# Inflation Modelling

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**Abstract**

This paper reviews the inflation experience in the UK since 1923. Annual series, starting in each calander month, are considered and ARIMA models are fitted. Evidence supporting the use of an AR(1) model, as used in the Wilkie model, is presented. Monthly data is used to develop seasonal models.

Keywords :- Inflation, autoregressive models, Wilkie model

# 1 Introduction

With the increasing application of stochastic asset models it is essential that the users of these models are confident that the scenarios generated are plausible and that the models fully utilise the available data. The paper represents work in progress to critically assess time series models for assets based on the most relevant and complete data available.

The UK's inflation experience has been chosen as a start point as the data are readily available, reliable and the inflation sub-model lies at the heart of the Wilkie stochastic asset model. Although this paper deals specifically with UK inflation the techniques are general and can be applied to any asset class.

## 1.1 Outline

Section 2 will examine the experience of the UK. Section 3 will propose and test ARIMA models for describing the annual changes in inflation. Section 4 will then consider what models best describe the monthly inflation process. Section 5 contains suggestions for further work in this area.

# 2 The U.K.'s Inflation Experience

The UK has been chosen as a comparitively long and reliable series is available and an accurate description of the UK's inflation is relevant to both domestic institutions and global investors.

## 2.1 Data

Monthly inflation data is available for inflation in the UK from July 1914 to June 1947 from the Cost of Living Index (CLI) and from June 1947 onwards from the Retail Price Index (RPI). Although series can be constructed to represent inflation before these dates, for example see [2], monthly data is not available and the accuracy of the data is open to question. In addition the globalisation of the world economy, especially in the later half of the Twentieth Century suggests that data from previous centuries will be of limited relevance if the model is to be generalised to describe the inter-relation of inflation in many countries.

In the UK the data gathered for various months during the period September 1914 until July 1919 are subject to errors of up to 2.5% of the recorded value. In addition Figure 1, which shows the rate of inflation over the previous year for each month, clearly demonstrates that 1919 was an exceptional year in the UK's inflationary experience this century. The economic after effects of the First World War clearly have a considerable impact with prices rising sharply in 1918 and 1920 before declining steeply in 1921 and 1922. It appears that 1923 is the first year of relative stability after these sharp inflationary swings. I will therefore use the UK's monthly inflation data from December 1923 onwards. I shall refer to this monthly series from December 1923 onwards as the UK inflation series and denote it $Q_{1223}$.

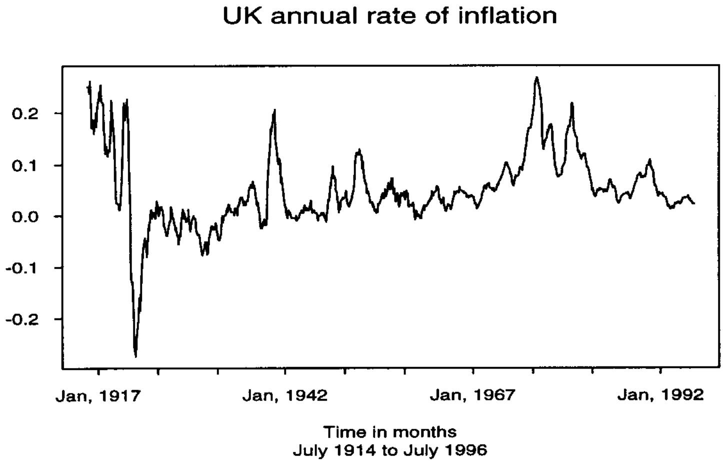


Figure 1: Annual Inflation Rate in the UK

Although the monthly inflation index should apply to the middle of the month during the period September 1914 until June 1947 there were 48 occasions when the Cost of Living Index (CLI) recorded actually related to the end of the previous month, also during this period the published figures were rounded to a significant degree. Since the introduction

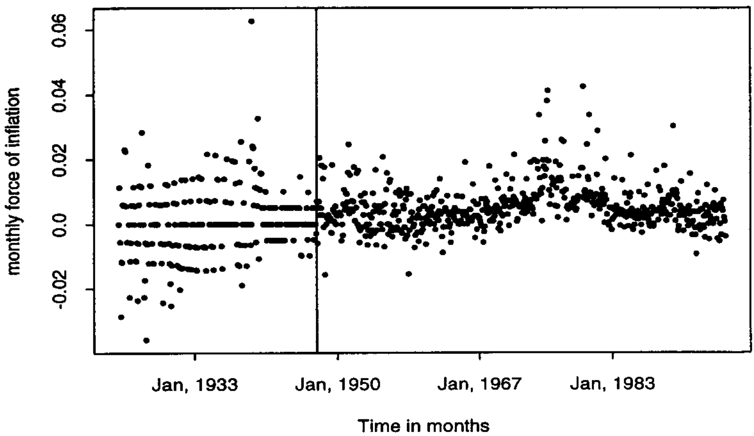


Figure 2: Monthly Force of Inflation in the UK

of the RPI in June 1947 the data has always been for a Tuesday near the middle of the month and there has been a marked increase in the level of accuracy of the published data. This can be seen in Figure 2 which shows the monthly force of inflation (the log of the monthly change in the index). Because of these features I shall also consider a subset of the UK inflation series running from June 1947 onwards which I shall refer to as the RPI index and denote $Q_{0647}$.

## 2.2 Initial Analysis

In the analysis that follows I present results based on the more comprehensive inflation series, $Q_{1223}$. I shall only cite results based on the RPI series, $Q_{0647}$, where they require comment.

The majority of probability theory of time series is concerned with stationary time series. In order to induce stationarity transformations and/or differencing of the series may be required. The inflation index of the UK shown in Figure 3 clearly has a non-linear trend. In order in induce linearity a log transform was applied, this also means that if first differences are taken we will be working with the force of inflation. The second picture in Figure 3 shows the effect of this transform. Further transforms did not appear beneficial.

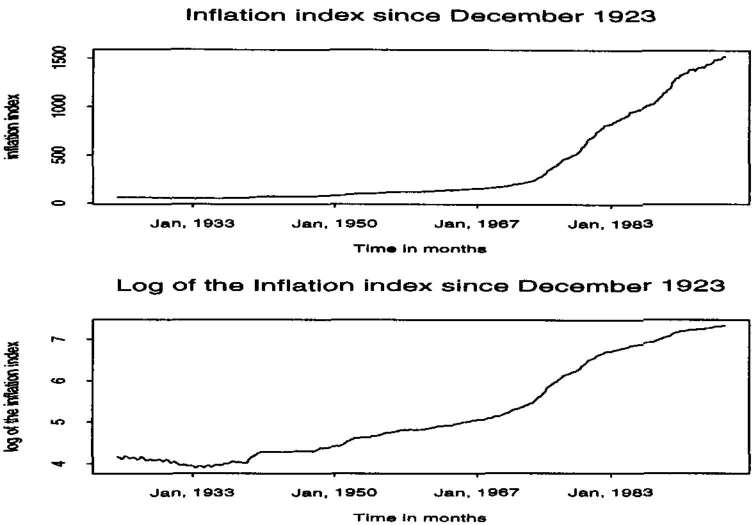


Figure 3: Inflation Index and the Log of the Inflation Index

The autocorrelation function of the of the Log of the inflation index shows high correlation between observations that slowly decays. This suggests that the series should be differenced. Taking first differences would be consistant with a linear trend in the log of the inflation index. Although there is clearly an increasing trend, it does not appear linear. Later analysis will show that further differencing is not beneficial.

Taking the first differences at a lag of one month of the Log of the inflation index gives the monthly force of inflation, $I^1_{1223}(t)$ where $t$ is a month from January 1924 onwards

$$I^1_{1223}(t) = \log Q_{1223}(t) - \log Q_{1223}(t-1)$$

or using the backwards difference operator, $\nabla$

$$I^1_{1223}(t) = \nabla \log Q_{1223}(t)$$

The autocorrelation function (acf) and the partial autocorrelation function (pacf) for $I^1_{1223}(t)$ are shown below in Figure 4. These show the characteristics of a time series with a seasonal component suggesting that the differencing should be at 12 monthly intervals. Differences in the log of the inflation index at lags l = 1,2, ...12, denoted $I^l_{1223}(t) = \nabla^l \log Q_{1923}(t)$, were taken and the acf and pacf calculated. These demonstrated the persistance of correlation at 12 monthly intervals, confirming the annual affect. Given this clearly identifiable annual affect it is worth considering what form of model is appropriate for annual inflation.

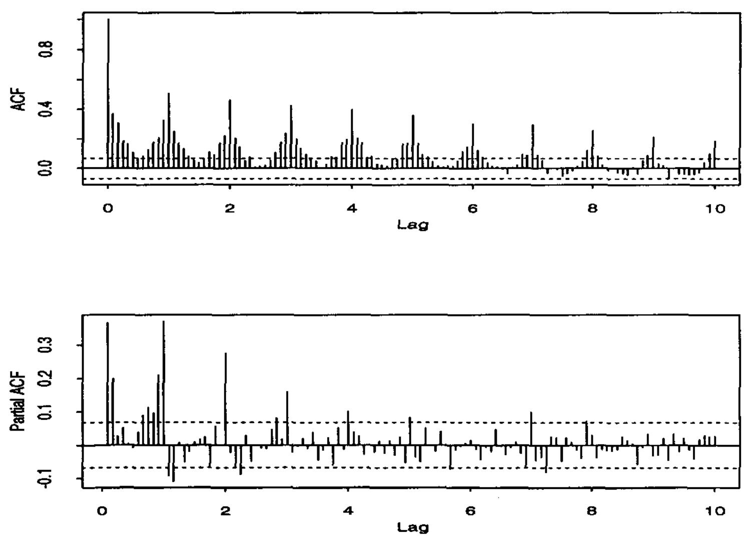


Figure 4: ACF and Partial ACF for the monthly force of inflation in the UK

# 3 Models for Annual Inflation in the UK

Having identified the annual effect in the data we can consider the 12 series of the form $I_{1223}^{l=12}(12t)$, each one representing the annual force of inflation starting from a different month. Hence in general we have

$$I_{series}^{l/m}(lt) = \nabla^l \log Q_{series}(lt)$$

Where $l$ is the lag at which the differencing take place, $m$ is the starting month for the series, and $t$ is the time measured in months. So, with $l = 12$, for our annual inflation series, $I_{1223}^{l=12/m=1}(12t)$ corresponds to the annual force of inflation from January to January. The autocorrelation and partial autocorrelation functions are shown in Figure 5 for the June annual force of inflation $I_{1223}^{l=12/m=6}(12t)$. The figure shows that the autocorrelation function (acf) decays rapidly. This pattern is consistant with a low order autoregressive (AR) process. The decay is not monotonic however with a surprising persistance in the autocorrelation at lags 3 and 4. The partial autocorrelation function (pacf) indicates that an autoregressive process is clearly required, however there is not clear evidence of higher order processes as only at lag 3 does the pacf exceed the 95% confidence interval around zero.

## 3.1 Proposed Annual Inflation Models

For each of the 12 annual series $I_{series}^{l=12/m}(12t)$ with m= 1 ... 12 ARIMA(p,d,q) models were fitted by maximum likelihood. In each case the first 4 observations were omitted so that the Akaike Information Criterion (AIC) calculated for different models with different numbers of parameters would be directly comparable. Models of orders up to p = 4, q = 3 and d = 2 were considered subject to the contraint $p + d < 5$. The AIC was used to calculate which model provided the best fit, the results are presented in Table 1.

From Table 1 it can be seen that for six of the annual inflation series $I_{1223}^{l=12/m}(12t)$ an AR(1) model was the most suitable. For the UK RPI series $I_{0647}^{l=12/m}(12t)$ six of the 12 series also indicate an AR(1) model should be used, however the start months, m, are not all the same as for $I_{1223}^{l=12/m}(12t)$. When choosing a model to describe annual inflation it makes little intuitive sense for the structure of the model to depend on which month the modelling starts. It is therefore natural to investigate whether the models suggested by the AIC provide a substantively better desciption of the process than a AR(1) model. This can be considered in two ways,

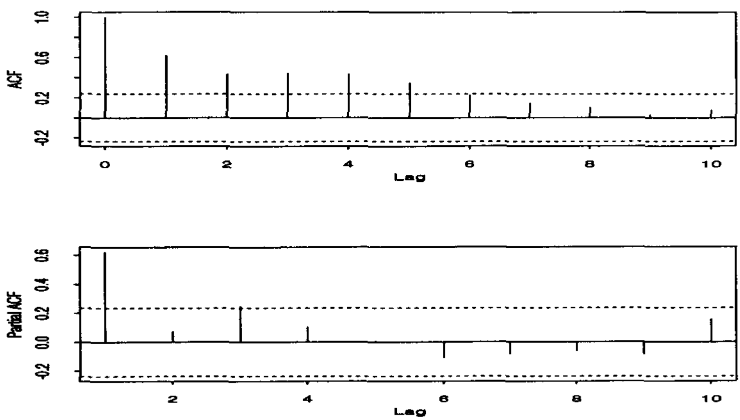


Figure 5: ACF and Partial ACF for the annual force of inflation in the UK, $I_{l=12|m=6}(t)$

1. Firstly, do the Loglikelihoods of the more general models indicate that they are a marked improvement on the AR(1) models?
2. Secondly, do the parameters of the more general model significantly differ from zero?

Using a generalisation of the likelihood ratio test, see for example [7] or [8], the test statistic $D_n$ is calculated:-

$$D_n = \frac{\ell(\underline{p}_n, \underline{x})}{\ell(p_1, \underline{x})} \quad (1)$$

$$2\log D_n = 2(\log \ell(\underline{p}_n, \underline{x}) - \log \ell(p_1, \underline{x})) \quad (2)$$

$$2\log D_n \sim \chi^2_{n-1} \quad (3)$$

where $\ell$ denotes the likelihood function , $\underline{p}_n$ is a vector of the n parameters required by the more general model and $\underline{x}$ is the vector of observations. The statistic $D_n$ is distributed

| | Results of fitting ARIMA models to | | | |
|---|---|---|---|---|
| | the UK inflation series $Q_{1223}$ | | the UK RPI series $Q_{1947}$ | |
| Month | Best Model | Reduction in AIC of Best From AR(1) | Best Model | Reduction in AIC of Best From AR(1) |
| Jan | AR(1) | | ARMA(4,2) | 3.270 |
| Feb | AR(1) | | AR(1) | |
| Mar | AR(1) | | AR(1) | |
| Apr | AR(1) | | AR(1) | |
| May | AR(1) | | ARMA(4,2) | 0.451 |
| Jun | AR(3) | 1.179 | ARMA(3,3) | 0.822 |
| Jul | ARMA(1,1) | 2.496 | ARMA(3,3) | 0.269 |
| Aug | AR(3) | 0.312 | AR(1) | |
| Sep | AR(3) | 0.207 | AR(1) | |
| Oct | AR(1) | | AR(1) | |
| Nov | AR(3) | 0.353 | ARMA(3,2) | 3.418 |
| Dec | ARMA(3,3) | 3.870 | ARMA(3,2) | 4.551 |

Table 1: Results of maximum likelihood fit of ARIMA models to the annual force of inflation

exactly $\chi^2_{n-1}$ if the residuals are normal, otherwise $D_n$ is asymptotically distributed $\chi^2_{n-1}$. Table 2 gives the distributions of $2\log D_n$ and the associated probabilities of the observations. High probabilities indicate that we should reject the hypothesis that the AR(1) model describes the data as satisfactorily as the more general model. At the 95% confidence level the table suggests that for the UK inflation series $I_{1223}^{l=12/m}$ the more general models in July ARMA(1,1) and December ARMA(3,3) provide a substantively better description of the data than an AR(1) model. The probabilities for the UK RPI series $I_{0647}^{l=12/m}$ suggest that the more general models should be retained for January, November and December. It is interesting to note that the results did not suggest that an integrated model provided a better description, this demonstrates that further annual differencing of the data is not justified when trying to induce stationarity, as claimed in section 2.2. Finally it is worth noting that if the Bayesian Information Criterion (BIC) is used, which penalises the use of extra parameters more severely than the AIC, then an AR(1) is considered to provide the best description of the data for all 12 of the models for both the UK inflation series $I_{1223}^{l=12/m}$ and the UK RPI series $I_{0647}^{l=12/m}$.

Having considered what models to use based on the AIC the parameters estimated are checked to see whether they are significantly different from zero. Table 7, in Appendix A,

| | Results of fitting ARIMA models to | | | | | |
|---|---|---|---|---|---|---|
| | the UK inflation series $Q_{1223}$ | | | the UK RPI series $Q_{1947}$ | | |
| Month | $2\log D_n \sim$ | $2\log D$ | $P[\chi^2_{n-1} \leq 2\log D]$ | $2\log D \sim$ | $2\log D$ | $P[\chi^2_{n-1} \leq 2\log D]$ |
| January | | | | $\chi^2_5$ | 13.270 | .97902 |
| February | | | | | | |
| March | | | | | | |
| April | | | | | | |
| May | | | | $\chi^2_5$ | 10.451 | 0.93658 |
| June | $\chi^2_2$ | 5.179 | .92494 | $\chi^2_5$ | 10.822 | 0.94497 |
| July | $\chi^2_1$ | 4.496 | .96603 | $\chi^2_5$ | 10.269 | 0.93204 |
| August | $\chi^2_2$ | 4.312 | .88421 | | | |
| September | $\chi^2_2$ | 4.207 | .87797 | | | |
| October | | | | | | |
| November | $\chi^2_2$ | 4.353 | .88656 | $\chi^2_4$ | 11.418 | .97775 |
| December | $\chi^2_5$ | 13.870 | .98354 | $\chi^2_4$ | 12.551 | .98631 |

Table 2: Testing whether the models suggested by AIC are substantively better than AR(1) models

shows the estimated parameters and their standard errors for each of the months where models other than AR(1) were suggested by the AIC for the UK inflation series $Q_{1223}$. The parameter values shown in bold type are those which differ from zero by at least two times their standard errors, those which are significantly different from zero. This shows that, with the exception of December, all the first autoregressive parameters are significant. Although the second autoregressive parameters are not significantly different from zero three of the five autoregressive parameters for lag 3 are significant. This appears to be the influence of the correlation at lag 3 which was shown in Figure 5 and commented on in section 2.2. Overall there appears to be limited statistical evidence for using ARMA models that are more general than the AR(1) annual model advanced by Wilkie in [1] and [2]. There does however appear to be some evidence for an autocorrelation parameter at a lag of three years. This is interesting as the "Full Standard Basis" proposed by Wilkie in [1] used lags up to 3 years for the Consols sub-model.

For comparison purposes Table 8, in Appendix B, also shows the estimated parameters and their standard errors for different months' models based on the UK inflation series $I_{1223}^{l=12/m}$. The model suggested by the AIC is shown in bold type. This provides further evidence for the use of an AR(1) model.

Table 9, in Appendix C, gives the values of the parameters and their standard errors of the ARIMA models fitted to the series $I_{0647}^{l=12/m}$.

It is interesting to note that the coefficients of the AR(1) models fitted to both $I_{1223}^{l=12/m}(12t)$ and $I_{0647}^{l=12/m}(12t)$ depend on the starting month. The estimated coefficients are smaller in the mid-Year months (April-October) and their standard error is higher. In general if $X(t)$ is AR(1), with coefficient $\beta$ and $\epsilon_t$ is the random innovation at time t with variance $\sigma^2$, then, writing the backwards shift function as $B$, we have:-

$$\begin{aligned} X(t) &= \beta X(t-1) + \epsilon_t \\ (1-\beta B)X(t) &= \epsilon_t \\ \text{which leads to} \qquad Var[X(t)] &= \frac{\sigma^2}{1-\beta^2} \end{aligned}$$

So as beta $\beta$ gets smaller the variance in $I_{1923}^{l=12/m}(12t)$, the annual force of inflation decreases. The data clearly suggests that there is less variance in the annual force of inflation measured up to the mid-Year months than there is in the annual force of inflation measured up to the Year-end/start months. This is surprising, as the annual force of inflation will have the inflationary effects of each calendar month and it just the start/end point that causes the difference in parameter values and standard errors.

## 3.2 Confirming the stationarity of the series

Although the previous section presents a standard approach to analysing time series it would preferable to confirm that the series analysed were indeed stationary. The augmented Dickey-Fuller regressions, as described by Holden and Perman 1994 [3], provide a procedure for testing for unit roots. The following regression equations are estimated, where $\nabla$ is the backwards difference operator, so that $\nabla x_t = x_t - x_{t-1}$:-

$$\nabla x_t = \alpha_0 + \alpha_1 x_{t-1} + \sum_{k=1}^{p} \gamma_k \nabla x_{t-k} + \epsilon_t \tag{4}$$

$$\nabla x_t = \alpha_0 + \alpha_1 x_{t-1} + \alpha_2 t + \sum_{k=1}^{p} \gamma_k \nabla x_{t-k} + \epsilon_t \tag{5}$$

The critical values for determining whether we can reject the existance of a unit root depends on whether the values of $\alpha_2$ , the time trend, and/or $\alpha_0$ , the intercept term can be assumed to be zero. Therefore a sequential testing approach under which joint hypotheses are considered needs to followed. The approach given below is set out in more detail in [3].

- **Step 1** Estimate the regression equation (5).
- **Step 2** Calculate the statistic $\Phi_3$ as described in Fuller 1976, [5]. The null hypothesis $H_0$ is tested against the alternative hypothesis $H_A$ where

  $$H_0 : (\alpha_0 \neq 0, \alpha_1 = 0, \alpha_2 = 0) \qquad \text{against} \qquad H_A : (\alpha_0 \neq 0, \alpha_1 \neq 0, \alpha_2 \neq 0)$$

  If the null can not be rejected then go to step 5.
- **Step 3** As the hypothesis in step 2 was rejected we know that

  $$\begin{pmatrix} \alpha_1 = 0 \\ \alpha_2 \neq 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \alpha_1 \neq 0 \\ \alpha_2 = 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \alpha_1 \neq 0 \\ \alpha_2 \neq 0 \end{pmatrix}$$

  Test the hypothesis

  $$H_0 : (\alpha_1 = 0)$$

  If the hypothesis is rejected then proceed to step 4.
- **Step 4** As the hypothesis in step 3 was rejected we know that

  $$\begin{pmatrix} \alpha_1 \neq 0 \\ \alpha_2 = 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \alpha_1 \neq 0 \\ \alpha_2 \neq 0 \end{pmatrix}$$

  In either case $\alpha_1 \neq 0$ and so there is no unit root. We can now consider a t-test on $\alpha_2$ to see if there is a trend in the series. We can also test the hypothesis that the intercept is zero, $\alpha_0 = 0$. This completes the testing process.
- **Step 5** As step 2 did not allow us to reject the null hypothesis we claim that $\alpha_1 = 0$ and $\alpha_2 = 0$. Test the null hypothesis $H_0 : (\alpha_1 = 0)$
- **Step 6** Now test for non-zero drift, $\alpha_0 \neq 0$
- **Step 7** Finally set $\alpha_2 = 0$ and try and re-confirm the conclusions reached by calculating the statistic $\Phi_1$ as described in [5]. Here the null hypotheses is $H_0 : (\alpha_0 = 0, \alpha_1 = 0)$, i.e. there is a unit root and no drift.

| Series Tested | Significance Levels | Null Hypothesis, $H_0$:- series is non-stationary |
|---|---|---|
| $\log Q_{1223}(12t)$ | 10%, 5%, 2.5%, 1% | unable to reject the $H_0$ |
| $\nabla^{12} \log Q_{1223}(12t)$ | 2.5%, 1% | unable to reject the $H_0$ |
| | 5% | Reject $H_0$ for May, June, August and September |
| | 10% | Reject the $H_0$ |
| $\nabla^{12}\nabla^{12} \log Q_{1223}(12t)$ | 10%, 5%, 2.5%, 1% | Reject $H_0$ for all start months |

Table 3: Results of Augmented Dickey-Fuller Tests for stationarity

The procedure outlined above was applied to the log of the annual inflation index series starting in different months. Table 3 summarises the results for $\log Q_{1223}(12t)$,

Table 3 shows the hypothesis that the second difference of the log of the annual inflation index series has a unit root, in other words is non-stationary, can be rejected at a 1% confidence level. It may be tempting to work with this series; however we must be careful to avoid over-differencing as this means that the model will have a more complex structure than is necessary (and will be based on fewer data points). Given the rejection of the existance of a unit root at the 10% significance level for the first difference of the log of the annual inflation index series for all months and the rejection for 8 of the 12 series at the 5% level I prefer to work with first differences, the force of inflation. This decision is supported by the autocorrelation function and the partial autocorrelation functions considered in the previous section as well as the results from fitting models using the Akaike Information Criterion (AIC).

Overall there does not appear to be convincing evidence that a more complex model than an AR(1) process is required to model annual inflation in the UK.

## 3.3 Analysis of the Residuals for Annual Inflation Models in the UK

To date however we have made the assumption (either implicitly or explicitly) that the residuals are normally distributed. This assumption needs to be examined. First consider the AR(1) models fitted to the annual data series from December 1923, $Q_{1223}$, onwards.

| | Jan | Feb | Mar | Apr | May | Jun |
|---|---|---|---|---|---|---|
| mean | 0.0002 | 0.0004 | 0.0004 | 0.0003 | 0.0003 | 0.0002 |
| std error | 0.036 | 0.0352 | 0.0375 | 0.0393 | 0.0406 | 0.043 |
| coefficent of skew | 0.596 | 0.8969 | 1.2754 | 0.9127 | 1.305 | 1.1296 |
| P[coef of skew ≤ observed value] | 0.9798 | 0.999 | 1 | 0.9992 | 1 | 0.9999 |
| kurtosis | 4.9821 | 5.2976 | 6.5188 | 4.8327 | 6.0121 | 5.0795 |
| P[kurtosis ≤ observed value] | 1 | 1 | 0.9992 | 1 | 0.9998 | 1 |
| Jacques-Bera Statistic | 15.8254 | 25.1344 | 55.8788 | 19.7939 | 46.9934 | 27.893 |
| P[J-B statistic≤ observed value] | 0.9996 | 1 | 1 | 0.9999 | 1 | 1 |

| | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|
| mean | 0.0002 | 0.0004 | 0.0004 | 0.0005 | 0.0008 | 0.0002 |
| std error | 0.0416 | 0.0414 | 0.0413 | 0.036 | 0.0358 | 0.036 |
| coefficient of skew | 1.5553 | 1.4765 | 1.5835 | 0.8129 | 0.5103 | 0.4978 |
| P[coef of skew ≤ observed value] | 1 | 1 | 1 | 0.9973 | 0.9593 | 0.9566 |
| kurtosis | 6.6946 | 5.9688 | 6.9615 | 4.1413 | 4.2835 | 4.4664 |
| P[kurtosis ≤ observed value] | 1 | 1 | 1 | 0.9744 | 0.9858 | 0.9942 |
| Jacques-Bera Statistic | 69.0053 | 51.8726 | 76.0987 | 11.6722 | 7.9549 | 9.2934 |
| P[J-B statistic≤ observed value] | 1 | 1 | 1 | 0.9971 | 0.9813 | 0.9904 |

Table 4: Results of testing the residuals of the AR1 models fitted to the $Q_{1223}$ series for normality

Table 4 shows the results of testing these residuals for normality.

Table 10, in Appendix D, shows the results of testing the residuals of the series from June 1947 $Q_{0647}$ onwards after AR(1) models have been fitted.

The tables 4 and 10 clearly show that the residuals are not normally distributed.

The largest 3 residuals for each of the 12 series comprising $Q_{1223}$ were considered. These residuals were found to correspond to 1975 (11 times) , 1940 (9), 1980 (6) ,1979 (4), 1974 (4) , 1939 (1) and 1951 (1). After these residuals were removed from the series only the July , August and December series gave Jacques-Bera statistics that were significant at the 5% level. There appears to be some evidence that the lack of normality of the residuals is due to occasional severe shocks to the system.

Table 5 repeats the analysis of residuals for the models selected using the AIC. As before the 3 largest residuals were considered for each of the six series. These residuals could be identified as predominantly coming from certain years 1975 (6 times), 1940

| | Jun | Jul | Aug | Sep | Nov | Dec |
|---|---|---|---|---|---|---|
| mean | 0.0013 | 0.0007 | 0.0017 | 0.0016 | 0.0006 | 0.0004 |
| std error | 0.0403 | 0.0406 | 0.0398 | 0.0349 | 0.0355 | 0.0338 |
| coefficient of skew | 1.4465 | 1.4985 | 1.5244 | 0.7709 | 0.6281 | 0.7039 |
| P[coef of skew $\leq$ observed value] | 1 | 1 | 1 | 0.9953 | 0.9834 | 0.9915 |
| kurtosis | 6.4699 | 6.4388 | 6.997 | 4.0535 | 4.6025 | 4.5325 |
| P[kurtosis $\leq$ observed value] | 1 | 1 | 1 | 0.9619 | 0.9967 | 0.9953 |
| Jacques-Bera Statistic | 60.3777 | 61.554 | 74.7601 | 10.3156 | 12.2646 | 12.8108 |
| P[J-B statistic $\leq$ observed value] | 1 | 1 | 1 | 0.9942 | 0.9978 | 0.9983 |

Table 5: Results of testing the residuals of the best models fitted to the $Q_{1223}$ series for normality

(5 times) and 1974 (5 times) the other 2 residuals came from 1939 and 1980. If these values were removed from the residuals before the tests for normality were carried out then there was no evidence to reject the hypothesis that the (remaining) residuals were normally distributed. This was also found to be true if just the 2 largest residuals were removed. Clearly for the models providing the best fit using the AIC the residuals were non-normal. This non-normality was caused by a few extreme residuals which correspond to dates when large inflationary shocks occurred.

Overall an AR(1) model is found to be most suitable for the annual force of inflation. The non-normality of the residuals makes it tempting to consider other models (e.g. ARCH). However, further consideration of the residuals shows that the non-normality is caused by a few large changes in the force of inflation. These shocks correspond to significant historical events, for example the 1974 oil crisis. Improvements in the modelling of inflation must take account of these changes in the inflationary conditions.

To date all the data has been used, but, by considering annual models from different calendar months, the data has been used in construct 12 separate (albeit closely correlated) models. In the next section I will consider monthly data and see what evidence exists for a monthly model of inflation that uses the full data set.

| Model | AR(1) parameter | $AR(1)_{12}$ parameter | AIC |
|---|---|---|---|
| $AR(1)_{12}$ | | 0.6702 | -3026.953 |
| standard error | | (2.590e-02) | |
| $AR(1) \times AR(1)_{12}$ | 0.9935 | -0.4546 | -5748.184 |
| standard error | (3.972e-03) | (3.115e-02) | |

Table 6: Comparison of $AR(1)_{12}$ and $AR(1) \times AR(1)_{12}$ models

# 4 Analysis of monthly Inflation

I now consider the monthly time series for the inflation indices which were introduced in section 2.1, $Q_{1923}$ and $Q_{1947}$. As before I shall work with the log of these series. Given the clear annual affect identified in section 2.2 I shall retain the structure of the AR(1) annual model and try to identify what additional monthly components are required. The autocorrelation function shown in Figure 2 suggests that an AR(1) monthly component could provide a better fit to the monthly data. Table 6 compares the models $AR(1)_{12}$ and $AR(1) \times AR(1)_{12}$ fitted to the series $\nabla^{12} \log Q_{1223}$. Where the $AR(1)_{12}$ model represents an autoregressive model with one parameter at a lag of 12 months, and the $AR(1) \times AR(1)_{12}$ model represents an autoregressive model with one parameter at a lag of one month and 1 parameter at a lag of 12 months. The models have been fitted so that the Akaike Information Criterion values are directly comparable and the standard errors of the parameter values are shown in brackets.

The considerably lower AIC value for the $AR(1) \times AR(1)_{12}$ model indicates a significant improvement. The high value of the AR(1) parameter for this model of 0.9935 is not surprising as the annual differencing will cause high correlation between successive values of the series $\nabla^{12} \log Q_{1923}$. Having found a substantive improvement by including a monthly AR(1) term it is natural to ask whether further improvements can be achieved by adding higher order monthly AR parameters and adding monthly moving average (MA) parameters.

Figure 6 shows the surface of AIC values generated by fitting models of the form $ARMA(p,q) \times AR(1)_{12}$ for p and q in the range 0 to 11. The minimum of this surface indicates which model provides the best fit according to the AIC. Although adding an AR(1) monthly parameter dramatically improves the fit adding extra AR or MA parameters leads to apparently minimal further improvement as shown by the apparent

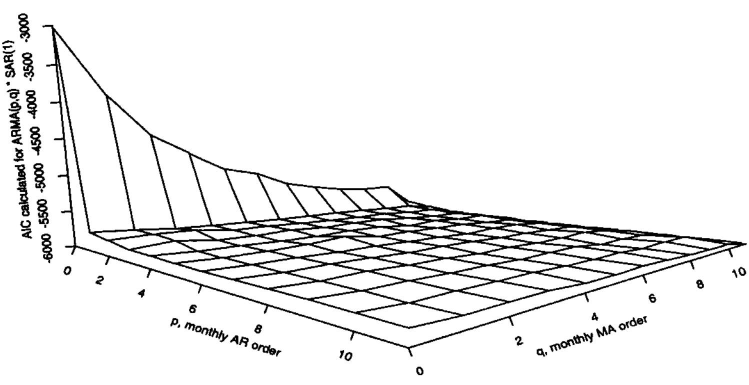


Figure 6: The calculated values for the Akaike Information criteria (AIC) for models of the form ARMA(p,q) $\times$ $AR(1)_{12}$

flatness of the surface for $p \geq 1$. Even so, the model that minimises the AIC value is ARMA(11,11)$\times AR(1)_{12}$, and the improvement is significant using a $\chi^2$ test. The model ARMA(11,11)$\times AR(1)_{12}$ does however include 8 parameters which are not significantly different from zero (if compared to twice their standard errors).

Dropping the insistance on the annual structure and selecting the optimal order for a pure autoregressive process using the Akaike Information Criterion (AIC) leads to a model with AR parameters at lags up to 25 months. The calculated value for the AIC for this model was -5926.372. This, however is not an improvement as many of the models of the form ARMA(p,q)$\times AR(1)_{12}$ gave lower AIC values while requiring fewer parameters.

Unfortunately there does not appear to be a simple seasonal ARMA model that provides the best description of the data and retains the AR(1) annual structure that was found to be preferable for modelling annual inflation. The simplest seasonal model AR(1)$\times AR(1)_{12}$ can be improved upon by adding further parameters which are not obviously spurious. Further the monthly AR(1) parameter for this model is within 2 standard errors of 1, raising the possibility that the model may be non-stationary. This leads to the conclusion that the AR(1) annual structure as used in the Wilkie model will have to be superceded if we are to build models using the full monthly series.

# 5 Further Research

The introduction mentioned that this paper is the result of work in progress and as such there are numerous areas left to consider. Set out below are a few ideas of how this work could be advanced.

- In the light of the parameter value for the monthly AR(1) component being so close to 1, the stationarity of the series should be checked.
- The residuals should be examined for normality. This should provide a better idea of what other model structures could be useful.
- Work with the series $\nabla \log Q_{1223}$, the monthly force of inflation, to see if an ARMA model, either seasonal of non-seasonal, can be developed that provides a better description of the data.
- As prices are continually changing it is reasonable to suppose that inflation is a continuous time process that (in the UK) is only observed monthly. Given the available data it is of interest to find what the plausible continuous time processes are that could lead to the observed monthly inflation values.
- Many of the items included in the "basket of goods" that comprise the inflation measure are traded in some form on futures markets, for example oil and foodstuffs. It would therefore be of interest to find if these futures markets provide useful predictors of inflation.
- Data is available on a monthly basis for many other countries. It is reasonable to expect that some inflationary pressures are common to countries and that inflation is imported and exported to a certain degree. Therefore a multivariate model simultaneously modelling the inflation in many countries should be investigated.
- The methodology could be applied to different asset classes, in particular, analysis should be extended to include currency movements and the correlation of domestic and overseas asset classes.

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# A Estimated Parameters and their standard errors

| Month | | AR1 | AR2 | AR3 | AR4 | MA1 | MA2 | MA3 |
|---|---|---|---|---|---|---|---|---|
| June | parameter values | **0.5364** | -0.0619 | **0.2540** | NA | NA | NA | NA |
| | (standard error) | 0.1173 | 0.1339 | 0.1173 | NA | NA | NA | NA |
| July | parameter values | **0.8610** | NA | NA | NA | **0.4452** | NA | NA |
| | (standard error) | 0.0915 | NA | NA | NA | 0.1611 | NA | NA |
| August | parameter values | **0.5473** | -0.0343 | 0.2262 | NA | NA | NA | NA |
| | (standard error) | 0.1190 | 0.1364 | 0.1190 | NA | NA | NA | NA |
| September | parameter values | **0.5580** | -0.0486 | 0.2273 | NA | NA | NA | NA |
| | (standard error) | 0.1190 | 0.137 | 0.1190 | NA | NA | NA | NA |
| November | parameter values | **0.7468** | -0.1965 | **0.2398** | NA | NA | NA | NA |
| | (standard error) | 0.1186 | 0.1477 | 0.1186 | NA | NA | NA | NA |
| December | parameter values | 0.1777 | 0.3702 | **0.5597** | -0.2899 | -0.5988 | 0.0706 | 0.0706 |
| | (standard error) | 0.4636 | 0.2557 | 0.1268 | 0.3048 | 0.4258 | 0.5767 | 0.5767 |

Table 7: Components and standard errors of the ARMA models suggested by the AIC fitted to the annual series $I_{1223}^{l=12/m}(12t)$

# B Estimated Parameters and their standard errors

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AR(1) | | | | | | | | | | | | |
| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| AR1 | 0.709 | 0.715 | 0.6851 | 0.665 | 0.6452 | 0.6104 | 0.6144 | 0.6343 | 0.6355 | 0.7131 | 0.7298 | 0.7127 |
| s.e. | 0.0855 | 0.0848 | 0.0883 | 0.0906 | 0.0926 | 0.0961 | 0.0957 | 0.0944 | 0.0943 | 0.0857 | 0.0835 | 0.0851 |
| AR(2) | | | | | | | | | | | | |
| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| AR1 | 0.6956 | 0.6981 | 0.6407 | 0.6208 | 0.6027 | 0.5576 | 0.4998 | 0.5702 | 0.5786 | 0.6862 | 0.7444 | 0.7029 |
| s.e. | 0.1212 | 0.1212 | 0.121 | 0.121 | 0.121 | 0.1208 | 0.1193 | 0.1216 | 0.1217 | 0.1221 | 0.1221 | 0.1212 |
| AR2 | 0.0184 | 0.0231 | 0.0628 | 0.065 | 0.0647 | 0.085 | 0.1816 | 0.0987 | 0.0868 | 0.0365 | -0.0193 | 0.0134 |
| s.e. | 0.1212 | 0.1212 | 0.121 | 0.121 | 0.121 | 0.1208 | 0.1193 | 0.1216 | 0.1217 | 0.1221 | 0.1221 | 0.1212 |
| AR(3) | | | | | | | | | | | | |
| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| AR1 | 0.6873 | 0.6909 | 0.6285 | 0.6083 | 0.5882 | 0.5364 | 0.4608 | 0.5473 | 0.558 | 0.6774 | 0.7468 | 0.6923 |
| s.e. | 0.1189 | 0.1196 | 0.1201 | 0.1192 | 0.1183 | 0.1173 | 0.1185 | 0.119 | 0.119 | 0.119 | 0.1186 | 0.1186 |
| AR2 | -0.1216 | -0.0948 | -0.0286 | -0.0535 | -0.0732 | -0.0619 | 0.0712 | -0.0343 | -0.0486 | -0.1217 | -0.1965 | -0.1323 |
| s.e. | 0.1444 | 0.1456 | 0.1422 | 0.14 | 0.1379 | 0.1339 | 0.1308 | 0.1364 | 0.137 | 0.1442 | 0.1477 | 0.1444 |
| AR3 | 0.1981 | 0.1649 | 0.139 | 0.183 | 0.2197 | 0.254 | 0.211 | 0.2262 | 0.2273 | 0.2251 | 0.2398 | 0.2085 |
| s.e. | 0.1189 | 0.1196 | 0.1201 | 0.1192 | 0.1183 | 0.1173 | 0.1185 | 0.119 | 0.119 | 0.119 | 0.1186 | 0.1186 |
| ARMA(1,1) | | | | | | | | | | | | |
| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| AR1 | 0.7399 | 0.7461 | 0.7771 | 0.8017 | 0.812 | 0.8459 | 0.861 | 0.8396 | 0.8368 | 0.7797 | 0.6979 | 0.7359 |
| s.e. | 0.1151 | 0.1129 | 0.1105 | 0.1068 | 0.1063 | 0.0988 | 0.0915 | 0.0991 | 0.1002 | 0.107 | 0.1195 | 0.1152 |
| MA1 | 0.0648 | 0.0665 | 0.1871 | 0.2696 | 0.318 | 0.4284 | 0.4452 | 0.3881 | 0.3837 | 0.1458 | -0.0707 | 0.0489 |
| s.e. | 0.1707 | 0.1692 | 0.1725 | 0.172 | 0.1726 | 0.1673 | 0.1611 | 0.1681 | 0.1691 | 0.169 | 0.1664 | 0.1700 |

Table 8: The estimated parameters and their standard errors for various ARMA models fitted to the annual series $I_{1223}^{l=12/m}(12t)$

# C Estimated Parameters and their standard errors

| AR(1) | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AR1 | 0.7555 | 0.7592 | 0.7558 | 0.6936 | 0.6736 | 0.6535 | 0.6928 | 0.7006 | 0.7171 | 0.7406 | 0.7583 | 0.7573 |
| s.e. | 0.0988 | 0.0981 | 0.0987 | 0.1086 | 0.1114 | 0.1128 | 0.1075 | 0.1076 | 0.1051 | 0.1013 | 0.0983 | 0.0985 |

| AR(2) | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AR1 | 0.8181 | 0.8271 | 0.83 | 0.692 | 0.635 | 0.6122 | 0.6019 | 0.6444 | 0.6738 | 0.7293 | 0.793 | 0.8283 |
| s.e. | 0.1502 | 0.1501 | 0.15 | 0.1508 | 0.1505 | 0.1488 | 0.1477 | 0.1502 | 0.1505 | 0.1507 | 0.1506 | 0.1500 |
| AR2 | -0.0856 | -0.0918 | -0.1004 | 0.0024 | 0.0578 | 0.0654 | 0.1368 | 0.0843 | 0.0633 | 0.016 | -0.0475 | -0.0973 |
| s.e. | 0.1502 | 0.1501 | 0.15 | 0.1508 | 0.1505 | 0.1488 | 0.1477 | 0.1502 | 0.1505 | 0.1507 | 0.1506 | 0.1500 |

| AR(3) | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AR1 | 0.8362 | 0.8507 | 0.8551 | 0.6945 | 0.6314 | 0.6082 | 0.6004 | 0.6416 | 0.6719 | 0.7286 | 0.7984 | 0.8476 |
| s.e. | 0.1485 | 0.1476 | 0.1475 | 0.1487 | 0.1502 | 0.1483 | 0.1491 | 0.1507 | 0.1507 | 0.1506 | 0.1501 | 0.1484 |
| AR2 | -0.2232 | -0.2585 | -0.2712 | -0.1104 | 0.006 | 0.003 | 0.1301 | 0.0629 | 0.0446 | -0.0191 | -0.119 | -0.2403 |
| s.e. | 0.1919 | 0.1916 | 0.1916 | 0.1811 | 0.1779 | 0.1738 | 0.1728 | 0.1788 | 0.1814 | 0.1863 | 0.1916 | 0.1924 |
| AR3 | 0.1707 | 0.2024 | 0.2068 | 0.1636 | 0.0829 | 0.1018 | 0.0116 | 0.0353 | 0.0294 | 0.0508 | 0.0934 | 0.1777 |
| s.e. | 0.1485 | 0.1476 | 0.1475 | 0.1487 | 0.1502 | 0.1483 | 0.1491 | 0.1507 | 0.1507 | 0.1506 | 0.1501 | 0.1484 |

| ARMA(1,1) | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AR1 | 0.6863 | 0.6807 | 0.6697 | 0.697 | 0.7451 | 0.7505 | 0.8108 | 0.7788 | 0.7703 | 0.7529 | 0.7256 | 0.6814 |
| s.e. | 0.1442 | 0.1441 | 0.1464 | 0.1559 | 0.1479 | 0.1481 | 0.1219 | 0.1328 | 0.1329 | 0.1338 | 0.1368 | 0.1445 |
| MA1 | -0.1546 | -0.1791 | -0.1954 | 0.0065 | 0.1324 | 0.1706 | 0.2258 | 0.1497 | 0.1058 | 0.0258 | -0.0734 | -0.1707 |
| s.e. | 0.1959 | 0.1935 | 0.1933 | 0.2173 | 0.2198 | 0.2209 | 0.2028 | 0.2093 | 0.2072 | 0.2033 | 0.1982 | 0.1946 |

| ARMA(3,2) | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AR1 | -0.0733 | -0.1614 | 0.1103 | 0.2670 | 1.3847 | 1.2904 | 0.6395 | 0.1518 | 0.0577 | -0.1201 | 0.0829 | 0.1849 |
| s.e. | 0.2965 | 0.3476 | 0.1256 | 1.0015 | 0.3182 | 0.3948 | 0.1410 | 0.7709 | 0.5531 | 0.1399 | 0.1418 | 0.1297 |
| AR2 | 0.1311 | 0.1671 | -0.2663 | 0.6086 | -1.2232 | -1.0845 | -0.5906 | -0.0279 | 0.0379 | 0.001 | -0.1169 | -0.3045 |
| s.e. | 0.2843 | 0.3194 | 0.1200 | 0.1651 | 0.4303 | 0.5213 | 0.1457 | 0.6605 | 0.4973 | 0.1411 | 0.1413 | 0.1245 |
| AR3 | 0.2306 | 0.2706 | 0.5528 | -0.1320 | 0.6064 | 0.5489 | 0.3245 | 0.2379 | 0.2298 | 0.3724 | 0.3397 | 0.5093 |
| s.e. | 0.2627 | 0.2878 | 0.1256 | 0.5976 | 0.1983 | 0.2247 | 0.1410 | 0.5721 | 0.4434 | 0.1399 | 0.1418 | 0.1297 |
| MA1 | -1.1082 | -1.1790 | -0.7679 | -0.5459 | 0.7109 | 0.6673 | 0.0800 | -0.5652 | -0.7233 | -1.0803 | -1.0043 | -0.8820 |
| s.e. | 0.2368 | 0.2942 | 0.0008 | 0.9726 | 0.3597 | 0.4317 | 0.0009 | 0.7284 | 0.5051 | 0.0016 | 0.0010 | 0.0010 |
| MA2 | -0.7062 | -0.6854 | -0.9999 | 0.4540 | -0.6484 | -0.6017 | -0.9999 | -0.5276 | -0.5708 | -0.9999 | -0.9999 | -0.9999 |
| s.e. | 0.2183 | 0.2520 | 0.0008 | 0.9726 | 0.3487 | 0.4023 | 0.009 | 0.694 | 0.5015 | 0.0016 | 0.0010 | 0.0010 |

Table 9: Parameters and their standard errors for the ARMA models fitted to the annual series $I_{0647}^{l=12/m}(12t)$

# D Testing the residuals for normality after fitting AR(1) models to the annual series $I_{0647}^{l=12/m}(12t)$

| | Jan | Feb | Mar | Apr | May | Jun |
|---|---|---|---|---|---|---|
| mean | -0.0001 | 0.0002 | 0.0003 | 0.0006 | 0.0001 | -0.0013 |
| std error | 0.0317 | 0.0307 | 0.0309 | 0.0348 | 0.0362 | 0.0392 |
| coefficient of skew | 1.1293 | 1.0299 | 1.0064 | 0.6041 | 0.7578 | 0.394 |
| P[coef of skew $\leq$ observed value] | 0.9992 | 0.998 | 0.9976 | 0.9546 | 0.983 | 0.8674 |
| kurtosis | 4.3294 | 4.0104 | 4.2481 | 3.7677 | 3.8553 | 3.0997 |
| P[kurtosis$\leq$ observed value] | 0.9686 | 0.9213 | 0.9596 | 0.8587 | 0.8843 | 0.5561 |
| Jacques-Bera statistic | 20.3189 | 15.5712 | 16.5939 | 6.0623 | 8.9598 | 1.8662 |
| P[J-B statistic$\leq$ observed value] | 1 | 0.9996 | 0.9998 | 0.9517 | 0.9887 | 0.6067 |

| | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|
| mean | -0.0008 | -0.0007 | -0.0006 | -0.0006 | -0.0002 | -0.0001 |
| std error | 0.036 | 0.0361 | 0.0349 | 0.033 | 0.0317 | 0.0317 |
| coefficient of skew | 0.6553 | 0.9159 | 1.0086 | 1.0641 | 1.2745 | 1.2225 |
| P[coef of skew $\leq$ observed value] | 0.9681 | 0.9948 | 0.9976 | 0.9986 | 0.9998 | 0.9997 |
| kurtosis3.3629 | 3.7447 | 3.9943 | 4.314 | 4.45 | 4.4051 | |
| P[kurtosis$\leq$ observed value] | 0.6961 | 0.8513 | 0.918 | 0.967 | 0.9788 | 0.9754 |
| Jacques-Bera statistic | 5.4712 | 11.5666 | 14.9632 | 18.5071 | 25.4419 | 23.5248 |
| P[J-B statistic$\leq$ observed value] | 0.9351 | 0.9969 | 0.9994 | 0.9999 | 1 | 1 |

Table 10: Results of testing the residuals for normality of the AR1 models fitted to the annual series $I_{0647}^{l=12/m}(12t)$