Friedrich-Alexander-Universität
Erlangen-Nürnberg

Analyzing Surplus Appropriation Schemes in Participating Life Insurance from the Insurer's and the Policyholder's Perspective

AFIR Colloquium
Madrid, Spain
June 22, 2011

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Introduction
Motivation

- Participating life insurance contracts:
  - Important product design in German life insurance market
  - Include interest rate guarantees and bonus mechanisms through which profits are distributed and appropriated to the policyholders

- Focus on:
  - Analysis of surplus appropriation schemes, i.e. different ways of how a given amount of surplus, determined by a reserve based distribution system, can be credited to the policyholders' contracts

Introduction
Aim of paper

- Examine surplus appropriation schemes often inherent in participating life insurance contracts:
  - Bonus system: surplus increases death and survival benefit
  - Interest-bearing accumulation: accumulates surplus on a separate account, death benefit is kept constant
  - Shortening the contract term: death and survival benefit is kept constant, survival benefit is paid earlier

  With respect to their impact on:
  - Insurer’s shortfall risk
  - Net present value from a policyholder’s viewpoint

- Conduct this analysis by considering mortality risk as well as market risk

Model framework

Insurance contract and modeling mortality probabilities

- Pool of traditional participating life insurance products:
  - Actuarially priced based on a mortality table (DAV 2008 T)

- Constant annual premium is given by equivalence principle:

$$P \cdot \ddot{a}_{x:\overline{n}|} = S_1 \cdot A_{x:\overline{n}|} \text{ with } A_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1} \cdot {}_k p_x \cdot q_{x+k} + v^n \cdot {}_n p_x \text{ and } \ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k \cdot {}_k p_x$$

- Actual mortality rates for risk measurement derived by Lee-Carter (1992) model:

$$\ln\left[\mu_x \;\; \tau \;\right] = a_x + b_x \cdot k_\tau + \varepsilon_{x,\tau} \Leftrightarrow \mu_x \;\; \tau \;\; = e^{a_x + b_x \cdot k_\tau + \varepsilon_{x,\tau}}$$

- Modification by Brouhns, Denuit, and Vermunt (2002):

$$D_{x,\tau} \sim Poisson \;\; E_{x,\tau} \cdot \mu_x \;\; \tau \quad \text{with } \mu_x \;\; \tau \;\; = e^{a_x + b_x \cdot k_\tau}$$

Model framework
Policy reserves

- Actuarial reserve for individual contract is given by

$$ {}_tV_x = S_{t+1} \cdot A_{x+t:\overline{n-t|}} - P \cdot \ddot{a}_{x+t:\overline{n-t|}} $$

- Total portfolio policy reserve is determined by

$$ PR_{t^-} = \left(N - \sum_{i=1}^{t} d_i \right) \cdot {}_tV_x $$

where N = initial number of contracts sold,
$\sum_{i=1}^{t} d_i$ = number of deaths until year t

- Development of payments over time

time	- 0	0 +	- 1	1 +	…	- t	t +	…	- n-1	n-1 +	- n	n +
age	x		x+1		Dec. 31st	x+t		Jan. 1st	x+n-1		x+n	
sum insured	0	0	S_1	0		S_t	0		S_{n-1}	0	S_n	0
premium	0	$P_0=P$	0	$P_1=P$		0	$P_t=P$		0	$P_{n-1}=P$	0	0
dividend	0	0	0	D_1		0	D_t		0	D_{n-1}	0	D_n

Model framework

Development of the asset base

- Asset portfolio follows a geometric Brownian motion

$$dA(t) = \mu \cdot A(t) \cdot dt + \sigma \cdot A(t) \cdot dW^P(t)$$

- Portfolio is composed of bonds and stocks, with a continuous one-period return of the portfolio, given by

$$r_t = a \cdot r_S + (1-a) \cdot r_B, \; with \; E(r_t) = m = \mu - 0.5\sigma^2$$

- Assets at the end of year *t*, after accounting for decrements in the portfolio of policyholders due to death, results to

$$A_{t^-} = A_{(t-1)^+} \cdot \exp(r_t) - S_t \cdot d_t, \; with \; A_{0^-} = 0, A_{0^+} = E_0 + P \cdot N$$

payment of death benefits,
S_t = sum insured, depends on surplus scheme,
d_t = number of deaths in year t

Model framework
Surplus appropriation schemes

- Actual policy interest rate credited to the policyholders for period *t*-1 until *t*, based on a smoothing scheme by Grosen and Jørgensen (2000), is given by

$$r_t^P = \max\left\{ r^G, \alpha \cdot \left(\frac{B_{t-1}^{+}}{PR_{t-1}^{-} + IA_{t-1}^{-} + RD_{t-1}^{-}} - \gamma \right) \right\}$$

where α = surplus distribution ratio

γ = target buffer ratio

r^G = guaranteed interest rate

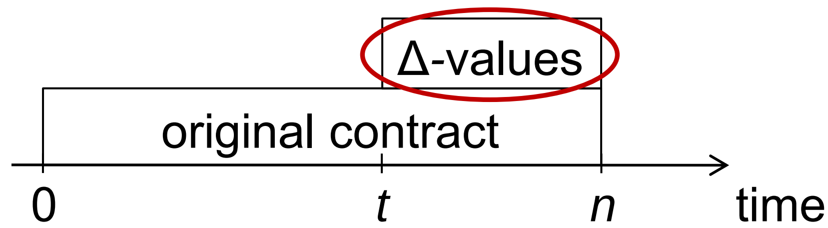
- Surplus for the *t*-th year results to $PR_{t-1}^{-} \cdot \left(r_t^P - r^G \right)$

→ amount is used differently within each of the 3 companies depending on the concrete appropriation scheme

Model framework

Appropriation scheme: bonus system

1. Bonus system:
   - Surplus is used to increase the initially guaranteed sum insured S_1 (death and survival benefit)
   - Done by using the surplus as a single premium for an additional contract of the same type with same maturity:



   - Surplus per insured results in an additional sum insured of

$$\Delta S_t \cdot A_{x+t:\overline{n-t}|} = PR_{t-1}^{-} \cdot r_t^P - r^G \Big/ N - \sum_{i=1}^{n} d_i$$

➡ increased sum insured is given by $S_{t+1} = S_t + \Delta S_t$

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Model framework

Appropriation scheme: interest-bearing accumulation

2. Interest-bearing accumulation:
   - Sum insured is kept constant, i.e. $S_t = S_1, \forall t = 1, \ldots, T$
   - Surplus is accumulated on a separate account, IA_t
   - Forward projection of the interest-bearing accumulation account is given by

$$IA_t^- = IA_{t-1}^- \cdot 1 + r^{IA} \cdot 1 - d_t \Big/ \; N - \sum_{i=1}^{t-1} d_i + PR_{t-1}^- \cdot r_t^P - r^G$$

Adjustment for death:
funds that belonged
to policyholders that died
within the *t*-th year, are
passed to the collectivity
of policyholders

Model framework

Appropriation scheme: shortening the contract term

3. Shortening the contract term:
   - Surplus is used to decrement the remaining years to maturity (contract term *n*(*t*) is a function of time *t*)
   - Reduce the contract term for full years only

$$RD_{t^-} = RD_{t-1^+} \cdot \; 1 + r^{RD} \; \cdot \; 1 - d_t \Big/ \; N - \sum_{i=1}^{t-1} d_i \;\; + PR_{\;t-1^-} \cdot \; r_t^P - r^G \;\; , RD_0 = 0$$

   - Policy reserve incl. surplus for an individual insured

$${}_tV_x^{surplus} \;\; n \;\; t-1 \;\; = {}_tV_x \;\; n \;\; t-1 \;\; + RD_{\;t-1^-} \Big/ \; N - \sum_{i=1}^{t-1} d_i$$

   - Determine the years to reduce the contract term

$$k_{\max} \;\; t \;\; = \max_{k \in K \;\; t} \;\; k : {}_tV_x^{surplus} \;\; n \;\; t-1 \;\; - {}_tV_x \;\; n \;\; t-1 \; - k \;\; \geq 0$$

$$\text{with } K \;\; t \;\; = \;\; 0,\ldots,n \;\; t-1 \;\; - t$$

new policy period is given by $n \;\; t \;\; = n \;\; t-1 \;\; - k_{\max} \;\; t$

Model framework

Evaluating the surplus appropriation schemes

- Shortfall probability (assets not sufficient to cover liabilities):

$$SP = P\left(T_s \le T\right), with\ T_s = \inf\left\{t : A_{t^-} < PR_{t^-} + IA_{t^-} + RD_{t^-}\right\},\ t = 1,\ldots,T$$

- Net present value from a policyholder's viewpoint = expected value of insurance benefits - premiums

$$NPV = E^Q\left(\sum_{t=0}^{T-1}\left({}_t p'_x \cdot q'_{x+t} \cdot S_{t+1} \cdot e^{-(t+1)\cdot r_f} - {}_t p'_x \cdot P \cdot e^{-t\cdot r_f}\right)\cdot 1\left\{T_s > T\right\}\right)$$

$$+E^Q\left({}_T p'_x \cdot \left(S_T + \left(IA_{T^-} + RD_{T^-} + TB_T\right)\cdot \frac{1}{N - \sum_{i=1}^{T} d_i}\right)\cdot e^{-T\cdot r_f}\cdot 1\left\{T_s > T\right\}\right)$$

$$+E^Q\left(\sum_{t=0}^{T-1}\left({}_t p'_x \cdot \left(A_{t^+}\cdot e^{r_{t+1}}\right)\cdot\left(1-c\right)\cdot \frac{1}{N - \sum_{i=1}^{t} d_i}\cdot e^{-(t+1)\cdot r_f} - {}_t p'_x \cdot P \cdot e^{-t\cdot r_f}\right)\cdot 1\left\{T_s = t+1\right\}\right)$$

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Numerical results
Input parameters

- Assets

	stocks	bonds
Expected one-period returns	8.00%	6.02%
Volatility	21.95%	3.30%
Correlation between stocks and bonds	-0.1648	
Stock portion	10%	
Risk-free rate	3%	

- Liabilities

	r^G	r^{IA}	r^{RD}
Rate of interest	2.25%	0%	0%
Number of contracts sold	100,000		
Sum insured in t = 0	1		
Level premium for T = 30	0.0247		
Contract term	30		
Age of the policyholders in t = 0	35		

Numerical results
Shortfall risk

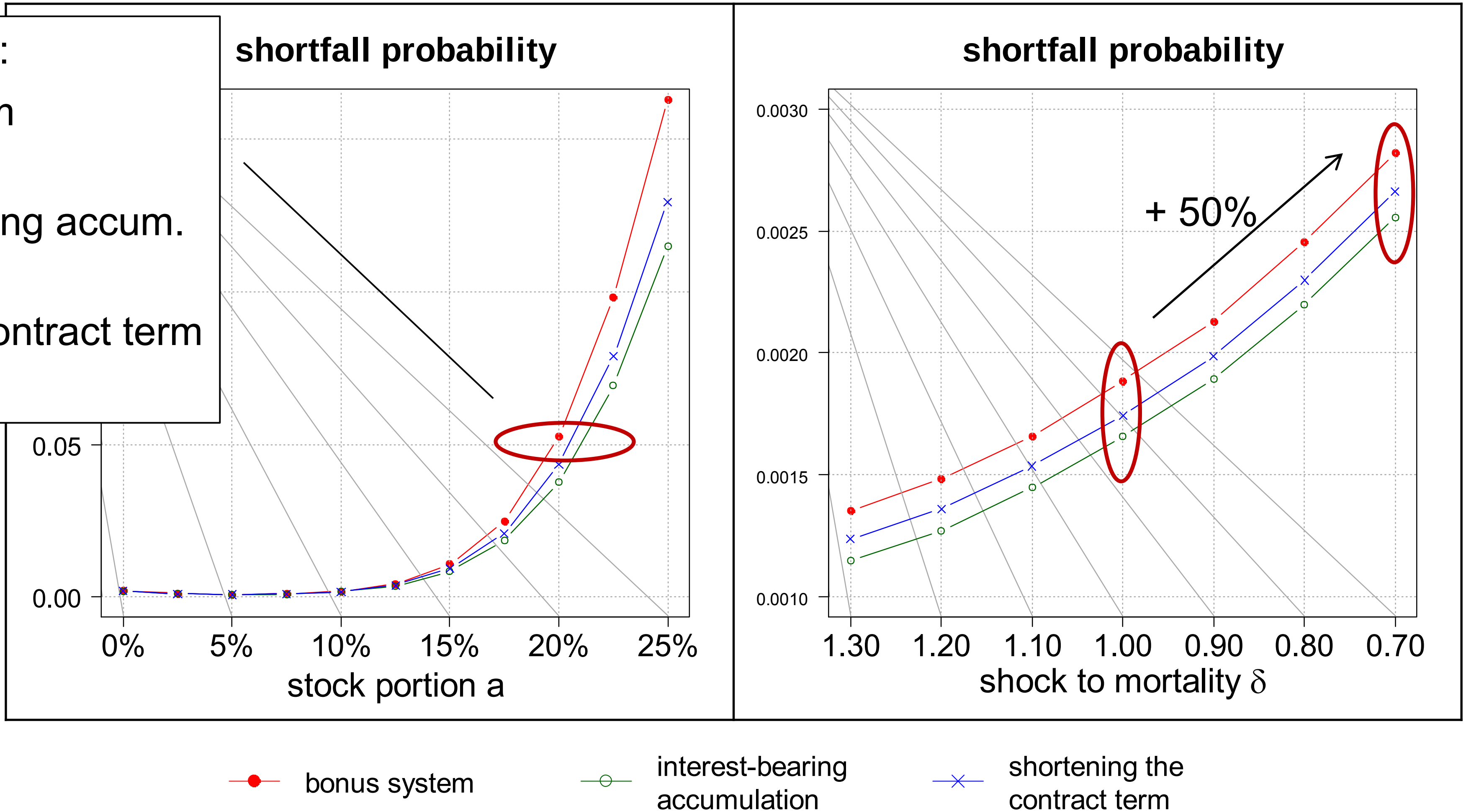
- *SP* as a function of stock portion and shock to mortality

SP = 0.05 for:

Bonus system
a = 19.81%

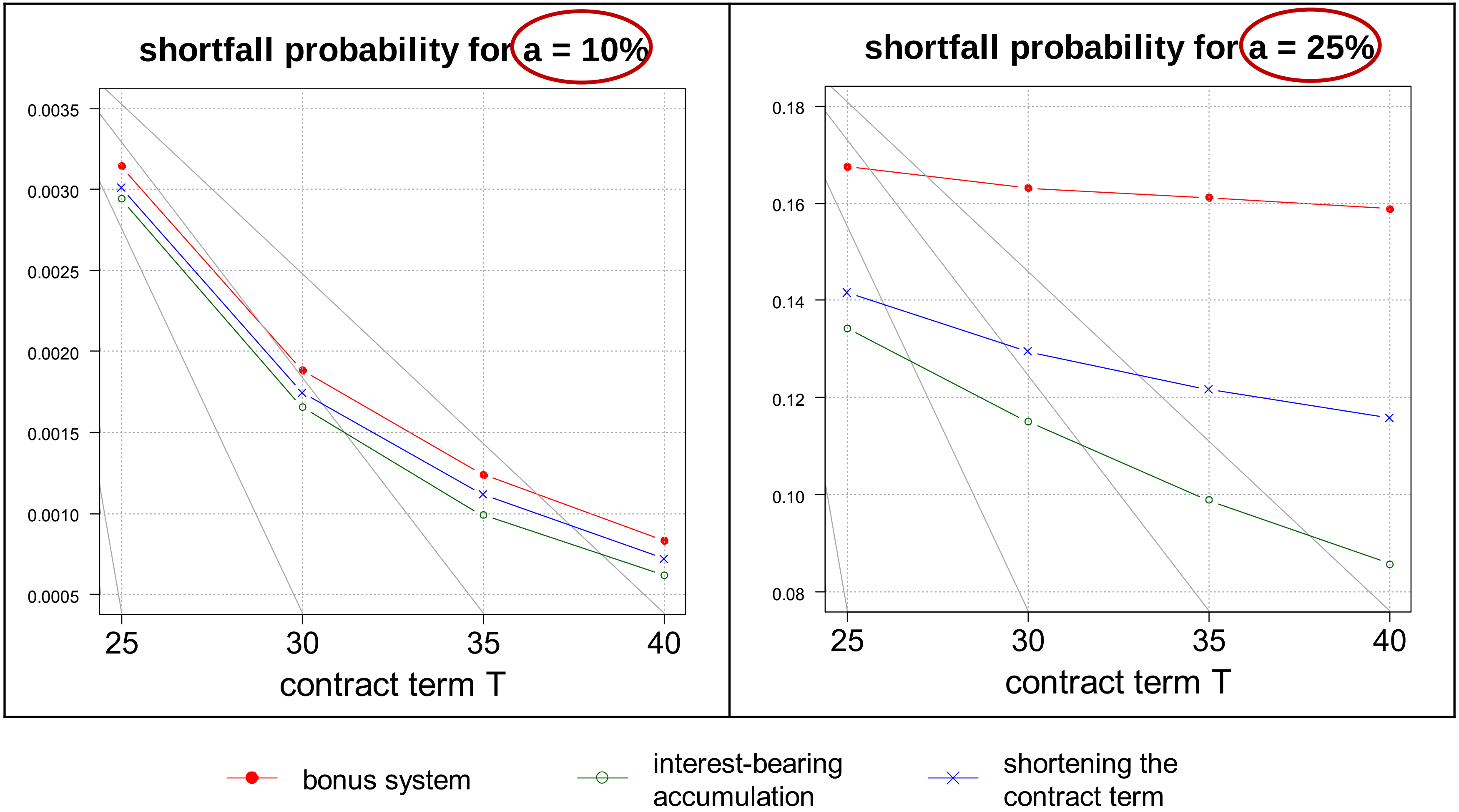
Interest-bearing accum.
a = 21.11%

Shortening contract term
a = 20.54%



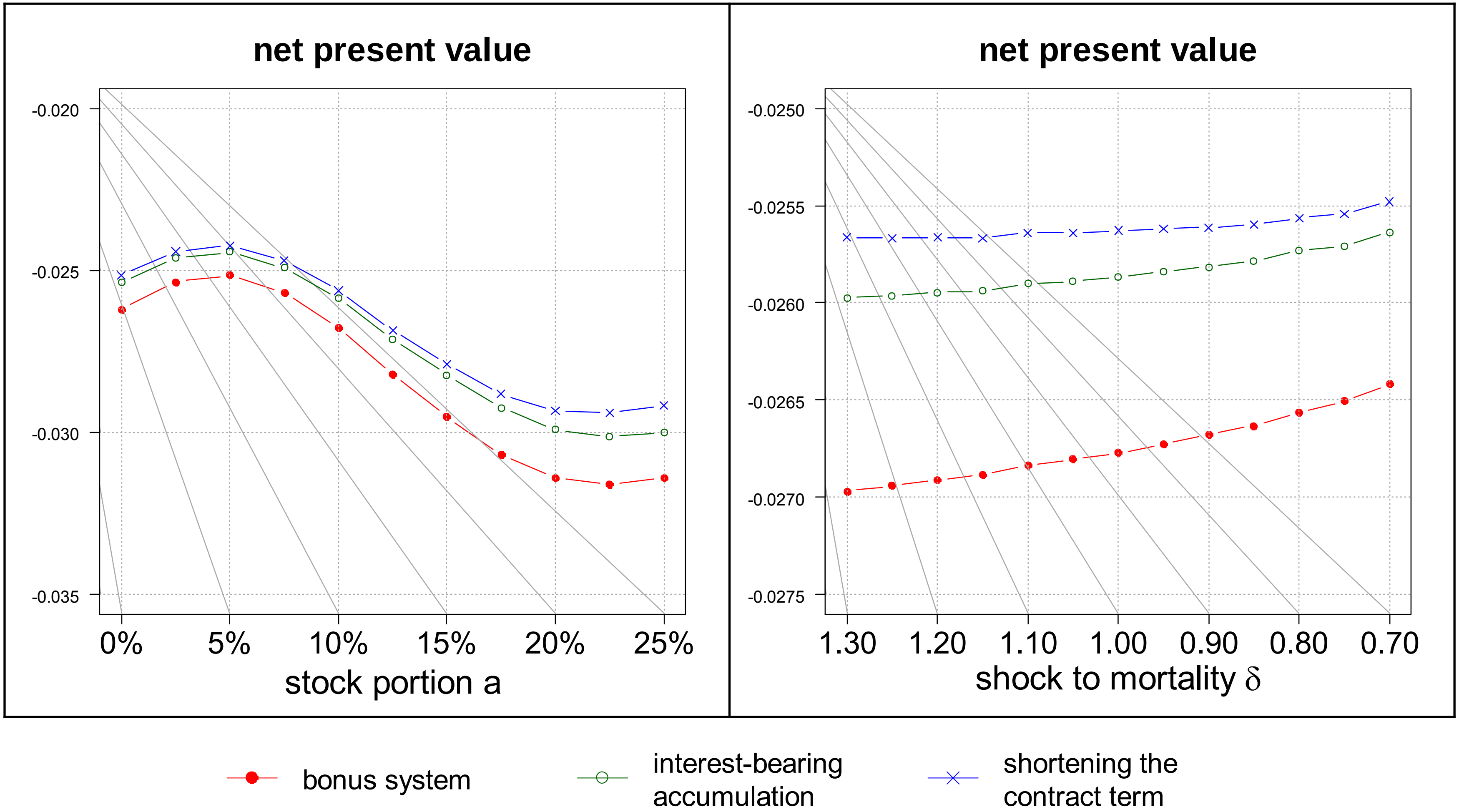
Numerical results
Shortfall risk

- *SP* as a function of contract term *T*



Numerical results
Net present value

- *NPV* as a function of stock portion and shock to mortality



Summary

- Results show: Even if the smoothing surplus distribution scheme is the same, the impact of the concrete surplus appropriation (with respect to guaranteed death/survival benefits) differs substantially:
  - Insurer's risk situation, from highest to lowest: 1) bonus system – 2) shortening contract term – 3) interest-bearing accumulation
  - Net present value from policyholder's viewpoint: 1) shortening contract term – 2) interest-bearing accumulation – 3) bonus system
- Increasing gap in shortfall risk between 3 schemes for higher stock portions and higher distributed surplus
- In contrast: shock to mortality implies similar increase in risk
- Risk reduction for longer contract periods not as effective in case of the (most common) bonus system, especially for high stock portion

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Thank you very much for your attention!

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