2nd AFIR Colloquium 1991, 4: 407-429

Solvency Standards

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Summary

Actuaries and others need a consistent framework for evaluating the financial condition of insurance companies. Although the importance of cash flows was first recognized in 1979 and the recognition of their importance has grown steadily since then, cash flows are not usually analyzed in the course of the evaluation of insurance enterprises. Expected values of cash flows, however, are not enough. Risk must be dealt with explicitly if actuaries and others are to arrive at consistent valuations of insurance enterprises.

The importance of adequate reserves and the historical roots of non-life actuarial science in North America have obscured the truth that the solvency of an insurance enterprise arises from the ability of its assets to throw off cash in amounts greater than is required by its obligations.

The principal result reported here is that if certain requirements are met then there is but a single framework for analyzing cash flows and drawing conclusions about the solvency of an enterprise. This framework includes five elements:

1. Explicit statement of the projected cash flows arising out of both assets and obligations and the risks associated with each.

2. Emphasis on the concept of scenarios.

3. Explicit statement of utility functions to reflect risk.

4. A formula to tie together all cash flows over all time.

5. A result, the risk-adjusted present value of the insurance enterprise, this is a function of the evaluator's aversion to risk. While solvency itself is not technically impaired unless cash outflows exhaust funds available for payments, solvency could be said to be at risk whenever the risk-adjusted present value is negative for reasonable levels of risk aversion.

Résumé

Normes de Solvabilité

Les actuaires et d'autres personnes ont besoin d'un cadre uniforme pour évaluer la condition financière des compagnies d'assurance. Bien que l'importance des "cash-flows" ait été reconnue pour la première fois en 1979 et que depuis lors on se soit rendu de plus en plus compte de leur importance, les "cash- flows" ne sont pas généralement analysés lors de l'évaluation des compagnies d'assurance. Cependant les valeurs de "cash-flow" prévues ne sont pas suffisantes. Le risque doit être traité de façon explicite si les actuaires et d'autres souhaitent aboutir à des évaluations cohérentes de compagnies d'assurance.

L'importance de réserves adéquates et les racines historiques de la science actuarielle ne traitant pas de la vie en Amérique du Nord ont voilé la vérité selon laquelle la solvabilité d'une compagnie d'assurance vient de la capacité de ses actifs à se débarrasser d'argent dans des quantités supérieures à celles exigées par ses obligations.

Le principal résultat rapporté ici est que si certaines exigences sont satisfaites, il n'y a qu'un seul cadre pour analyser les "cash-flows" et tirer des conclusions sur la solvabilité d'une entreprise. Ce cadre comprend cinq éléments:

1. Etat explicite des "cash-flows" projetés provenant à la fois des actifs et des obligations et les risques associés à chacun.

2. Insistance sur le concept des scénarios.

3. Etat explicite des fonctions d'utilité pour refléter le risque.

4. Une formule pour relier ensemble tous les "cash-flows" de tous temps.

5. Un résultat, la valeur actuelle corrigée par le risque de la compagnie d'assurance, qui est fonction de l'aversion de l'évaluateur vis-à-vis du risque. Tandis que la solvabilité elle-même n'est pas techniquement compromise à moins que les sorties d'argent épuisent les fonds disponibles pour les paiements, on peut dire que la solvabilité est en danger chaque fois que la valeur actuelle corrigée par le risque est négative pour des niveaux raisonnables d'aversion de risque.

INTRODUCTION

Actuaries and others need a consistent framework for evaluating the financial condition of insurance companies. Current evaluation efforts are inconsistent from one type of insurance enterprise to the next, from one actuary to the next, and from one audience to the next. Regulators in the United States request one type of evaluation. Regulators in the United Kingdom request another. Other types of evaluations are used in the course of mergers and acquisitions. Still other types of evaluations are used for Federal income tax purposes. This inconsistency is under pressure because of the increased focus on the role of the actuary in solvency regulation.

Although the importance of cash flows was first recognized at least as early as 1979, and although the importance of cash flows has been increasingly recognized since then, cash flows are not usually analyzed in the course of the evaluation of insurance enterprises. Most evaluations rely on accounting conventions. These in turn reflect generally expected value calculations, although various sources of optimism and pessimism have an effect. Assets are generally valued at book or market. Liabilities are often valued as expected value. In the United States, liabilities are subject to infrequent regulatory review. Cash flows have not yet developed into a standard method of evaluating insurance enterprises.

And there is also no standard for reflecting risk. Most evaluations make little reference to risk. Risk is considered only at a sort of summary level, if it is considered at all. This broad-brush treatment of risk is a source of substantial inconsistency from one evaluation to the next. Risk must be dealt with more explicitly if actuaries and others are to arrive at consistent evaluations of insurance enterprises.

Honest asset values and adequate reserves are, of course, important. Unfortunately, the importance of adequate reserves and the historical roots of non-life actuarial science in North America have obscured the truth that the solvency of an insurance enterprise arises from the ability of its assets to throw off cash in amounts greater than as required by its obligations.

The actuarial literature on the subject of evaluations using cash flows has become substantial. At this time, it is appropriate to point out the framework for analyzing cash flows and draw conclusions about solvency that has emerged. The principal result reported here is that if certain requirements are met then there is but a single framework for analyzing cash flows and drawing conclusions about the solvency of an enterprise.

This single framework includes five elements:

1. Explicit statement of the projected cash flows arising out of both assets and obligations and the risks associated with each.

2. Emphasis on the concept of scenarios.

3. Explicit statement of utility functions to reflect risk.

4. A formula to tie together all cash flows over time.

5. A result, the risk-adjusted present value of the insurance enterprise, that is a function of the evaluator's subversion to risk and that reflects substantial details about expected payoffs and their uncertainties.

While solvency itself is not technically impaired unless cash outflows exhaust funds available for payments, solvency could be said to be at risk whenever the risk-adjusted present value is negative for reasonable levels of risk aversion.

This paper concludes with an example of the calculation of the risk-adjusted present value of an insurance enterprise. This example begins by analyzing 17 scenarios. Each scenario has associated with it an estimate of cash flows from the portfolio of obligations and from the portfolio of assets. The calculations rely on a particular utility function.

The paper concludes with a brief discussion of the implications of the theoretical results.

DEFINITIONS AND NOTATION

The expected value of a set of cash flows

$$x_i \qquad i=1,\dots, n$$

with probabilities

$$p_i \qquad i=1,\dots, n$$

is defined as

$$EV = \sum_{i=1}^{n} p_i x_i \tag{1}$$

If cash flows may occur in the future, with n_t possible outcomes at time t, denoted by

$$x_{i,t} \quad i=1,\dots, n_t; \; t=0,\dots$$

the expected value and the present value of the cash flows are defined as

$$EV = \sum_{t=0}^{\infty} \sum_{i=1}^{n_t} p_{i,t} x_{i,t} \qquad (2)$$

$$PV = \sum_{t=0}^{\infty} \sum_{i=1}^{n_t} p_{i,t} v_t x_{i,t} \qquad (3)$$

where

$p_{i,t}$ = probability of event i leading to cash flow x at time t

v_t = present value of $1 cash flow at time t.

The probabilities in these formulas are subjective. However much they are based on data and careful analysis of the factors leading to the cash flows, they necessarily reflect the analyst's personal decisions about how to represent the future.

To be tractable, cash flows should have two properties described by Mateja and Geyer [1]:

1. After-Tax. Cash flows must be evaluated after-tax and discounted at after-tax rates of return if they are to provide consistent results.

2. <u>End Conditions</u>. The present value of a company's cash flow is the sum of its current cash, the present value of future dividends, and the present value of the increase in its cash.

When these possibilities exist for each of several scenarios ($j=1,.., m$) with events denoted by:

$$x_{i,j,t} \qquad \begin{array}{l} i=1,\ldots, n_{j,t} \\ j=1,\ldots, m \\ t=0,\ldots, \infty \end{array}$$

we say

$$EV = \sum_{j=1}^{m} \sum_{t=0}^{\infty} \sum_{i=1}^{n_{j,t}} p_j p(i,j,t \mid j) x_{i,j,t} \qquad (4)$$

$$PV = \sum_{j=1}^{m} \sum_{t=0}^{\infty} v_{j,t} \sum_{i=1}^{n_{j,t}} p_j p(i,j,t \mid j) x_{i,j,t} \qquad (5)$$

These can be read as follows:

The expected value of a set of cash flows over a range of points in time for each of several scenarios is the sum over all such flows of the probability of the cash flow times the amount of the cash flow. The probability, in turn, is the probability of the scenario times the probability of the cash flow of the stated amount at time t given the occurrence of the scenario.

The present value of a set of cash flows over a range of points in time for each of several scenarios is a similar sum, but the total at each point in time is discounted at an after-tax rate of return appropriate for the scenario.

In practice, the discount factors $v_{j,t}$ can be found by observing the current price of government-backed notes with various maturities, with adjustments for income taxes. Discount rates observed today can be thought of as averages over several scenarios of high interest rates, medium interest rates, and low interest rates.

Let the utility function U(x) be a function of x with a few obvious properties:

- U(x) is continuous and differentiable everywhere
- U(x) is monotonically increasing, that is, $U(x)' > 0$
- U(x) is concave everywhere, that is,

$$U''(x) < 0 \qquad -\infty < x < \infty$$

- an inverse function exists such that

$$U^{-1}U(x) = ax + b \qquad \text{a and b are constants}$$

Example:

$$U(x) = e^{-x/c} \tag{6}$$

$$U^{-1}(x) = -c \ln x \tag{7}$$

(ln denotes the natural logarithm)

Whenever a utility measure is in the same units as the cash flow, it makes intuitive sense to call it the Risk-Adjusted Value, or RAV, of the set of outcomes. We can without loss of generality define

$$RAV = U^{-1}\left[\sum_{i=1}^{n} p_i \, U(x_i)\right] \tag{8}$$

The expected value calculation (1) is a special case of utility in which the evaluator's aversion to risk is negligible.

In classic decision theory, the RAV of a lottery is the amount the rational evaluator would be willing to pay to participate in the lottery.

The calculation of the risk-adjusted value in (8) is not an arbitrary formula. The pioneers of decision theory (e.g., Raiffa[2]) showed that consistent decisions could be developed from a given set of estimates of probabilities only when the probabilities and utilities were combined in this way.

Correct evaluation of the solvency of the insurance enterprise should reflect the following constraints:

1. The risk-adjusted value for the entire set of outcomes does not depend on how finely the evaluator enumerates the possible outcomes. The risk-adjusted values of the various components of the analysis should display the associative property of algebra.

2. To the extent the company has cash inflows exactly equal to all cash outflows for various times under a given scenario, the net cash flow is zero and the company has removed the element of risk from both the asset portfolio and the portfolio of obligations.

3. As the probability of any one event increases toward certainty, the risk-adjusted value of all outcomes should approach the present value of that event at the risk-free rate of return for the scenario in which that event arises.

4. An evaluator with very large risk capacity (in relation to the outcomes) is an expected-value decision-maker.

5. An evaluator with a very limited risk capacity (in relation to the outcomes) will behave as if the worst possible outcome were certain to occur. This is the premise of Von Neumann game theory.

   In practical applications insurance enterprises take risks that are significant in relation to their risk capacities, yet keep the risks assumed to a small enough level to avoid having to behave as if the worst possible outcome were certain to occur.

Some evaluators (including the author) would prefer a framework in which a sixth constraint is met:

6. Amounts that are absolutely certain may be handled outside the RAV calculation.

The first constraint restricts the way interest can be reflected in the calculation of risk-adjusted value. The formula over time is

$$RAV = \sum_{t=0}^{\infty} v_t U^{-1} \left[\sum_{i=1}^{n_t} p_{i,t} \, U(x_{i,t}) \right] \tag{9}$$

$$= \sum_{t=0}^{\infty} v_t \; RAV_t \tag{10}$$

That is, each discount factor must be analyzed into a risk-free component and a utility component which explicitly states the probabilities and amounts that might be paid or received. Then all possible cash flows must be analyzed together at each point in time. This is the risk-adjusted value at time t, and reflects all risk. These should be treated as if they were certain to occur, and their present value totalled up at the risk-free rate of return. (Recall that all cash flows are to be discounted at after-tax rates of return.)

The present value calculation in equation (3) is a special case of utility in which the evaluator's aversion to risk is negligible.

The sixth constraint limits the choice of utility functions to the class of exponential utility functions in equation (6).

RESULTS

For a particular scenario at a particular time,

$$RAV_{j,t} = U^{-1} \sum_{i=1}^{n_{j,t}} p(i,j,t|j)\, U(x_{i,j,t}) \tag{11}$$

For the scenario as a whole,

$$RAV_j = \sum_{t=0}^{\infty} v_{j,t}\; RAV_{j,t} \tag{12}$$

and for the entire set of scenarios

$$RAV = U^{-1} \sum_{j=1}^{m} p_j\; U(RAV_j) \tag{13}$$

In summary,

$$RAV = U^{-1} \sum_{j=1}^{m} p_j\, U\left[\sum_{t=0}^{\infty} v_{j,t}\, U^{-1} \sum_{i=1}^{n_{j,t}} p\,(i,j,t|j)\, U(x_{i,j,t})\right] \tag{14}$$

As an example, if U(x) = exp(-x/c)

$$RAV(c) = - c \ln \sum_{j=1} p_j e^{\sum_{t=0}^{\infty} v_{j,t} \ln \sum_{i=1}^{n_{j,t}} p(i,j,t|j)\, e^{-\frac{x_{i,j,t}}{c}}} \qquad (15)$$

These calculations can be done readily on spreadsheets once the probabilities and risk-free rates of return have been chosen.

EXAMPLE

Exhibit 1 provides an overview of a calculation of risk-adjusted value. The example involves a workers' compensation insurance company. The scenarios reflect varying assumptions about litigation, inflation, interest rates, defaults in payments on bonds, and calls on callable bonds.

In Exhibit 1, the present value of the insurance enterprise is summarized as $107 million and the risk-adjusted value is summarized as a negative $46 million.

Exhibit 1 also shows three critical assumptions, the precision associated with the estimate of loss payments in each year, the evaluator's risk capacity, and the enterprise's initial cash.

Exhibit 2 shows additional assumptions. The expected loss payments in each year under various assumptions of litigation and inflation range from $3 million in the tenth year for a low cost scenario to $135 million in the third year for the high cost scenario. Asset payments on bonds are assumed to be steady over ten years unless callable bonds are called. This is assumed to happen at the earliest date after two years from the date of evaluation. In addition, under some scenarios, 10% of the callable bonds or 10% of the non-callable bonds (or both) are assumed to default on payments in years 3 to 10.

The evaluation is done using exponential utility. This risk capacity shown in Exhibit 1 is the constant c in equation (6).

The loss payments in a particular year under a given scenario are assumed to be distributed with a gamma distribution. This was selected only for convenience of calculation for this hypothetical example. The risk-adjusted value of losses distributed with a gamma distribution is a function only of the product of the precision of the loss estimate, α, and the risk capacity. The formula is:

$$RAV = c\alpha \ln \left(1 + \frac{\text{mean}}{c\alpha}\right)$$

Exhibit 3 provides information about the risk-adjusted value under other assumptions. If the risk capacity is low, the risk-adjusted value of the enterprise is about equal to the risk-adjusted value of the worst scenario. The enterprise's value is not sensitive to changes in interest rates.

Exhibit 4 shows a calculation of a single scenario.[1]

The gamma distribution is an inappropriate choice if the product of risk capacity and the precision of the loss estimate is less than the expected value of the loss payments. In this example, therefore, the product of risk capacity and the precision of the loss estimates must not be less than about 135. Even if the risk capacity is as great as 100, however, there is a substantial difference between the risk-adjusted value and the present value. If the initial cash were $75 million instead of $150 million, the present value of the enterprise would be $32 million, while the risk-adjusted value would be a negative $23 million.

IMPLICATIONS

These results have several implications:

1. Immunization against interest rate risk is not possible unless the cash flows are reasonably predictable.

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[1] A full set of the exhibits can be obtained from the author.

2. The risk-adjusted value of an insurance enterprise may be enhanced by the purchase of reinsurance. These results suggest new theoretical models for the reinsurance portfolio problem.

3. The risk-adjusted value of a given insurance company will increase if it underwrites policies that are equally as profitable as the current policies but have results negatively correlated with the results of the current portfolio.

4. Using debt to acquire investible assets will seldom increase the risk-adjusted value of the enterprise because the new obligations will (generally) be more certain than the new investment results.

5. Issuing stock to raise funds for investible assets may increase the risk-adjusted value of the original owners' equity if the new assets and liabilities increase the firm's risk capacity relative to its risks. (See constraints 4 and 5).

BIBLIOGRAPHY

[1] James A. Geyer and Michael E. Mateja, "Cash Flow Based Surplus," unpublished, dated November 11, 1985.

[2] Raiffa, H., "*Decision Analysis - Introductory Lectures on Choices Under Uncertainty*", Addison-Wesley Publishing Company, Inc., Reading, Massachusetts.

Exhibit 1

OVERVIEW OF ALL SCENARIOS

	Litigation	Inflation	Interest	Defaults		Disc.Rate	Probability	PV(i)	RAV(i)
				Callables	Non-Call's				
1	low	low	moderate	no	no	4.5%	21.50%	168.2	97.7
2	moderate	moderate	moderate	no	no	4.5%	34.40%	117.1	15.0
3	high	high	high	no	no	7.0%	16.80%	62.8	-82.0
4	high	high	low	no	no	3.0%	13.50%	59.4	-91.0
5	low	low	moderate	yes	no	4.5%	0.50%	153.1	82.6
6	moderate	moderate	moderate	yes	no	4.5%	0.80%	102.0	-0.1
7	high	high	high	yes	no	7.0%	0.60%	48.1	-96.7
8	low	low	moderate	called	no	4.5%	1.00%	126.7	56.2
9	moderate	moderate	moderate	called	no	4.5%	1.60%	75.6	-26.4
10	high	high	low	called	no	3.0%	1.35%	17.4	-133.0
11	low	low	moderate	no	yes	4.5%	0.50%	162.1	91.6
12	moderate	moderate	moderate	no	yes	4.5%	0.80%	111.0	9.0
13	high	high	high	no	yes	7.0%	0.80%	56.9	-87.9
14	high	high	low	no	yes	3.0%	0.15%	53.3	-97.1
15	low	low	moderate	yes	yes	4.5%	1.50%	147.0	76.5
16	moderate	moderate	moderate	yes	yes	4.5%	2.40%	95.9	-6.1
17	high	high	high	yes	yes	7.0%	1.80%	42.2	-102.6
precision of loss est	4			Total, Average, and RAV			100.00%	107.1	-45.6
risk capacity	50								
initial cash	150								

Exhibit 2

ADDITIONAL ASSUMPTIONS

	Expected Loss Payments			Asset Payments					
	Litigation and Inflation			Callable Bonds			Non-Callable Bonds		
Year	Low	Moderate	High	Base	Called	Default	Base	Default	Treas.'s
1	60	67.3	74.9	25	25	25.0	10	10.0	10
2	75	85.8	97.3	25	25	25.0	10	10.0	10
3	100	116.7	135.0	25	125	22.5	10	9.0	10
4	50	59.5	70.2	25	0	22.5	10	9.0	10
5	25	30.4	36.5	25	0	22.5	10	9.0	10
6	15	18.6	22.8	25	0	22.5	10	9.0	0
7	9	11.4	14.2	25	0	22.5	10	9.0	0
8	6	7.7	9.9	25	0	22.5	10	9.0	0
9	4	5.3	6.8	25	0	22.5	10	9.0	0
10	3	4.0	5.3	25	0	22.5	10	9.0	0
Total	347	406.7	472.9	250	175	230.0	100	92.0	50

Exhibit 3

EFFECTS OF CHANGES IN ASSUMPTIONS

A. Variation in Risk and Risk Capacity

Risk Capacity	Precision of Loss Estimate	Product	RAV
33.775	4.00	135.1	-758
50	4.00	200.0	-46
100	4.00	400.0	52
200	4.00	800.0	83
49	4.00	196.0	-52
50	3.92	196.0	-51
200	2.00	400.0	58

B. Variation in Interest Rate

Risk Capacity	Precision of Loss Estimate	Interest	RAV
50	4	Base Case	-46
50	4	1.1 x as great	-45

Exhibit 4

		Scenario	1	probability	0.215	RAV(j)	97.7		
						PV(j)	168.2		
		litigation	low						
		inflation	low						
		interest	moderate	defaults:	none	disc. rate	4.5%		
		precision of loss estimate		4					
		risk capacity		50					
		initial cash		150					

Year	Expected Loss	RAV(Loss)	Asset Cash Flows: Callables	Asset Cash Flows: Non-Call's	Asset Cash Flows: Treas.'s	RAV(j,t)	v(j,t)	v(j,t)RAV(j,t)	Value Ignoring Risk
1	60	-71.3	25	10	10	-26.3	0.9569	-25.2	-14.4
2	75	-94.0	25	10	10	-49.0	0.9157	-44.9	-27.5
3	100	-138.6	25	10	10	-93.6	0.8763	-82.0	-48.2
4	50	-57.5	25	10	10	-12.5	0.8386	-10.5	-4.2
5	25	-26.7	25	10	10	18.3	0.8025	14.7	16.0
6	15	-15.6	25	10		19.4	0.7679	14.9	15.4
7	9	-9.2	25	10		25.8	0.7348	19.0	19.1
8	6	-6.1	25	10		28.9	0.7032	20.3	20.4
9	4	-4.0	25	10		31.0	0.6729	20.8	20.9
10	3	-3.0	25	10		32.0	0.6439	20.6	20.6
Total	347	-426.2	250	100	50	-26.2		-52.3	18.2