

**NON-STATIONARITY IN SOME SOUTH AFRICAN
FINANCIAL AND ECONOMIC SERIES**

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Abstract

This paper examines various forms of non-stationarity in some South African financial and economic time series useful for asset-liability modelling. It examines a number of univariate models for dealing with each type of non-stationarity and lays the foundation for a multivariate modelling of these series.

Keywords: stochastic models, non-stationary, GARCH, unit roots, markov-switching models.

1. Introduction

Thomson (1994, 1995, 1996) developed a stochastic investment model of inflation rates, short-term and long-term interest rates, dividend growth rates, dividend yields, rental growth rates and rental yields based on annual data from 1960 (or later) to 1993. The model was intended to be used for the long-term projection of assets and liabilities of a South African defined benefit pension fund.

Using the methods described by Box and Jenkins (1970), univariate models were identified for each series. Analysis of the autocorrelation coefficients indicated that six out of the nine series should be differenced. These univariate models were then used to identify the transfer function cascade structure through the analysis of pre-whitened cross-correlation functions. In estimating the model's functional form and parameters, however, each variable was assumed to be stationary. In other words, no response variables were differenced in the transfer functions even though the univariate analysis indicated the need for differencing. Univariate models for variables with no inputs were revised, if necessary, by not differencing in an attempt to impose stationarity on the series.

This paper formally tests three of the nine series modelled by Thomson (1994, 1995, 1996) for unit roots against a number of alternative hypotheses. Specification and stability tests, recursive least squares tests and residual based tests, including tests for GARCH effects, are also carried out on each series. The analysis is based on annual data over the period 1960 to 1993. The series tested are

- *INFL*, defined as the mean force of inflation in year *t*;
- *EQDY*, defined as the natural logarithm of the All Share Index Dividend Yields per cent; and
- *EQDG*, defined as the mean force of equity dividend growth in year *t*.

2. ARMA models

In the stochastic investment model developed by Thomson (1996), the variables *INFL*, *EQDY* and *EQDG* are at the top of the cascade structure and are modelled as follows:

$$(1) \quad EQDY_t = 0.310 + 0.810EQDY_{t-1} + 0.198\eta_{a,t}$$

$$(2) \quad EQDG_t = 0.093 + 0.116\eta_{b,t} + 0.076\eta_{b,t-1}$$

$$(3) \quad INFL_t = 0.008 + 0.899INFL_{t-1} + 0.088EQDG_t - 0.079EQDG_{t-1} + 0.077EQDG_{t-2} - 0.069EQDG_{t-3} + 0.020\eta_{c,t}$$

where $\eta_{a,t}$, $\eta_{b,t}$ and $\eta_{c,t}$ are independent and identically distributed $N(0,1)$ random variables for all *t*. The models were estimated by maximum likelihood after centering each series.

Each series is modelled as a stationary series. The asymptotic means for $EQDY$, $EQDG$ and $INFL$ are 1.632, 0.093 and 0.095 respectively while the corresponding asymptotic variances are 0.113998, 0.019232 and 0.002873. The input series $EQDG$ contributes 27.4% to the asymptotic variance of $INFL$. Using equations (1), (2) and (3), forecast means and confidence intervals for $EQDY$, $EQDG$ and $INFL$ are illustrated in Figures 1, 2 and 3.

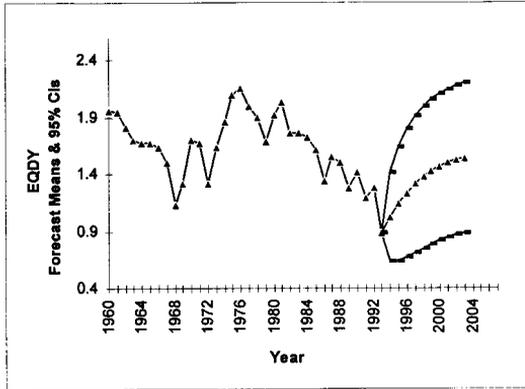


Figure 1. $EQDY$, 1960-1993, forecast means & 95% confidence intervals, 1994-2003.

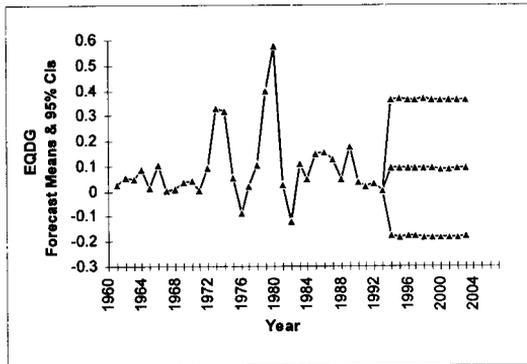


Figure 2. $EQDG$, 1961-1993, and forecast means & 95% confidence intervals, 1994-2003.

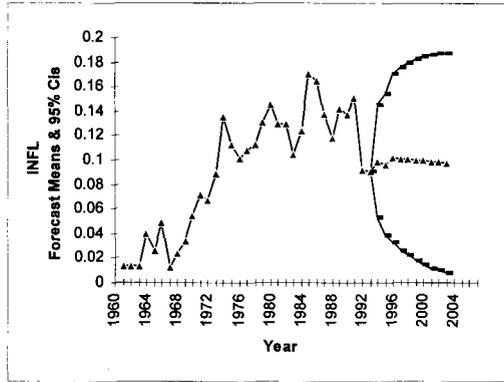


Figure 3. *INFL*, 1961-1993, and forecast means & 95% confidence intervals, 1994-2003.

Following the decision not to difference made by Thomson (1994, 1995, 1996) and using the methods described by Box and Jenkins (1970), the best univariate ARMA model for *INFL* is

$$(4) \quad INFL_t = 0.015899 + 0.853885 INFL_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim N(0; 0.022138^2)$. The model is stationary with an asymptotic mean and variance equal to 0.109 and 0.001809 respectively.

Both models for *INFL* have similar forecasts except that the forecast variance of the transfer function model is larger than the forecast variance of the univariate AR(1) model. This is caused by the variance of the input variable *EQDG* and it is interesting to note that the standard deviation of the error process in (3) is not much less than the standard deviation of the error process in (4) despite the larger coefficient of $INFL_{t-1}$ in (3) compared with (4).

In building a multivariate time series model, the purpose of developing univariate models for each of the variables is to guide subsequent multivariate modelling. How best to proceed hinges on knowing whether the individual series are $I(0)$ or $I(1)$. The decision of not to difference has a profound effect on the model's long-term forecast means and confidence intervals. In fitting an ARMA model to a series, there is an implicit assumption of weak stationarity in the series: the asymptotic forecast mean and variance are constant and equal to the unconditional mean and variance. Further, each series is mean reverting so that if a variable is currently a long way from its asymptotic mean, mean reversion will affect short term forecasts as well, as illustrated by Figure 1.

On the other hand, the implication of a unit root in a time series is that shocks to the system are permanent, trends are stochastic and forecast variances increase linearly as the lead time of the forecast increases. In the section 3, each series is formally tested for unit roots.

3. Unit root tests

Under the null hypothesis that ρ equals zero and for zero and non-zero values of α and β , Dickey and Fuller (1979, 1981) derive the limit distributions of the regression t test for ρ in the process $x_t = \alpha + \beta t + \rho x_{t-1} + \varepsilon_t$, when the equation being estimated is the above equation with various combinations of zero and non-zero values for α and β . $\{\varepsilon_t\}$ is assumed to be a sequence of independent normal random variables with mean zero and variance σ^2_ε .

Said and Dickey (1984) generalise these results in the case of ARMA(p,q) error process by fitting an appropriate augmented Dickey Fuller regression. Perron (1988) shows that neither the Dickey-Fuller τ_μ nor the Phillips-Perron $Z(\tau_\mu)$ are capable of distinguishing a stationary process around a linear trend from a process with a unit root and drift since the rejection of a unit root is unlikely if the series is stationary around a linear trend and becomes impossible as the sample size increases. For this reason, it is advisable to begin one's tests for a unit root by fitting the equation

$$(5) \quad \nabla x_t = \alpha + \beta t + \varphi x_{t-1} + \sum_{i=1}^l \theta_i \nabla x_{t-i} + \varepsilon_t,$$

where $\varphi = (1-\rho)$, α and β are unrestricted, and, where l is less than or equal to some function of the sample size. The appropriate function of the sample size is discussed in Diebold and Nerlove (1990), Mills (1993), Said and Dickey (1984) and Schwert (1987).

Dickey and Fuller (1981) show that the statistic Φ_3 is the most powerful test of the tests that permit the null model to contain a drift. Φ_3 is used to test the null $(\alpha, \beta, \rho) = (\alpha, 0, 1)$ against the alternative $(\alpha, \beta, \rho) \neq (\alpha, 0, 1)$. It should be tested prior to testing τ_τ (which is used to test the null $\rho=1$ against the alternative $\rho<1$) so that the alternative of a process stationary around a linear trend can be considered. For a full description of data-based, sequential testing procedures for unit roots, see Dolado, Jenkinson and Sosvilla-Rivero (1990), Mills (1993), Holden and Perman (1994) or Sherris, Tedesco and Zehnwrith (1996).

Equation (5) has been fitted to each series with the appropriate value of l in each case. Parameters have been estimated by conditional maximum likelihood since these estimates are consistent even when the series is non-stationary. This is not true of the unconditional maximum likelihood estimates, Hamilton (1994). The parameter estimates for equation (5) both with and without the trend term as well as the statistics Φ_1 , Φ_2 and Φ_3 (see Dickey Fuller (1981),) are reported in Table 1. Unless otherwise stated, all hypothesis tests are conducted at the 5% significance level.

$\nabla X_t =$	α	β	ϕ	θ_1	Φ_1	Φ_2	Φ_3
(1) <i>INFL</i>	0.014	0.001	-0.272			1.5237	2.0976
(2) <i>INFL</i>	0.016		-0.146		2.3533		
(3) <i>EQDY</i>	0.421	-0.004	-0.229			1.3768	1.6162
(4) <i>EQDY</i>	0.264		-0.179		1.3987		
(5) <i>EQDG</i>	0.075	0.001	-0.945	0.489		8.0661	12.0961
(6) <i>EQDG</i>	0.090		-0.936	0.481	13.248		

Table 1. Parameter estimates and statistics for equation (5).

Line (1) in Table 1 shows the results of fitting equation (5) to *INFL* with a time trend but with no lagged first differences. Φ_3 equals 2.0976 which is less than 5.91, the 10% critical value for a sample of size 25 given in Table VI by Dickey and Fuller (1981). Thus, we cannot reject $H_0: (\alpha, \beta, \rho) = (\alpha, 0, 1)$; there is no significant trend and the series possibly has a unit root with or without drift.

To support the conclusion that $\rho=1$ and since β can be assumed to be zero, we can test τ_τ against the non-standard distribution described by Fuller (1976), Table 2. This table only gives critical values for a limited number of sample sizes however MacKinnon (1991) estimates response surface regressions in which critical values depend on the sample size. τ_τ equals -1.706450 which is greater than the MacKinnon 10% critical value of -3.2109 so we cannot reject the null of a unit root. This confirms the results of our previous test.

To establish whether the series has a non-zero drift, we carry out the F-test Φ_2 described in Dickey and Fuller (1981). This tests $H_0: (\alpha, \beta, \rho) = (0, 0, 1)$ against $H_1: (\alpha, \beta, \rho) \neq (0, 0, 1)$ but since we can assume that $\beta=0$ and $\rho=1$, it is effectively a test of $\alpha=0$. For *INFL*, Φ_2 equals 1.5237 which is less than the 10% critical value of 4.67 given by Dickey and Fuller (1981), Table V. We cannot reject the null hypothesis and so α is not significantly different from zero.

The power of this test is improved by dropping the trend term in equation (5), Holden and Perman (1994); the results are shown in line (2) of Table 1. If $\alpha \neq 0$, τ_μ has a standardized normal distribution; otherwise, it has the non-standard distribution described in Fuller (1976), Table 8.5.2, middle panel, and MacKinnon (1991). For *INFL*, τ_μ equals -1.838761 which is greater than the MacKinnon 10% critical value of -2.6164 under the assumption that $\alpha=0$ so that we cannot reject the null hypothesis of a unit root.

To reestablish our conclusions regarding ρ and α , we carry out the F-test Φ_1 described in Dickey and Fuller (1981) which tests $H_0: (\alpha, \rho) = (0, 1)$ against $H_1: (\alpha, \rho) \neq (0, 1)$. For *INFL*, Φ_1 equals 2.3533 which is less than the 10% critical value of 4.12 given by Dickey and Fuller (1981), Table IV. Dropping both the intercept and time trend terms in equation (5) gives $\tau = -0.309757$ which is less than the MacKinnon 10% critical value of -1.6213, confirming our previous results. Unit root tests on the differenced series are not reported here but confirm that no further differencing is required.

Based on samples of size 100, the empirical power of the tests Φ_3 , τ_r , Φ_2 , τ_μ and Φ_1 for various values of α and ρ is given by Dickey and Fuller (1981), Table VII. With $\rho=0.8$ and $\alpha=0$, the empirical powers are 0.57, 0.46, 0.41, 0.71 and 0.78 respectively. It is clearly well worth reestablishing one's conclusions regarding $H_0: (\alpha, \rho)=(0, 1)$ by estimating equation (5) with β restricted to zero.

It should be noted that although the estimate of the intercept, α , in line (2) of Table 1 is almost significant (with a p-value of 0.0655), the model being estimated is in fact the mean reverting model (since $\rho < 1$) so that α is actually a measure of the mean level of the series. When ρ is set equal to zero, α measures the drift and is not significant (with a p-value of 0.5578).

The preceding results suggest the model

$$(6) \quad INFL_t \approx INFL_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim N(0, 0.022972^2)$. That is, the mean force of inflation in year t is a random walk with zero drift. The time trend is stochastic and the forecast variance increases linearly with the lead time.

The results of fitting equation (5) to $EQDY$ with no lagged differences are shown in lines (3) and (4) of Table 1. Φ_3 is not significant and τ_r , which equals -1.668810, is also less than the MacKinnon 10% critical value of -3.2081. There is no evidence against a unit root and we proceed by dropping the trend term. Φ_2 is not significant, nor is $\tau_\mu = -1.369578$ and Φ_1 is considerably less than the 10% critical value suggesting that the model $\nabla EQDY_t = \varepsilon_t$, $\varepsilon_t \sim N(0; 0.156381^2)$, is not inappropriate.

A quick glance at the series in Figure 1 however suggests that, at least after 1975, $EQDY$ is trended. The results of fitting equation (5) to $EQDY$ for the period 1975 to 1993 confirm this suspicion. Φ_3 equals 10.5742 which is significant at the 2.5% level. We are left with the following alternatives:

$$\begin{array}{ccccc} \beta \neq 0 & & \beta = 0 & & \beta \neq 0 \\ \rho = 1 & \text{or} & \rho \neq 1 & \text{or} & \rho \neq 1 \end{array}$$

We then test $\rho=1$ using $\tau_r \sim N(0, 1)$. Since τ_r equals -4.247466 which is significant, we are left with the two possibilities:

$$\begin{array}{ccccc} \beta = 0 & & \beta \neq 0 \\ \rho \neq 1 & \text{or} & \rho \neq 1 \end{array}$$

In either case $\rho \neq 1$ so conventional test procedures can be used. We thus fit the model

$$(7) \quad EQDY_t = \alpha + \beta t + \rho EQDY_{t-1} + \varepsilon_t$$

and test the null hypothesis that $\beta=0$. β is significant but ρ is not. When the term $EQDY_{t-1}$ is dropped, the equation becomes

$$(8) \quad EQDY_t = 3.059 - 0.057t + \varepsilon_t$$

(0.136) (0.005)

where $\varepsilon_t \sim N(0; 0.126935^2)$. Both terms are highly significant and this model seems to fit well over the sub-period 1975 to 1993. The year 1975 was chosen after inspection of the data and is thus highly correlated with the data. Christiano (1992) criticizes this method of choosing the break point since the finite sample and asymptotic distributions of the statistics depend on the extent of the correlation between the choice of breakpoints and the data, Perron (1994). The effects of structural breaks is considered in more detail in the next section.

Lines (5) and (6) in Table 1 shows the results of fitting equation (5) to $EQDG$ with and without the trend term respectively. Both Φ_3 and τ_τ , which equals -4.905872 , are significant at the 1% level. However, since we can reject the null hypothesis that $\rho=1$, $EQDG$ does not have a unit root and we can use conventional test procedures to fit a model. The results of fitting the MA(1) model are discussed in section 6 together with tests for GARCH effects.

The tests described by Phillips and Perron (1988) are more appropriate than the augmented Dickey-Fuller tests if normality, autocorrelation or heterogeneity is a problem. Phillips (1987) shows that non-parametric test statistic $Z(\tau_\mu)$ has the same asymptotic power as τ_μ . However, if the underlying process has a unit root and a moving average structure, $\varepsilon_t - \theta\varepsilon_{t-1}$, $\theta > 0$, the finite sample properties are markedly worse for $Z(\tau_\mu)$ than for τ_μ . Agiakloglou (1996) shows that there are similar problems with the statistic proposed by Bierens (1993) in that the probability of a *type I error* is much larger than suggested by the critical level of the test. If $\theta < 0$, the power of the $Z(\tau_\mu)$ statistic is greater than the power of the τ_μ statistic.

If one fits an ARIMA(0,1,1) model to the series, θ is negative suggesting that the Phillips Perron tests might be more powerful than the Dickey Fuller tests. Further, since the residuals of the MA(1) process and the series for $EQDG$ are both found to be heterogeneous, the Phillips Perron tests are more appropriate. However, since the results of the Phillips Perron tests on $EQDG$ are no different from the augmented Dickey Fuller test results, they are not reported here.

None of the Phillips-Perron tests were significant when applied to either $INFL$ or $EQDY$ supporting the results of the Dickey-Fuller tests reported above. There was, however, no evidence in either series of non-normality, autocorrelation or heterogeneity against the use of the Dickey-Fuller tests and these results are just mentioned in passing.

The purpose of the desired stochastic investment model is the long-term projection of assets and liabilities. Thus, both the short and long-run forecasts are important. The variance of the long-term forecasts resulting from the random walk model, however, casts doubt on the validity of a unit root in the models for $EQDY$ and $INFL$. It is for this reason that Thomson (1994, 1995, 1996) does not difference either univariate series or response variables. Mean reversion in short-term forecasts, however, results in excessively high initial returns for property trusts and

extremely low initial returns for equities in the Thomson model, as shown by Maitland (1996). This is equally undesirable and ARMA models do not provide the answer. One solution to the problem of a long term equilibrium when each series is $I(1)$ is the use of cointegration but it is not the intention of this paper to identify cointegrating relationships and cointegration is not discussed further in this paper.

4. Interventions

The oil shocks of the 1973 and 1979 lead to a period of high, entrenched inflation in the 80's with inflation remaining above 10%. Chow (1960) suggests a forecast test and a breakpoint test to test for a structural change in parameters. The likelihood ratio statistic for the forecast test using $INFL$ up to and including 1973 to forecast $INFL$ from 1974 to 1993 is 51.0, with a p-value of 0.000264. The likelihood ratio statistic for the breakpoint test with a break point at the end of 1973 is 8.015, with a p-value of 0.018177. Both of these tests indicate the presence of a structural break in 1973.

Recursive estimates of the parameters are illustrated in Figures 4a and 4b and also indicate the presence of a structural break at the end of 1973.

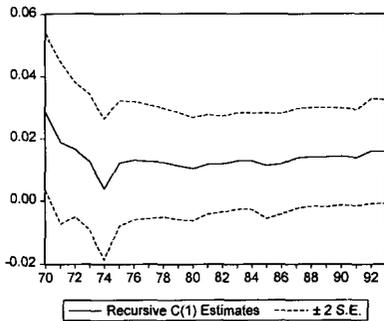


Figure 4a. Recursive estimates of the intercept in equation (3).

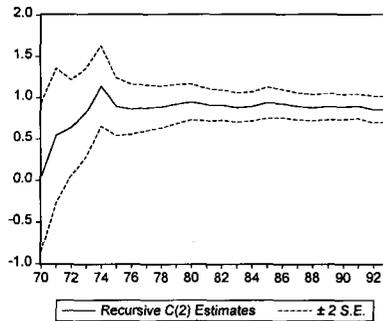


Figure 4b. Recursive estimates of the coefficient of $INFL_{t-1}$ in equation (3).

The structural break is caused by the change in the level of inflation following the 1973 oil shock and invalidates the assumption of weak stationarity. The results of fitting an AR(1) process are an AR(1) parameter which is too high, an arbitrary long-run mean of 9.5% and a forecast variance which is too large.

Box and Tiao (1975) introduce the concept of an intervention so that 'aberrant' events can be separated from the noise function and modelled as a separate component in the deterministic part of the general time series model. They model the intervention using either the "additive outlier" or "innovational outlier" model. In the additive outlier model, the transition caused by the

intervention occurs instantaneously while in the innovational outlier model the transition between one regime and the next is gradual.

The general form of the intervention model can be expressed (see SAS/ETS (1993)) as the ARIMAX model

$$(9) \quad W_t = \mu + \sum_i \frac{\omega_i(B)}{\delta_i(B)} B^{k_i} X_{i,t} + \frac{\theta(B)}{\phi(B)} \varepsilon_t$$

where

- t is an element of $\{1, 2, \dots, T\}$ and indexes time;
- W_t is the response series;
- μ is the mean term;
- B is the backshift operator;
- $\phi(B)$ is the autoregressive operator;
- $\theta(B)$ is the moving average operator;
- $X_{i,t}$ is the i^{th} input variable at time t;
- k_i is the pure time delay for the effect of the i^{th} input series;
- $\omega_i(B)$ is the numerator polynomial for the transfer function of the i^{th} input series; and,
- $\delta_i(B)$ is the denominator polynomial for the transfer function of the i^{th} input series.

$X_{i,t}$ can either be another series or an intervention variable taking the form of a pulse function, $D(TB)_t$, a step function, DU_t , or broken trend function, DT_t , where

$$\begin{aligned} D(TB)_t &= 1 \text{ if } t = T_B, 0 \text{ otherwise;} \\ DU_t &= 1 \text{ if } t > T_B, 0 \text{ otherwise;} \\ DT_t &= t \text{ if } t > T_B, 0 \text{ otherwise;} \end{aligned}$$

and where T_B refers to the time of intervention.

In the case of a single intervention, if either $\phi(B)$, $\omega(B)$ or $\delta(B)$ are polynomials of degree greater than or equal to 1, the intervention is equivalent to the innovational outlier model; otherwise, the additive outlier model describes the intervention.

5. Unit root tests revisited

Perron (1989) shows that standard unit root tests which do not allow for the presence of a structural break have little power against the alternate of no unit root when the underlying series has a structural break but no unit root. The power of these tests decreases as the magnitude of the intervention variables increases. Perron extends the augmented Dickey Fuller regression to include a pulse term, a step term and a broken trend term as follows:

$$(10) \quad \nabla x_t = \alpha + \xi DU_t + \beta t + \gamma DT_t + \delta D(TB)_t + \varphi x_{t-1} + \sum_{i=1}^l \theta_i \nabla x_{t-i} + \varepsilon_t$$

Under the null hypothesis, $\varphi=0$ and $\gamma=\beta=0$; δ measures a one-time shock to the system which persists because of the unit root and ξ measures the change in drift following the intervention. Under the alternative hypothesis of a process stationary about a broken mean level or trend, ρ is less than one and at least one of β , ξ and γ is non-zero while δ should be close to zero.

The results of estimating equation (10) for *INFL* with an intervention in 1973 are shown in Table 2, line (1). Perron (1989), Table VI.B, provides critical values for the t-statistic from the least squares estimation of (10) under the null hypothesis of a unit root. With $\lambda \approx 0.4$ where $\lambda = T_B/T = 13/33$ is the "break fraction" (see Perron (1989)), the t-statistic of -3.426504 is greater than the 10% critical value of -3.95. At this stage, we cannot reject the null hypothesis of a unit root process but an examination of the series for *INFL* suggests we should be estimating equation (10) with β and γ restricted to zero. This gives a more powerful test when the series in each regime are not trended or are without drift.

	α	ξ	β	γ	δ	φ	σ_ε
(1) <i>INFL</i>	-0.00116 (0.01633)	0.039785 (0.01909)	0.003465 (0.00208)	-0.00360 (0.002123)	0.019913 (0.02378)	-0.67559 (0.19717)	0.0199
(2) <i>INFL</i>	0.023018 (0.00828)	0.048736 (0.01811)	0	0	0.036873 (0.02200)	-0.56555 (0.17389)	0.0202
(3) <i>EQDY</i>	1.31497 (0.34375)	0.654388 (0.171344)	-0.02863 (0.014804)	-0.01165 (0.014286)	0.327018 (0.182049)	-0.70321 (0.166237)	0.1549
(4) <i>EQDY</i>	1.397165 (0.326676)	0.698910 (0.161441)	-0.03843 (0.008591)	0	0.388980 (0.164443)	-0.70849 (0.165115)	0.1539
(5) <i>EQDY</i>	1.457725 (0.350525)	0.643609 (0.171930)	-0.03678 (0.009216)	0	0	-0.73560 (0.177287)	0.1657

Table 2. Parameter estimates for equation (10) with standard errors in brackets below.

Under the null hypothesis of a unit root, Perron (1989), Table IV.B, provides critical values for the t-statistic of φ from the least squares estimation of (10) with γ restricted to zero. For *INFL*, the results of estimating equation (10) with $\beta=\gamma=0$ are shown in Table 2, line (2). The t-statistic of -3.252370 is greater than the 10% critical value of -3.44. Despite the fact that $\varphi=-0.56555$, the t-statistic is not large enough to reject the null hypothesis that $\varphi=0$ due to the small sample size. Nonetheless, it is worth fitting the intervention model assuming no unit roots and the results are as follows:

With $T_B = 13$ (1973-1960), the appropriate intervention model for *INFL* is the innovational outlier model

$$(11) \quad INFL_t = 0.023275 + 0.038309 DU_t + 0.514727 INFL_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim N(0, 0.020864^2)$. All the parameters are significant at the 5% level.

After including the intervention term, the ϕ_1 parameter reduces to 0.514727 which is considerably lower than the estimate of 0.853885 in the univariate AR(1) model given in equation (4). The unconditional forecast variance within any one regime is 0.000592 compared with 0.001809 in (4), and the mean level of the series reflects the intervention rather than a weighted average of the mean levels before and after the intervention.

The results of estimating equation (10) for $EQDY$ are shown in Table 2, line (3). The t-statistic for φ is -4.230129 which is less than the 5% critical value of -4.22 given by Perron (1989), Table VI. We are thus able to reject the null hypothesis of a unit root and conventional test procedures can be used. Since the parameter γ is not significant, there is no significant change in the trend of $EQDY$ and to reestablish our conclusions regarding φ , we re-estimate equation (10) with γ restricted to zero. The results are shown in Table 2, line (4).

The t-statistic is -4.290899 which is less than the 2.5% critical value of -4.01 given in Perron (1989), Table IV, confirming our previous conclusion that the series does not have a unit root. Using standard testing procedures, all the coefficients are significant and the model appears to fit well. However, the term $D(TB)_t$ represents a single outlier and not a structural break. Removing this term results in the model whose coefficients appear in line (5) of Table 2. All the coefficients are highly significant and this model should be preferred above the model in line (4), Table 2, since it is more parsimonious and does not require the modelling of outliers when forecasting.

The intervention model (10) is a more appropriate description of the $INFL$ and $EQDY$ series in the period 1960 to 1993 than the AR(1) models but cannot be used directly for forecasting. The intervention date was chosen ex-ante and was not modified ex-post. It is unreasonable however to assume that changes to the structural components of $INFL$ or $EQDY$ are in anyway deterministic and, in order to forecast these series, α needs to be modelled as a random variable. The distinction between this random variable describing the intercept of the series and the random error sequence $\{\varepsilon_t\}$ is that changes in regime are relatively rare.

Hamilton (1993a, 1994) describes a Markov-switching model in which the parameters of an ARMA model in regime s_t are the outcome of an unobserved N-state Markov chain and are independent of the random error sequence $\{\varepsilon_t\}$. Diebold, Lee and Weinbach (1994) generalise this model to a model in which the probability of switching from one regime, s_t , to the next, s_{t+1} , can depend on a vector of observed variables as well as s_t . Garcia and Perron (1996) apply the Markov-switching model to the U.S. real interest rate from 1961 to 1986. Estimating a Markov-switching model for the mean level in $INFL$, however, would lead to a two-state Markov chain in which state two is an absorbing state. This is considered to be unrealistic and it is felt that, for the purposes of forecasting, an N-state Markov chain could be used with the number of states, N, the mean level of each state and the transition probabilities based on future expectations. This topic is extensive particularly in the framework of multivariate time series and is not discussed further in this paper.

6. GARCH effects

The residuals for the MA(1) model given in equation (2) are shown in Figure 5 together with the actual series and the one-step-ahead forecasts.

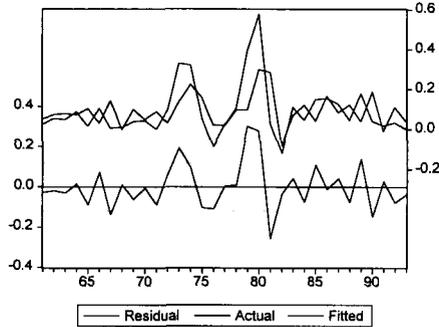


Figure 5. *EQDG*, 1961-1993, one-step-ahead forecasts and residuals for the MA(1) process.

The Jarque-Bera statistic (see Jarque and Bera (1981)), is 3.241433 with a p-value of 0.197757 indicating that the residuals are not significantly non-normal. The Breusch-Godfrey serial correlation Lagrange multiplier test (see Breusch and Pagan (1978), Godfrey (1978)) shows no evidence of residual correlation. However, the correlogram of squared residuals shows significant spikes in the ACF and PACF at lag 1 and the Ljung-Box Q-statistic at lag 2 is 7.7103 with a p-value of 0.005 indicating significant serial correlation in the squared residuals. With one lag, the ARCH LM test developed by Engle (1982) has an F-statistic of 9.767517 (p-value = 0.0039) and a TR^2 of 7.8597 (p-value = 0.0051). This indicates the presence of autoregressive conditional heteroscedasticity and is not surprising given the correlogram of squared residuals and the volatility clustering evident in the residuals in Figure 5.

Standard ARIMA models are designed to model only the mean of a series and assume that the variance is constant. In the GARCH model introduced by Engle (1982) and Bollerslev (1986), the variance is not constant but is conditional on previous squared residuals and previous estimates of the variance. Thus, the GARCH model allows one to model both the mean of a series and its variance. It is found that the MA(1)-GARCH(1,1) is a good description of the process underlying *EQDG*. The variance and mean equations for *EQDG* are given in equations (12a) and (12b) respectively.

$$(12a) \quad \sigma_t^2 = 0.000442 - 0.187215\varepsilon_t^2 + 1.198413\sigma_{t-1}^2$$

$$(12b) \quad EQDG_t = 0.068119 + \varepsilon_t + 0.624301\varepsilon_{t-1}$$

where $\varepsilon_t \sim N(0; \sigma_t^2)$.

7. Conclusion

This paper has examined only three of the nine series used by Thomson (1994, 1995, 1996) in developing a South African stochastic investment model but it is felt that the results presented illustrate a number of the shortcomings of traditional Box-Jenkins time series models.

In order to adequately model long-term relationships within the framework of Box-Jenkins ARIMAX models, Thomson was forced to avoid differencing. One of the consequences of imposing weak stationarity on the model is that series are mean reverting and give rise to undesirable short term dynamics. If the series are in fact mean reverting, it would be possible to make large profits by judiciously buying and selling when the series are not equal to their unconditional means. This goes against the efficient market hypothesis. The purpose of a stochastic investment model is the long-term projection of assets and liabilities so that both short-term and long-term forecasts are relevant. This suggests that a cointegration model would be a more appropriate tool for stochastic investment modelling.

The assumption of weak stationarity has a few more drawbacks. Firstly, because of the change in the level of inflation following the 70's oil shocks, both the transfer function model and the univariate model for *INFL* have an arbitrary long-run mean. The variance of the process is over-estimated, being much larger than the variance of *INFL* in either of the subperiods surrounding the intervention. Further, the importance of the previous year's inflation is over-estimated. For the *EQDY* series, the variance of the AR(1) model is unstable because the series is trended. The MA(1) model for *EQDG* assumes that the variance of dividend growth is constant. This is clearly not the case and results in forecast variances for equity returns which are larger than historical variances for most of the time. The important feature captured by the MA(1)-GARCH(1,1) model is the fluctuating variance of the growth rate. This has important consequences for short term risk management and the determination of excess volatility in the equity market.

This research lays the foundation for the multivariate modelling of these series and the nature of the relationships which should be built into a multivariate model. It also highlights the need to include time varying parameters into a multivariate model. These issues are critical in understanding the inter-relationships between the series and building a more realistic stochastic investment model for actuarial use in South Africa.

References

- Agiakloglou, C. (1996), 'A note on the unit root test based on the sample autocorrelations,' *The University of Piraeus Journal of Economics, Business, Statistics and Operations Research*, **46**, 97-106.
- Bierens, J.H. (1993), 'Higher-order sample autocorrelations and the unit root test,' *Journal of Econometrics*, **57**, 137-160.

Box, G.E.P. and Jenkins, G.M. (1970), *Time series analysis, forecasting and control*, Holden Day, San Francisco.

Box, G.E.P. and Tiao, G.C. (1975), 'Intervention analysis with applications to economic and environmental problems,' *Journal of the American Statistical Association*, **70**, 70-79.

Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroscedasticity," *Journal of Econometrics*, **31**, 307-327.

Breusch, T.S. and Pagan, A.R. (1978), 'A simple test for heteroscedasticity and random coefficient variation,' *Econometrica*, **46**, 1287-1294.

Chow, G. (1960), 'Tests for equality between sets of coefficients in two linear regressions,' *Econometrica*, **28**, 591-605.

Christiano, L.J. (1992), 'Searching for break points in GNP,' *Journal of Business and Economic Statistics*, **10**, 237-250.

Dickey, D.A. and Fuller, W.A. (1979), 'Distribution of the estimators for autoregressive time series with a unit root,' *Journal of the American Statistical Association*, **74**, 427-431.

Dickey, D.A. and Fuller, W.A. (1981), 'Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root,' *Econometrica*, **49**, 1057-1072.

Diebold, F.X. and Nerlove, M. (1990), 'Unit roots in economic time series: a selective survey,' in Rhodes, G.F. and Fomby, T.B. (eds), *Advances in Econometrics*, Vol. 8. Greenwich, CT: JAI Press, 3-69.

Diebold, F.X.; Lee, J-H.; and Weinbach, G.C. (1994), 'Regime switching with time-varying transition probabilities,' in C.Hargreaves (ed.), *Nonstationary Time Series Analysis and Cointegration*. Oxford University Press.

Dolado, J.J.; Jenkinson, T.; and Sosvilla-Rivero, S. (1990), 'Cointegration and unit roots,' *Journal of Economic Surveys*, **4**, 249-273.

Engle, R.F. (1982), "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation," *Econometrica*, **50**, 987-1007.

EViews (1994), *User's Guide*, Quantitative Micro Software, Irvine, California, USA.

Garcia, R. and Perron, P. (1996), 'An analysis of the real interest rate under regime shifts,' *The Review of Economics and Statistics*, **?**, 111-125.

Godfrey, L.G. (1978), 'Testing against general autoregressive and moving average error models when the regressors include lagged dependent variables,' *Econometrica*, **46**, 1293-1302.

Hamilton, J.D. (1994), *Time Series Analysis*, Princeton University Press.

Hamilton, J.D. (1993a), 'Estimation, inference and forecasting of time series subject to changes in regime,' in G.S.Maddala; C.R.Rao; and, H.D.Vinod (eds), *Handbook of Statistics*, Vol. 11. New York: North-Holland.

Holden, D. and Perman, R. (1994), 'Unit roots and cointegration for the economist,' in Rao, B.B. (ed.), *Cointegration for the Applied Economist*, MacMillan, London.

Jarque, C.M. and Bera, A.K. (1981), 'An efficient large sample test for normality of observations and regression residuals,' *Working papers in Economics and Econometrics*, No. 47, Australian National University.

Maitland, A.J. (1996), 'A review of Thomson's stochastic investment model,' *Transactions of the Actuarial Society of South Africa*, forthcoming.

MacKinnon, J.G. (1991), 'Critical values for cointegration tests,' in Engle, R.F. and Granger, C.W.J. (eds), *Long-Run Economic Relationships: Readings in Cointegration*, Oxford University Press.

Mills, T.C. (1993), *The econometric modelling of financial time series*, Cambridge University Press.

Perron, P. (1988), 'Trends and random walks in macroeconomic time series: further evidence from a new approach,' *Journal of Economic Dynamics and Control*, **12**, 297-332.

Perron, P. (1989), 'The Great Crash, the Oil Price Shock and the Unit Root Hypothesis,' *Econometrica*, **57**, 1361-1401.

Perron, P. (1993), 'The Great Crash, the Oil Price Shock and the Unit Root Hypothesis: erratum,' *Econometrica*, **61**, 248-249.

Perron, P. (1994), 'Trend, unit root and structural change in macroeconomic time series,' in Rao, B.B. (ed.), *Cointegration for the Applied Economist*, MacMillan, London.

Phillips, P.C.B. (1987), 'Time series regression with a unit root,' *Econometrica*, **55**, 227-301.

Phillips, P.C.B. and Perron, P. (1988), 'Testing for a unit root in time series regression,' *Biometrika*, **75**, 335-346.

Said, S.E. and Dickey, D.A. (1984), 'Testing for unit roots in autoregressive-moving average models of unknown order,' *Biometrika*, **71**, 599-607.

SAS/ETS (1993), *User's Guide. Version , second edition*. Copyright 1993 by SAS Institute Inc., Cary, NC, USA.

Sherris, M.; Tedesco, L.; and Zehnirith, B. (1996), 'Stochastic investment models: unit roots, cointegration, state space and GARCH models for Australian data,' *Research Paper Series*, Centre for Actuarial Studies, University of Melbourne, 33.

Schwert, G.W. (1987), 'Effects of model specification on tests for unit roots in macroeconomic data,' *Journal of Monetary Economics*, 20, 73-105.

Stock, J.H. (1993), 'Unit roots and trend breaks,' in Engle, R.F. and McFadden, D. (eds) *Handbook of Econometrics*, Amsterdam: North-Holland, Vol. 4.

Thomson, R.J. (1994), 'A stochastic investment model for actuarial use in South Africa,' Presented to the 1994 Convention of the Actuarial Society of South Africa.

Thomson, R.J. (1995), 'A methodology for the modelling of interest rates and other economic variables with reference to the money and capital markets of South Africa,' *Proceedings of the 5th AFIR Colloquium*.

Thomson, R.J. (1996), 'Stochastic investment modelling: the case of South Africa,' *British Actuarial Journal*, 2, 765-801.

