Derivatives and Pension Fund Asset-Liability Management

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Abstract

Financial derivatives can substantially alter the risk and return profile of investment portfolios. However, modeling the merits of derivatives and financial calculations involving the future performance of derivatives can be easily misleading. Arbitrage free pricing techniques are required to avoid exploitation of the scenario structure underlying the performance evaluation.

Further, for strategic asset allocation decisions compatibility with long term historical data is desirable. Since risk neutral pricing models eliminate asset risk premiums they do not represent expected developments correctly. Therefore, we should add back the risk premiums in a way that is consistent with the model.

In this article we consider the role of derivatives for pension fund asset-liability management. Using a new simulation approach that integrates a term structure model with an economic model, we consider the possibilities for pension fund risk reduction with derivatives. Here we define risk in terms of the downside volatility of the asset-to-liability funding ratio. We show that derivatives can indeed substantially reduce funding risk but that the relative attractiveness of derivatives strongly depends on the regulatory treatment.

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1. Introduction

In the past, pension fund investment policy for defined benefit plans has been subject to substantial discussion. At least three different points of view prevail. First of all there is the "prudent man" point of view, frequently held by regulators. This viewpoint focuses on the relative security of assets, and therefore advocates the investment in government debt type of securities. Substantial weight is put on asset return volatility, thereby rendering equity type of investments less attractive. Important within this viewpoint is the regulatory recognition of plan liabilities. Since, only accrued liabilities are legally binding, instead of projected future liabilities, pension plan liabilities are considered to resemble fixed income securities.

Second, there is the "pension cost reduction" point of view frequently encountered by pension plan sponsors. This viewpoint focuses on the nominal costs of the pension plan to the sponsor. Since equities should earn higher average returns, it is conjectured that a larger proportion of pension plan assets allocated to equities, should reduce the overall costs of funding the pension plan. Here, substantial weight is put on the long-run expected performance of equities versus fixed income instruments. Historical performance of equities favors equity type of investments, notably when inflation adjusted returns are considered. The higher returns on equity are sometimes referred to as long-term inflation hedging capabilities even though short term sensitivities of equity returns to inflation changes can be negative. Moreover, equities may be favored because of perceived long-term mean reversion suggesting reductions in the risk corresponding to long horizon equity investments.

Third, there is the "academic" viewpoint, focusing on present value considerations while assuming efficient capital markets. Here it is argued that the excess return on equities versus bonds and cash is merely a risk premium, and does not increase nor reduce the present value of plan contributions. Instead, timing of premium contributions (e.g. Black [1995]) and potential tax advantages that can be derived from fixed income investments in the pension plan are highlighted, e.g. Black

[1980] and Tepper [1981]. Further, since equity returns respond negatively to short term increases, the inflation hedging capabilities of equities are rendered to be illusory. A well known exception within this viewpoint motivating equity investments for pension plans is that of a pension plan with financially distressed sponsor. Moral hazard with respect to the limited liability of the plan (under original PBGC regulation) and with respect to its general liabilities, may rationalize risk seeking behavior. Summarizing the arguments prevailing within this viewpoint Bodie [1985, 1990] mentions three consistent rationales for equity investments:

- a desire for increases in nominal plan benefits to beneficiaries that can result from equity investments (i.e. the defined benefit plan is essentially managed as a defined contribution plan)
- a perceived net present value increase that may result from equity management due to inefficiencies in the security markets (i.e. markets are not considered efficient)
- the moral hazard argument for financially distressed firms.

Whatever, the particular point of view one is inclined to hold, it seems that financial derivatives are a means to reconcile many of the differences. It is frequently argued that by purchasing put options on an equity portfolio, the downside risk is eliminated and only upside potential remains. If this is true, than the use of financial derivatives in order to control downside should be favored by pension fund regulators. To the extent that the upside potential is unaffected by the purchase of put options, the perceived pension cost reduction remains feasible as are potential increases in the real level of future benefits.

The role of derivatives in portfolio problems has an extensive history. Merton et al. [1978], and Merton [1982] consider empirically the effects of call and put options on the return distribution of equity portfolios. Bookstaber and Clarke [1984] consider the effect of options on the return distribution using simulation methods. Ritchken [1985] considers the opportunities for risk reduction of strategies involving options relative to straight equity investments. Pelsser and Vorst [1995] examine the role of options in order to structure equity portfolios optimally in terms of shortfall constraints.

In this article we consider the role of derivatives in pension fund management. In accordance with the "academic" viewpoint, we will not argue that derivatives on equity portfolios (nor equity investments in general) contribute to the value of the fund to the sponsor. What we will consider is the extent to which derivatives are able to provide the perceived advantages of equity exposure given that we attempt to structure our portfolio optimally in the relevant measures of risk and return. Alternatively, we consider to what extent derivative strategies may serve to reduce the disadvantages of equities. In order to investigate these issues we will rely on simulation methods. In fact we suggest a simulation approach that integrates modern term structure models with a classic investment simulation approach. In this article we only consider relatively simple option strategies. Hence, the potential of option strategies may be somewhat understated. In a subsequent article we will consider more sophisticated option strategies.

The outline of the article is as follows. The next section is devoted to a description of the simulation model. Section three briefly considers the pricing of derivatives in the present model. Section four discusses the main assumptions on the pension fund and its portfolio constraints encountered in the simulation. Section five presents the results of the simulation. In section six we present the conclusions.

2. The simulation model

We consider a typical Dutch defined benefit plan with liabilities to present and past employees. The actuarial value of the liabilities is derived by discounting of the expected benefits at a fixed 4% interest rate. This interest rate can be thought of as a fixed 4% real interest rate that is applied to all vested benefits at a particular instant. Hence, the value of the technical reserve resembles a plan termination value with real benefits, that discounts vested benefits after the expected inflation increases.

Despite the indexing of benefits, this measure of the pension fund liabilities does not reflect all expected future increases in the value of the liabilities. In particular additional liabilities due to real salary increases, that are reflected in the more general concept of the projected benefit obligation (PBO), are ignored. The evolution of the liabilities is modeled using a stochastic model with respect to mortality, disability, and hiring and firing decisions, e.g. Boender [1995].

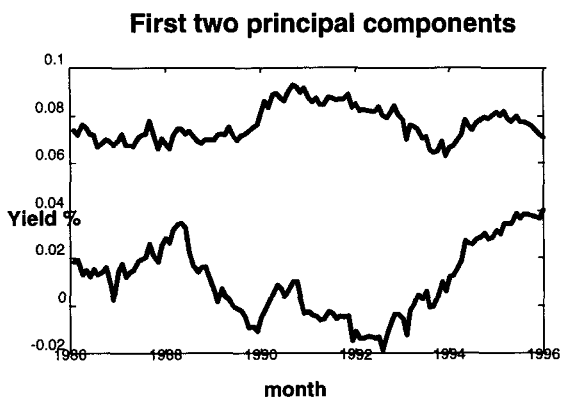
Economic quantities: interest rates, inflation and real wage increases are determined randomly using an Vector Autoregressive (VAR) Within the VAR model, the nominal interest rate evolution itself is determined using a term structure model. We can thereby incorporate both historic data (with respect to excess returns and interest rate volatilities) and current market conditions (with respect to the expected interest rate evolution). As such, our simulation approach follows the general lines of the classic Ibbotson and Sinquefield [1974] approach yet is much more sophisticated with respect to its actual implementation.

For simplicity, first consider the two-factor term structure model of Brown and Shaefer [1995]. Here, the evolution of the term structure is generated by two mean-reverting processes, one for the long interest rate and another for the spread between long and short rates. Their evolution is described by two Ornstein-Uhlenbeck processes:

$$dl = \kappa_l(\theta_l - l)dt + \sigma_l dW_t$$

$$ds = \kappa_s(\theta_s - s)dt + \sigma_s dW_s .$$

Here κ are mean reversion parameters, θ (possibly time dependent) long term means, and σ are the local standard deviations of the long term rate and the spread. We extract the unobservable long rate and spread by means of a principal components analysis on monthly zero coupon bond yield changes. These zero coupon bond yields have been estimated for the Dutch bond market over the 1986-1996 period on a monthly basis. The evolution of the estimated factors is presented in figure 1.



With respect to the excess returns, we only consider a broadly diversified stock index (Dutch All shares index) and medium term government bonds. In general we can expect that both of the term structure state variables affect asset returns. To incorporate this in the model, we consider the correlation between the excess returns on both indices and the idiosyncratic changes in the state variables.

Two additional economic variables are relevant for making projections of the pension fund evolution: inflation and real wage growth. Since expected inflation is included in the nominal interest rates (i.e. by applying the Fisher equation) we focus on the real interest rate. We let this real rate (denoted as *RR*) be represented by the difference between the short term nominal rate and realized inflation.

As suggested above, the set of economic variables is modeled as a first order Vector Autoregressive (VAR) model. The properties of VAR models have been discussed extensively (e.g. Judge et al. [1988]). By implementing an iterative Seemingly Unrelated Regression (SUR) approach we can eliminate insignificant coefficients from the system of equations:

$$X_t = m + \Omega(X_{t-1} - m) + \varepsilon_t$$

where $X_t = [l_t, s_t, ,RR_t, WR_t]$, m is the vector of historical means and ε is a vector of disturbances with mean *0* and covariance matrix Σ. The estimated coefficients are presented in table 1.

Table 1.
Regression coefficients

	constant	**spread**	**long rate**	**real rate**	**real wages**
spread	0.007	0.978	-0.091		
	(1.937)	*(54.681)*	*(-1.861)*		
long rate	0.003		0.966		
	(1.305)		*(37.776)*		
equities	0.006				
	(1.446)				
bonds	0.001				
	(0.659)				
real rate	0.004	-0.055		0.184	
	(7.353)	*(-3.838)*		*(2.086)*	
real wages	-0.002			0.667	-0.251
	(-3.239)			*(4.460)*	*(-2.855)*

Note: t-values in parentheses.

We consider the excess return variables, ER_t, BR_t, as normally distributed random variables with stationary mean and variances. Note that the term structure model is nested straightforwardly within the VAR specification but instead of satisfying the assumptions of the Brown and Shaefer model, the estimated relationship fits in with the more general Langetieg [1980] model.

Scenarios are generated by sampling from the VAR model. We implement the model using the historically fitted parameters for the two interest rate state variables. We remark that we could have used actual term structure information instead. Given an observed term structure and estimates on the term premiums included therein, we have a consensus estimate for the future level of nominal interest rates. This is similar to the Ibbotson and Sinquefield approach except that we recognize the term premiums involved. From the two factor term structure model we obtain for the yield on a zero coupon bond with time to maturity T:

$$P_t(S,L,T) = P_t(S,T)\,P_t(L,T),$$

with

$$P_t(X_t,T) = \exp\left(X_t T + \kappa_X\left(\theta^*{}_X - X_t\right)B_X(T) - \frac{1}{2}\sigma_X^2 B_X^2(T) \right)$$

and

$$B_X(T) = \frac{\left(1 - \exp(-\kappa_X T)\right)}{\kappa_T}.$$

In the more general, Langetieg, model the expression for the bond prices is slightly more complicated but resembles the structure of the equations above. Here θ^* reflects the risk neutral long term average value, i.e. the original average after a risk adjustment:

$$\theta_X^* = \theta_X + \lambda_X \frac{\sigma_X}{\kappa_X}.$$

In general we can resort to joint cross sectional and times series analysis of the term structure to estimate the particular market prices of risk. As noticed above, the term structure evolution is completely specified conditional on a set of estimated market price of risk parameters.

For the purpose of the simulation we do not specify the exact values for the market prices of interest rate risk but merely assume the historic value for the bond index. Implicitly we thereby assume a constant sensitivity to the changes in the state variables. Table 2 presents the correlation coefficients of the regression residuals for all variables defined above. From tabel 2 it is clear that in particular the long rate factor has a substantial effect on bond returns.

Table 2. Correlation matrix: unexpected changes in variables

	spread	**long rate**	**equities**	**bonds**	**real rate**	**real wages**
spread	1.0000	0.5873	-0.0155	-0.1834	-0.0371	0.0705
long rate	0.5873	1.0000	-0.1309	-0.6998	-0.1586	-0.0720
equities	-0.0155	-0.1309	1.0000	0.2051	0.0104	0.0175
bonds	-0.1834	-0.6998	0.2051	1.0000	0.1516	0.0574
real rate	-0.0371	-0.1586	0.0104	0.1516	1.0000	0.7650
real wage	0.0705	-0.0720	0.0175	0.0574	0.7650	1.0000

Since the market price of risk component causes a systematic drift in the evolution of the two interest rate variables, a first part of the average excess asset returns is explained. Of course, if the two factor model is accurate, excess bond returns are zero once we take account of their risk premium. These risk premiums should be proportional to the sensitivity with respect to unexpected changes in the state variables and to the corresponding market prices of risk. Given that we intend to use a government bond index as opposed to individual bonds, we should take into account that, in general, the index may have residual risk due to the intertemporal changes in the composition of the index.

3. The valuation of derivatives on interest rates and equity

We now briefly turn to the valuation of derivatives. First, for interest sensitive derivatives the valuation methodology can based on the two-factor interest rate model. From Jamshidian [1989] we know that, given an observed term structure of interest rates, the T-period forward rate corresponds to the T-period forward risk adjusted expected value of the state variables at time T. Further, for European style interest rate sensitive assets with pay offs at time $t+T$, the value of the asset at time t equals the present value of the expected pay off under the T-period forward risk adjusted measure. Also we can discount the expected value of the pay off at the yield to maturity of a T-period zero coupon bond. American style interest sensitive assets should be valued numerically.

For equity related derivatives an even simpler observation holds. Under no arbitrage relationships, the excess returns indicate the presence of risk premiums related to either interest rate or specific equity market risk Within the usual risk neutral valuation approach, these risk premiums vanish for the purpose of derivative pricing. Moreover, in the risk neutral valuation approach discounting takes place with respect to the money market account. Since we consider excess returns on equities relative to the money market account, we can eliminate the entire discounting procedure. Hence, we can simply calculate the expected pay off at time T using the Black-Scholes approach while assuming both a zero drift in the equity index and a zero interest rate.

4. The simulation: assumptions and constraints on the pension fund

We simulate the evolution of the pension fund over a 10 year projection period. In addition to the three basic asset classes, equities, government bonds and cash, we consider the relevance of equity strategies with put options to mitigate part of the downside risk involved with equities. The first 5 option strategies are periodically evaluated at market value basis and are defined by the exercise level and maturity period of the options.

Rebalancing of the strategy occurs at a monthly basis. In particular when we consider equities with 1 year to maturity put options purchased at "at the money" rates, we assume that at the end of each month the position is rebalanced into a new 1-year to maturity put option that is "at the money" at the new level of the stock index. Hence, gains and losses are settled at the end of each month.

The second 5 option strategies assume that 1 year options are purchased at the start of the year and held until maturity. These option strategies vary with respect to the assumed exercise level. Exercise levels, denoted by *X*, range from *90%-110%* of the stock index value. Except for the implementation of the first 5 option strategies, we consider all financial transactions of the pension fund and the second 5 option strategies to be executed at year end.

The appendix shows the average annual returns on the major asset classes and 10 different option strategies under 1,000 randomly generated scenarios. The 10 option strategies vary with respect to the time to maturity, denoted by *T*, of the options involved (1 month, 1 year and 5 years) and with respect to the exercise level of the options (*100%* of the index level, *90%* and *95%* as alternatives). From the table it is clear that rebalancing of the option strategies eliminates a considerable part of the skewness of the option strategy. Also, one of the most favorable characteristics is partly eliminated by the rebalancing procedure. Obviously, this is much less the case for the 5 strategies using options held to maturity.

With respect to the funding level of the pension fund we make the following assumptions. We assume that the initial funding level is *110%* of the technical provisions. At the end of each period mandatory contributions are required and paid into the fund if the funding level is below *100%*. When the funding level exceeds *135%* a premium reduction is granted such that the funding level returns to *135%*. Initially we place no restriction on the amount that can be withdrawn from the fund, in order to return to the *135%* funding level. Of course the particular assumptions on the funding level targets will significantly affect the risk and return position of the fund.

With respect to equity investments we assume that additional funding is required if the investment proportion allocated to equities exceeds *20%*. In that case, *30%* of the value of assets allocated to equities must be held within the fund as an asset valuation reserve. Hence, when the investment in equities equals *45%*, additional reserves equal to $30\% \times 25\% = 7.5\%$ should be maintained. This is in accordance with the present opinion of Dutch pension fund regulators.

As a consequence, equity investments in excess of *20%* induce additional funding at the first year of the projection period. This affects both the risk and the return of the fund and we will explicit consider the effects of this assumptions. First, to the sponsor the asset valuation reserve requirement amounts to a higher degree of leverage in the pension fund portfolio. This is easily observed from rewriting the equation:

$$A > MFR \times (L + 0.3 \times EQPR \times A)$$

where we use A for assets, L for liabilities, MFR to denote the mandatory minimum funding ratio level, and $EQPR$ to denote the proportion of assets invested in equities, into:

$$A > (MFR / (1\text{-}0.3{*}EQPR)) \times L.$$

For a fixed asset allocation a higher proportion in equities, $EQPR$, translates in a higher leverage ratio. Clearly, to plan participants some degree of risk reduction can be expected.

In order to facilitate the comparisons we assume that at the end of the projection period (at the end of year *10*) the funding level is restored to the initial *110%* in all cases. Clearly, additional contributions may be required, or alternatively, additional premium reductions may be granted in order to achieve the *110%* funding status at the end of year 10. We note that the timing assumption on the withdrawal of the asset valuation reserve can importantly affect the results of the equity investment. This is even more true when we realize that the asset valuation reserve may easily amount to several times the periodic premium contribution.

Before we turn to the discussion of the results we should elaborate on the evaluative criteria. As is more or less clear from the introduction it is theoretically hard to argue that the value of the fund is affected unless security market imperfections are taken into consideration. Some authors favor the use of present value calculations based on a external discount rate (e.g. the sponsor's cost of capital). In general, this is theoretically incorrect unless we assume that the financial structure of the sponsor is opaque to the security markets. Even though this can be true, it is hard to identify the correct discount rates on such an occasion. Obviously, it is far to simplistic to assume a constant discount rate regardless the choice of investments in the pension fund.

It is correct to consider the investment decisions of the in terms of risk and return. Of course there are many possible definitions on risk and return in the present context. We primarily focus on the following frequently encountered definitions:

- return: the average level of nominal contributions paid into the fund. Here we normalize by the level of actuarially fair contributions and we average across scenarios and time.
- risk: the average semi-standard deviation of the funding ratio both across scenarios and across time, with respect to a target of 1, we refer to this statistic as downside risk, e.g. Sortino and Van der Meer [1990].

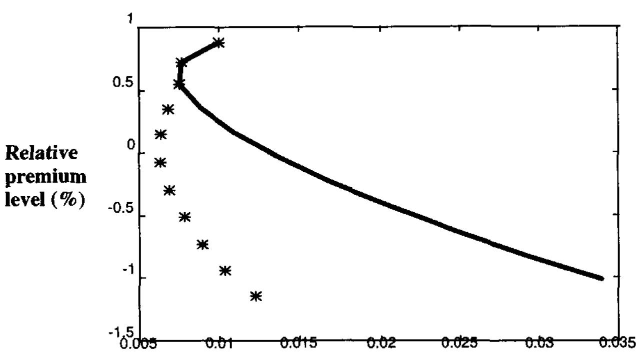
The relevance of downside risk is immediately clear form the possible non-symmetric return distributions and the possible non-symmetric effects of mandatory funding versus premium holidays.

5 Simulation results

First, we show the effect of the asset valuation reserve on the risk of equities to the beneficiaries of the fund. As shown in figure 2 this is a dramatic reduction in the downside risk of the fund. Here we show portfolios with equal spacing of an additional 10% invested in equities with the remainder in bonds. In the following we will only consider the equity and bond allocations. The results do not alter materially when we include cash investments. We do note that the risk of cash is generally lower than that of bonds, whereas the return of cash is slightly less favorable.

Note the kink in the curve due to the 20% floor with respect to the asset valuation reserve. Since initially additional funding is required, additional average return is generated by equities relative to the liabilities and the average level of premium contributions is slightly below the line where no mandatory valuation reserve is held.

Asset valuation reserve requirements:
effect on downside risk to technical provisions

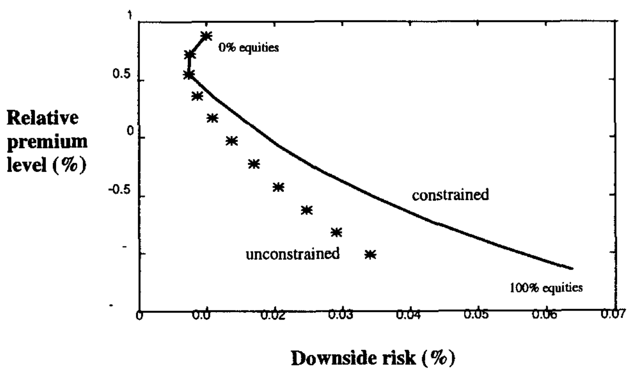


downside risk with respect to technical provisions
(-) unconstrained, (*) constrained ; total period

In fact, when measured in terms of risk to the plan participants, the asset valuation reserve provides stronger risk reduction than many of the put-option strategies can do. For instance, if we consider portfolios where equities are hedged using monthly rebalanced put options with 1 year maturity and exercise levels equal to the stock index value, than these optioned portfolios are inferior in terms of downside risk to participants versus average premium contributions.

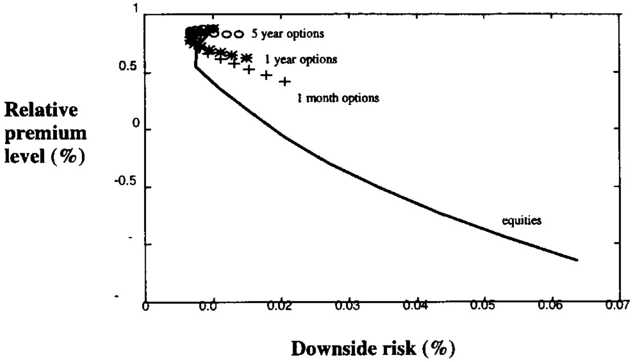
However, to the sponsor the downside risk increases due to the fact that additional leverage exits. This is illustrated in figure 3.

Asset valuation reserve requirements:
effect on downside risk to sponsor

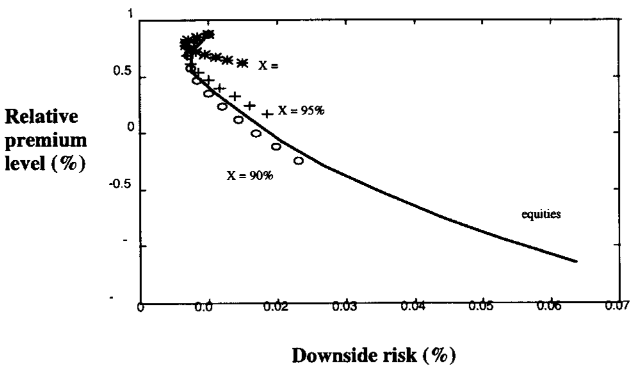


We will now consider the choice between equities and equities covered by the put options according to the previously described strategies. Here we consider the downside risk from the fund's sponsor point of view: i.e. in terms of the required level of assets relative to provisions including possible valuation reserves.

Rebalanced:
maturity variations, total period



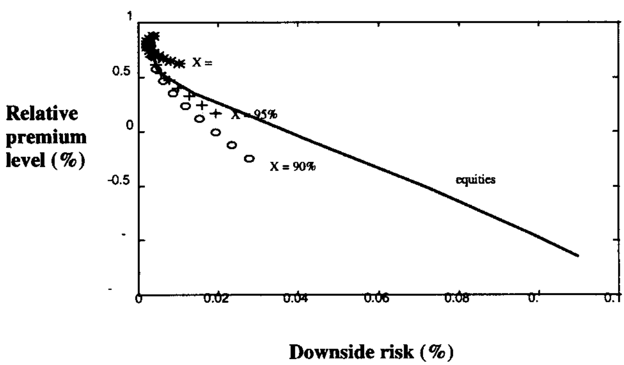
Rebalanced:
exercise level variations, total period



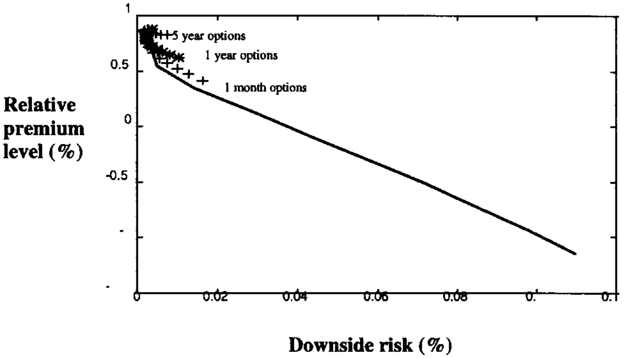
As is clear from figures 4a and 4b the risks and returns of the rebalanced put-optioned strategies vary considerably with respect to maturity and exercise ratio. For the exercise ratio held constant the return diminishes (i.e. the average level of contributions increases) with the length of the horizon of the options. On the other hand, for the maturity held constant, the return decreases (contributions increase) with the level of exercise. From this it is understood that in fact stocks without put-options provide more efficient combinations than do put-optioned strategies, given that no asset valuation reserve is required. However, due to the fact that additional funding is necessary for equity investments in excess of 20%, put-optioned strategies can be more favorable. Of course, this assumes that the level of exercise is high enough to warrant absence of additional reserve requirements.

To some extent the conclusions may be affected by the choice of the projection period. On some occasions it has been argued that the risk of stocks is notably present in the short run, and options might provide protection in the short run.

Rebalanced:
exercise level variations (single period)



**Rebalanced:
maturity variations (single period)**

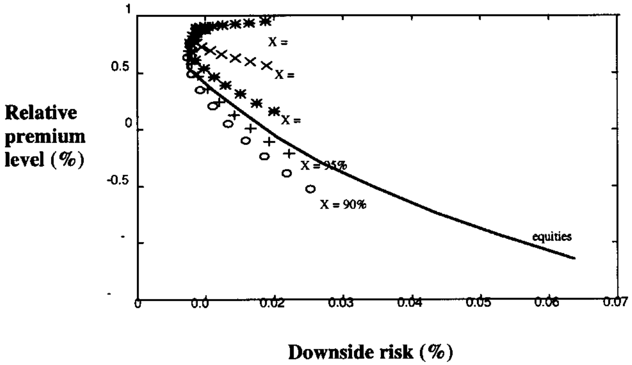


As is clear from figures 5a and 5b, this is true to some extent. When we consider the rebalanced option strategies for a projection period of only a single year, then indeed for limited equity allocations the optioned strategy may be even more favorable than uncovered equity investing.

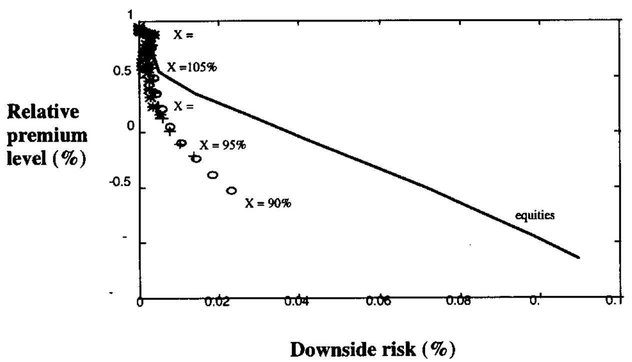
This brings us to the second 5 strategies, simply based on buy and hold. Since we seek to eliminate risk over the next period we consider only 1 year options in this strategy. Of course, even though the downside risk is limited for the period during which options are held, the risk is not limited for longer horizons.

This is illustrated in figures 6a and 6b. As is clear from figure 6a short term risk can almost entirely be eliminated by choosing appropriate exercise levels for the option strategies. In the short run, a more favorable risk return profile may result for the optioned strategies as compared to the uncovered equity investments. Similar to the rebalanced case put-optioned strategies may present a more desirable risk return trade off due to the lower capital requirements

buy/hold:
exercise level variations (total period)



buy/hold:
exercise level variations (single period)



6. Conclusions

In this article we have considered the role of derivatives in pension fund asset liability management. Using a new simulation framework we have examined the performance of equities both covered and uncovered by put options. The results indicate that for although the options can change the probability distribution of a pension funds' funding ratio significantly, the resulting distributions are not necessarily more favorable than those without options. In fact, put options are not particularly attractive for long term investors when no capital requirements related to equities are present. However, given that asset valuation reserves are required for equities the fund may use derivatives to exploit possible inconsistencies between the regulatory treatment of equities and equities covered by derivatives. Clearly, this is foremost of relevance to pension fund regulators who determine the risk based capital schemes.

We make two further qualifications. First of all, the required means for funding of a possible asset valuation reserve corresponding to equity investments may not be available. If funding can not be obtained or can not be obtained at zero cost, than derivatives may provide an answer if the additional risk of higher overall equity exposure can be born. However, when funding can not be obtained easily, this might lead one to question the desirability of additional downside risk in the first place.

Second, we assume that there is no immediate value consequence related to pension fund portfolio decisions. Of course, in realistic situations one of the critical issues may be to determine the appropriate level of risk that the plan sponsor can be bear with respect to premium contributions. If additional knowledge is available with respect to the present value consequences of unexpected increases in contributions, than more fine tuning of the portfolio is feasible. In particular, using derivatives and dynamic asset allocation strategies a much wider range of customized risk and return profiles is feasible. As suggested in the introduction to this article this will be addressed in a subsequent article.

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Appendix: Simulated return distributions

	Time	**1.00**	**2.00**	**3.00**	**4.00**	**5.00**	**6.00**	**7.00**	**8.00**	**9.00**	**10.00**
Asset class	**statistic**										
cash	*mean*	3.21%	3.55%	3.87%	4.22%	4.49%	4.67%	4.84%	4.93%	5.03%	5.11%
	st.deviation	0.60%	1.30%	1.86%	2.23%	2.48%	2.73%	2.91%	3.00%	3.01%	2.98%
	skewness	0.06	0.07	0.09	-0.10	-0.05	-0.01	0.02	0.00	0.03	0.10
	kurtosis	3.04	3.12	3.37	3.29	3.11	3.23	3.20	3.09	2.87	2.86
bonds	*mean*	3.86%	4.56%	4.90%	5.20%	5.91%	5.49%	5.96%	6.16%	6.50%	6.20%
	st.deviation	4.66%	4.86%	5.22%	5.45%	5.49%	5.33%	5.50%	5.73%	5.66%	5.56%
	skewness	0.20	0.18	0.23	0.16	0.07	0.07	0.16	0.11	0.26	0.09
	kurtosis	2.90	3.04	3.14	3.03	2.83	2.67	3.14	2.87	3.20	3.24
equities	*mean*	10.79%	10.14%	11.45%	11.47%	12.28%	11.61%	12.97%	12.18%	13.50%	12.56%
	st.deviation	17.15%	17.44%	17.65%	18.34%	17.93%	17.26%	18.24%	17.84%	17.94%	17.54%
	skewness	0.30	0.49	0.43	0.40	0.30	0.40	0.42	0.43	0.28	0.37
	kurtosis	3.03	3.33	3.03	3.04	3.31	3.24	3.30	3.49	2.97	2.90

Option strategy											
T=1;X=100;R	*mean*	5.80%	5.71%	6.35%	6.31%	6.58%	6.32%	6.83%	6.60%	7.19%	6.67%
	st.deviation	7.72%	7.87%	7.89%	8.20%	8.10%	7.78%	8.24%	7.92%	8.08%	7.80%
	skewness	0.33	0.51	0.48	0.44	0.31	0.47	0.46	0.47	0.34	0.41
	kurtosis	3.04	3.56	3.30	3.03	3.19	3.29	3.38	3.57	2.97	3.05
T=1/12;X=100;R	*mean*	5.17%	5.05%	5.56%	5.54%	5.79%	5.57%	6.01%	5.79%	6.29%	5.88%
	st.deviation	6.42%	6.53%	6.55%	6.81%	6.71%	6.44%	6.81%	6.58%	6.69%	6.48%
	skewness	0.23	0.40	0.38	0.34	0.20	0.35	0.34	0.35	0.23	0.31
	kurtosis	2.96	3.34	3.11	2.94	3.15	3.16	3.24	3.42	2.91	2.92
T=5;X=100;R	*mean*	6.10%	6.32%	7.21%	7.09%	7.26%	7.01%	7.48%	7.28%	8.14%	7.30%
	st.deviation	10.16%	10.45%	10.54%	10.79%	10.73%	10.48%	11.08%	10.46%	10.75%	10.32%
	skewness	0.57	0.76	0.66	0.61	0.54	0.73	0.68	0.73	0.59	0.67
	kurtosis	3.27	4.16	3.66	3.21	3.19	3.74	3.65	3.97	3.13	3.55
T=1;X=95;R	*mean*	7.18%	6.98%	7.81%	7.78%	8.19%	7.82%	8.54%	8.18%	8.97%	8.32%
	st.deviation	10.26%	10.44%	10.49%	10.91%	10.75%	10.32%	10.93%	10.56%	10.72%	10.40%
	skewness	0.31	0.49	0.46	0.42	0.29	0.44	0.43	0.45	0.31	0.39
	kurtosis	3.03	3.48	3.19	3.03	3.22	3.26	3.35	3.54	2.97	2.99
T=1;X=90;R	*mean*	8.41%	8.08%	9.09%	9.06%	9.60%	9.13%	10.06%	9.58%	10.54%	9.77%
	st.deviation	12.60%	12.83%	12.92%	13.43%	13.20%	12.69%	13.42%	13.02%	13.18%	12.82%
	skewness	0.31	0.49	0.45	0.42	0.29	0.43	0.43	0.44	0.30	0.38
	kurtosis	3.04	3.43	3.12	3.03	3.26	3.25	3.34	3.53	2.97	2.95
T=1;X=100;H	*mean*	7.11%	6.71%	7.56%	7.88%	8.44%	7.59%	8.79%	8.19%	9.18%	8.30%
	st.deviation	12.76%	12.99%	13.54%	13.84%	13.47%	13.21%	14.18%	13.63%	13.92%	13.66%
	skewness	1.12	1.33	1.13	1.17	1.15	1.17	1.14	1.26	0.95	1.02
	kurtosis	3.83	4.77	3.84	3.91	4.21	4.13	4.13	4.57	3.48	3.46
T=1;X=95;H	*mean*	8.31%	7.77%	8.73%	9.07%	9.73%	8.87%	10.15%	9.50%	10.57%	9.61%
	st.deviation	14.20%	14.52%	15.03%	15.29%	14.91%	14.62%	15.58%	15.03%	15.34%	15.11%
	skewness	0.88	1.05	0.89	0.95	0.90	0.92	0.91	1.00	0.74	0.79
	kurtosis	3.33	4.00	3.33	3.43	3.65	3.58	3.63	3.96	3.09	3.06
T=1;X=90;H	*mean*	9.30%	8.63%	9.73%	10.01%	10.78%	9.91%	11.27%	10.60%	11.73%	10.72%
	st.deviation	15.35%	15.77%	16.19%	16.51%	16.05%	15.75%	16.64%	16.11%	16.41%	16.19%
	skewness	0.68	0.82	0.71	0.75	0.70	0.72	0.74	0.81	0.57	0.62
	kurtosis	3.05	3.56	3.07	3.14	3.35	3.28	3.36	3.63	2.91	2.88
T=1;X=105;H	*mean*	5.84%	5.58%	6.27%	6.60%	7.00%	6.22%	7.32%	6.73%	7.64%	6.89%
	st.deviation	11.13%	11.29%	11.87%	12.18%	11.88%	11.60%	12.57%	12.08%	12.30%	11.99%
	skewness	1.40	1.67	1.40	1.44	1.44	1.46	1.40	1.55	1.20	1.29
	kurtosis	4.66	5.99	4.63	4.66	5.07	5.00	4.95	5.51	4.13	4.12
T=1;X=110;H	*mean*	4.60%	4.43%	5.04%	5.35%	5.51%	4.87%	5.87%	5.29%	6.11%	5.47%
	st.deviation	9.45%	9.61%	10.11%	10.46%	10.27%	9.94%	10.86%	10.46%	10.56%	10.25%
	skewness	1.70	2.05	1.70	1.72	1.76	1.79	1.71	1.87	1.50	1.59
	kurtosis	5.85	7.69	5.82	5.70	6.27	6.27	6.17	6.82	5.14	5.10

T = time to maturity

X = exercise price (% index value)

R = rebalance strategy

H = buy and hold strategy