



Index hedge; PC; No Poisson



Index hedge; Partial PC (recalibration risk); No Poisson

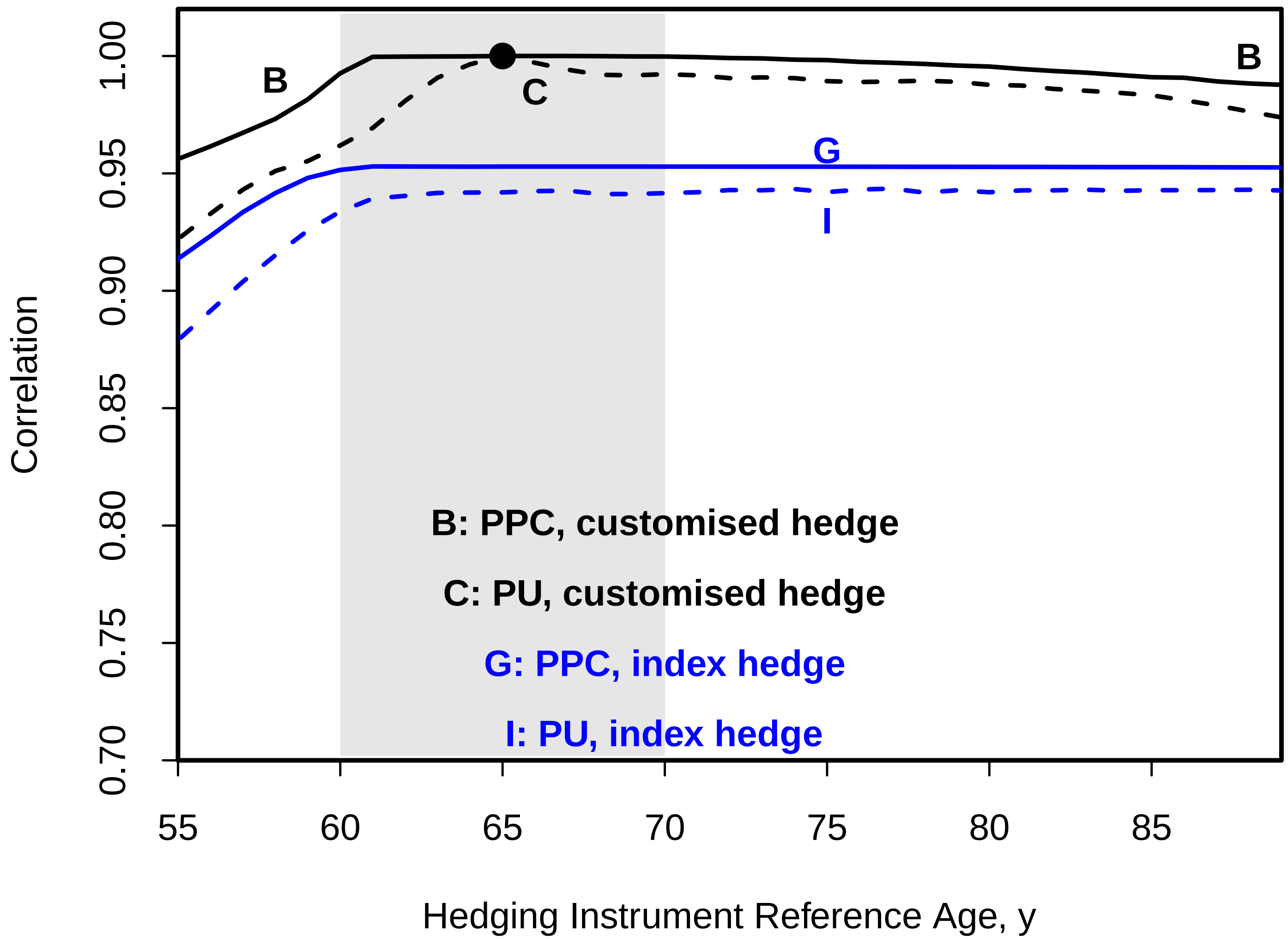


Recalibration risk explained

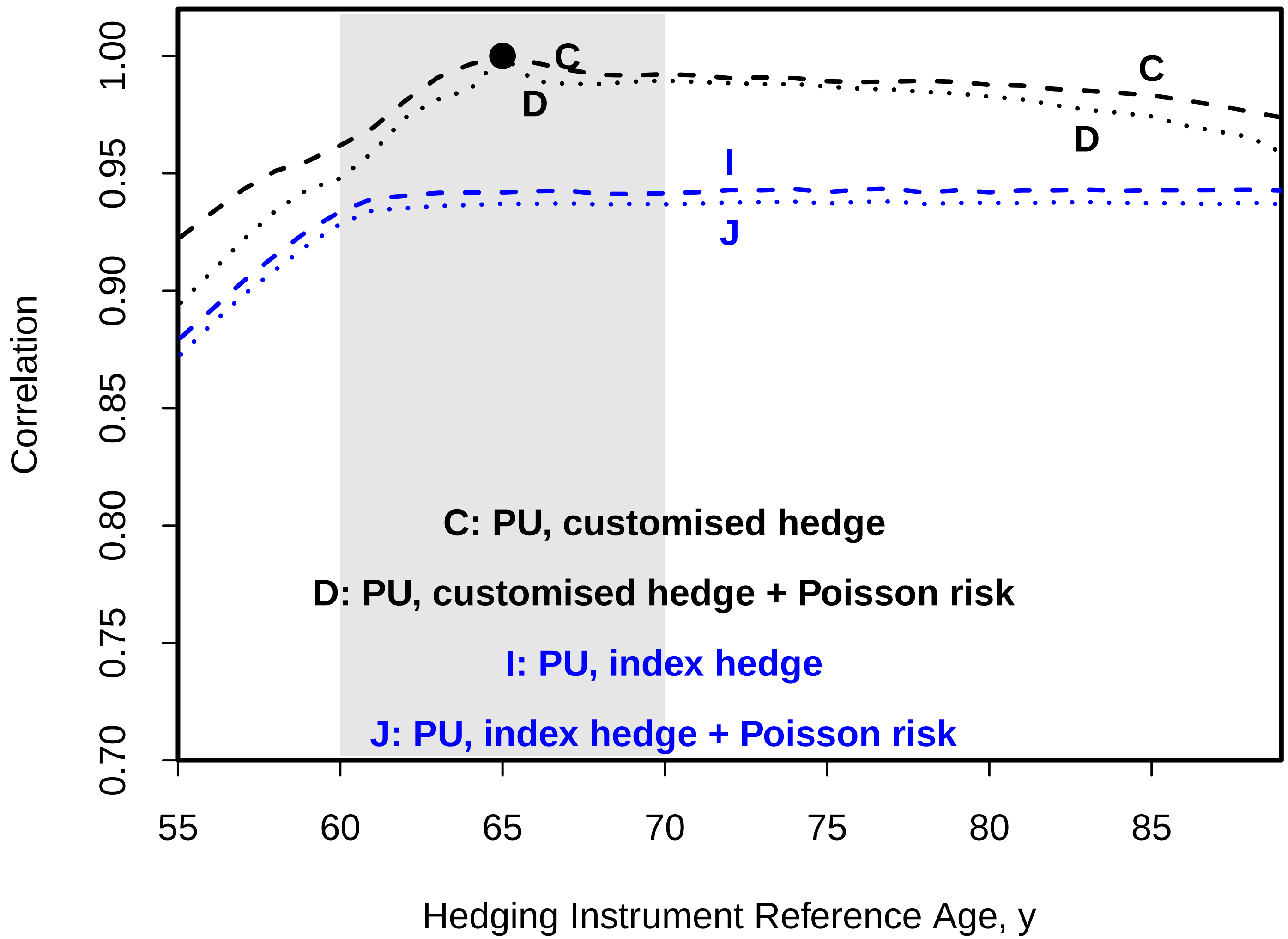
- $L \equiv a_2(T, x; \kappa^{(2)}(T), \mu, \ldots)$
- $H \equiv a_1(T, x; \kappa^{(1)}(T), \mu, \ldots) - \hat{a}_1^{\text{fxd}}(0, T, x)$
- Random $\mu \Rightarrow$ extra risk,

  BUT also higher correlation

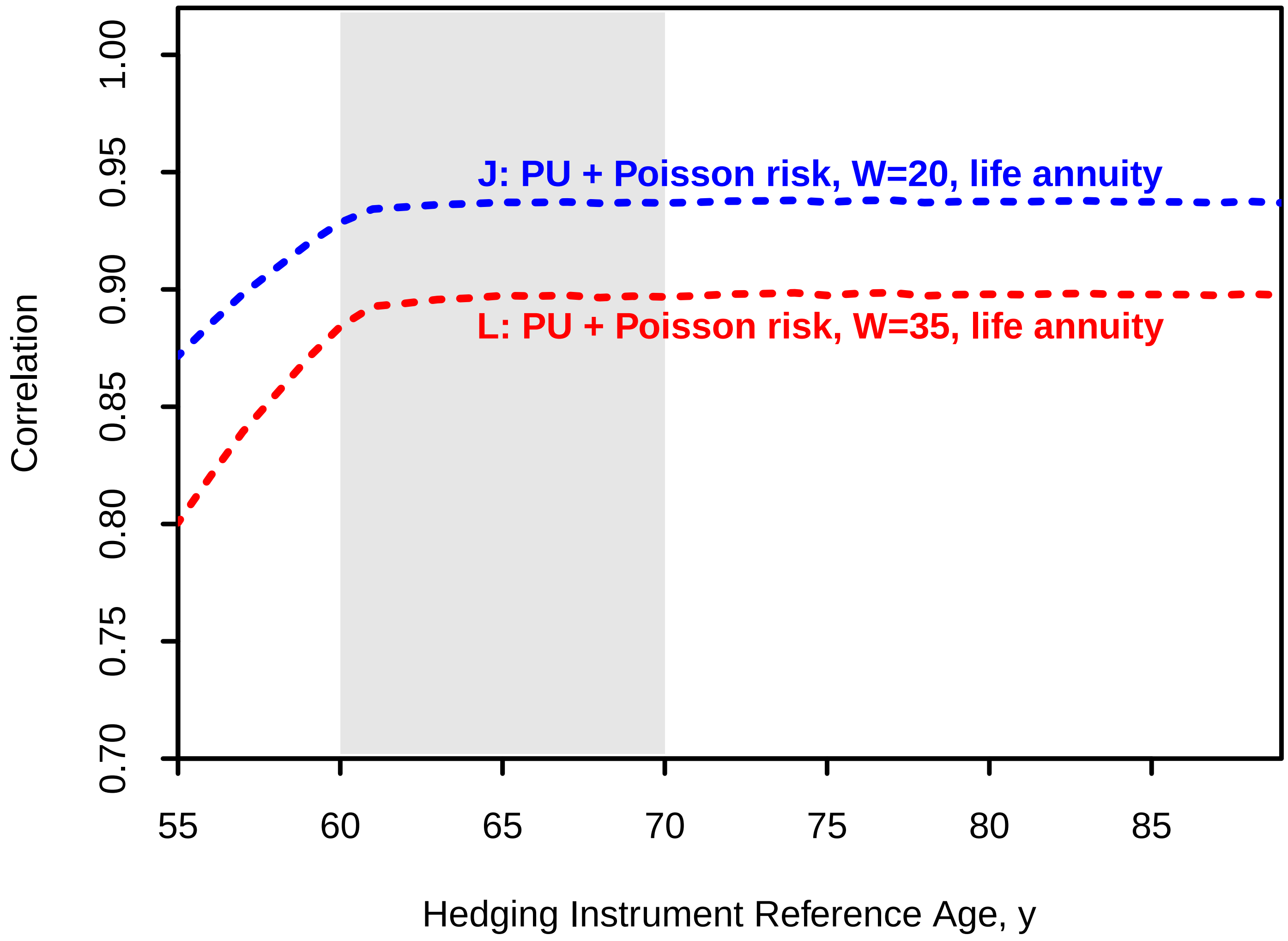
Index hedge; Full PU (recalibration risk); No Poisson



Index hedge; Full PU (recalibration risk); + Poisson



Recalibration window $W = 20$ or 35; Knightian Uncertainty!



Conclusions

- Population basis risk is only part of the story
- Parameter uncertainty is significant
  - especially recalibration risk, recalibration window
- Dependence on an uncertain but common trend $\Rightarrow$

  index hedges are more effective than you might think

Discussion + questions

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Further comments $+$ work

- Robustness of optimal hedge ratios
  - Impact of sub-optimal allocation
  - Sensitivity to PC/PU etc.
- Vega hedging;

  Use of more than one hedging instrument
- Use of more recent EW data
- Models with more complex correlation structure

Simple hedging problem (?)

$$\text{liability } L = \mu + \sigma\left(\rho Z_1 + \sqrt{1-\rho^2}Z_2\right)$$

$$\text{hedging instrument } H = \mu + \sigma Z_1$$

$$\text{hedged portfolio } P(h) = L + h \times H$$

Z_1 and Z_2 are i.i.d. $\sim N(0,1)$

Q1: What is $cor(L, H)$? ρ?

Q2: What h minimises $Var(P(h))$? $-\rho$?

Q3: What is the hedge effectiveness? ρ^2?

Relevance

NEWS: 31 January 2011 (Professional Pensions)

Pall scheme completes world's first longevity hedge for non-retired members

First for:

index-based longevity hedge (10-year q-forwards)

$+$ pre-retirement pension plan members

What is $M2(T)$?

- $M2(T)$ calibrated using data from $T - W$ to T
  - otherwise $\Rightarrow$ poor risk management
  - here: *calibration window* $W = 20$ years
    - $*$ relatively short?
    - $*$ but market practice to capture recent “trend”
    - $*$ source of *Knightian Uncertainty*

life or 25-year annuity