2nd AFIR Colloquium 1991, 2: 75-92

Risk Measurement for Asset Liability Matching
A Simulation Approach to Single Premium Deferred Annuities

Meye Smink

Faculteit de Economische Wetenschappen, Rijksuniversitet Groningen, Postbus 800,
9700 AV Groningen, The Netherlands

Summary

In this paper the effects of using different risk measures in portfolio analysis for insurance products are examined by a case study on SPDA's. Using a four parameter term structure model SPDA's and different bonds are simulated in order to determine yields and investment spreads. The outcomes of the simulation are placed into a risk/return framework, which is formulated to have a dynamic target. Two relevant risk measures: standard deviation and below target standard deviation are used. The impact of these two risk measures is analyzed and quantified in terms of opportunity costs.

Résumé

Mesure de Risque pour la Congruence des Engagements et des Actifs
Une Simulation des Annuités Différées de Prime Unique

Dans cet article, sont examinés les effets de l'utilisation de différentes mesures de risque dans l'analyse de portefeuille pour les produits d'assurance, grâce à une étude de cas sur les SPDA. En utilisant un modèle de structure de durée à quatre paramètres, les SPDA et différentes obligations sont simulées afin de déterminer les répartitions des rendements et des investissements. Les résultats de la simulation sont placés dans un cadre risque/rendement formulé pour avoir un objectif dynamique. Deux mesures de risque appropriées sont utilisées: l'écart type et l'écart type inférieur à l'objectif. L'impact de ces deux mesures de risque est analysé et quantifié en terme de coûts d'option.

1 Introduction

Volatile interest rates can influence profitability of particular insurance products quite dramatically. Embedded options in interest rate sensitive products, such as Single Premium Deferred Annuities (SPDA's), should be explicitly included in pricing and the asset allocation process.

For asset liability matching strategies it is required not only to recognize the interest rate sensitivity of both assets and liabilities, but also to specify what constitutes the risk associated with this sensitivity to interest rate changes. Once the case of deterministic or completely predictable cashflows from assets and liabilities is departed and the stochastic case is examined, it is understood that any asset liability matching problem deals with a risk/return trade off. Therefore, the question arises, how the risk/return framework should be structured, how different risk measures affect portfolio selection and what the associated opportunity costs are.

Recently, attention in research has refocused on the below-target risk measure, (Harlow & Rao, 1989, Clarkson, 1990, Meer van der, e.a., 1989). This risk measure seems to correspond more closely to investors risk attitudes and distinguishes between the pure investment risk, i.e. the below target returns, and uncertainty. One of the main criticisms against using below target variance is that it is necessary to specify a non-dynamic target, where usually asset/liability matching requires dynamic benchmarks.

In this paper a simulation approach is adopted for valuation of SPDA's and the selection of an optimal asset/liability portfolio. In this simulation a complete term structure is simulated using a parsimonious yield curve model. The resulting asset allocation process is structured in terms of a risk/return framework. However, instead of analyzing absolute returns, the expected spread of investment returns over liability return is examined here. The advantage of this approach is that the associated target, e.g. positive expected spread, implicitly is dynamic. The risk measures analyzed here are standard deviation and below-target standard deviation. For both measures optimal portfolios are analyzed and differences are discussed.

The first section provides a description and rationale for the term structure model used in the simulation. In the second section a SPDA model is formulated. The third section highlights a number of possible asset allocation strategies. The fourth section focuses on the risk/return framework, compares the two relevant risk measures and discusses their impact on portfolio selection. The fifth section presents the simulation results and section six provides the conclusions.

2 Term structure model

The term structure is modeled by a four parameter model as formulated by Nelson and Siegel (Nelson and Siegel, 1987). The rationale for this model is based on the expectations theory. That is, they show that by assuming that the spot rates are generated by a differential equation, the term structure can be represented by the following equation:

$$Y(t) = \beta_0 + (\beta_1 + \beta_2) * \frac{[1 - e^{-\frac{t}{\tau}}]}{\frac{t}{\tau}} - \beta_2 * e^{-\frac{t}{\tau}}$$

In this equation Y(t) stands for the yield to maturity, t for the time to maturity in days, the three beta's and τ are the model parameters.

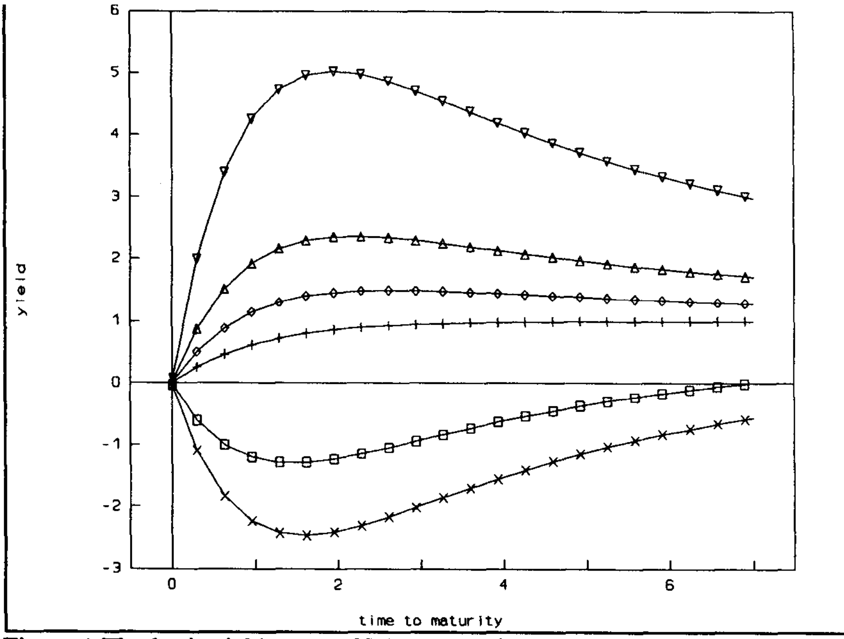

Figure 1 The basic yield curves, Nelson and Siegel model

As can be seen from figure 1, this is a very general model representing a very broad range of shapes, including monotonic, humped and S-shaped curves. Also, the three main movements in the term structure, i.e. the parallel upward or downward "jump", the widening of short and long rate differential or "tilt" and the steepening or flattening of the yield curve or "flex", which are generalizations of the additive and multiplicative processes (Khang, 1979), can easily be modeled by varying the four parameters.

For this, it should be noticed that Y(0) equals (β_0 + β_1) and Y(∞) equals β_0. Therefore, it can be concluded that the jump process is fully determined by β_0, the tilt process is determined by β_1 and the flex process is mainly characterised by β_2. The last parameter τ can be seen as a locational parameter reflecting the degree of time preference.

For the simulation purpose, it is necessary to specify the probability distribution for these four parameters. While it is generally assumed that interest rates follow a lognormal distribution, it is also known that multiperiod simulations using lognormal distributed variables tend to produce "runaway" interest rates unless a certain ceiling is imposed. The usual method for eliminating this effect is by assuming a certain degree of mean reversal or more general, assuming an autoregressive process for the term structure development. The process equation for this can be written as (Tilley, 1989):

$$\beta(t) = B + \sum_{i=1}^{n} M(i) * \beta(t-i) + E(t)$$

where: $\beta(t)$ = the vector of parameter values at period t,

B = the vector of expected parameter values,

M(i) = the matrix with autoregression coefficients of order i,

E(t) = the vector of residuals.

and

n = the order of the autoregressive process.

It should be noticed that there is weak mean reversion, i.e. the stochastic process is stable and there is a fixed point, when:

$$\det[I-\sum_{i=1}^{n} \mu_i M(i)] <> 0$$

for all μ such that $|\mu| <= 1$, (Brockwell and Davis, 1987). In this case, the properties of the process are that the expected value of the process is equal to the fixed point:

$$E(\beta) = [I-\sum_{i=1}^{n} \mu_i M(i)]^{-1} * B$$

and every ß(t) has an expected value that is closer to B then ß(t-1), i.e. there is mean reversal. In the simulations below, the autoregressive process will be applied to the first three parameters, while τ will be assumed to have a truncated normal distribution, because of it's non-negativity. The parameter values of the autoregressive process are chosen to correspond with historical data. It should be noticed however, that the model used in the simulations has not been empirically tested and results are merely indicative.

3 SPDA model

The SPDA is an interest rate sensitive insurance product that can be characterised as follows:

SPDA	=	Fixed rate accrual bond
	+	Insurer option to adjust credited rate

+ Policyholder option to lapse.

The bond compounds at the initial crediting rate, subject to an insurer option to change this rate after a guarantee period, usually between 1 and 10 years. The value of the policyholder lapse option will mainly depend on the crediting strategy of the insurance company and it's relation to competitive crediting rates, the costs of executing the option, i.e. surrender charges.

In the presented SPDA model, the yield on the SPDA is determined. In order to measure the yield it is necessary to make assumptions on policy features, insurer expenses, policyholder behaviour and competitor strategies.

The policy features of the typical SPDA product are:

1) the initial crediting rate CR(0), which will be assumed to be equal to a yield to maturity (YTM) for a selected maturity (m) minus a spread (s_0), i.e.:

$$CR(0) = Y(m,t) - S_0$$

2) the guarantee period during which the initial crediting rate prevails. This period will mostly be 1 year;

3) the reset crediting rates, assumed to be based on the current crediting rate, adjusted by an adjustment speed parameter times the yield differential between the current spread adjusted interest rate and the selected YTM, i.e.:

$$CR(t) = CR(t-1) + A * [Y(t,m) - S_0 - CR(t-1)]$$

Note that A=0 implies constant crediting rates and A=1 implies periodically varying rates;

4) the surrender charges, which are partly meant to offset the acquisition expenses and serve to lower policyholder lapses. Typically, the policyholder has the option to remove part of accumulated cash value free of surrender charges;

The insurer costs can be split into:

1) acquisition costs, which reduce the initial cash value of the product to the insurer, and,
2) maintenance costs.

The latter can be inflation adjustable but for sake of simplicity these, like the acquisition expenses, are assumed to be proportionally related to the SPDA value.

Policyholder behaviour is reflected in the occurring lapse rates. Although, from a purely economic point of view lapse behaviour is critically determined by the credited rate and the money market alternatives, including the competitor rates, adjusted for transaction costs, it is observed that lapse rates both have minimum and maximum values, reflecting resp. irrational lapsing (including lapse due to non-financial circumstances like death) and irrational non-lapsing. A convenient lapse function is the arctangent function:

$$LR(t) = \alpha_0 + \alpha_1 * Arctan[\alpha_2 * (R(t) - CR(t) - T) - \alpha_3]$$

In this function the four alpha's are parameters chosen so that the minimum and maximum lapse rate values are reached when the differential between the competitive rate, R(t) and the credited rate, CR(t), adjusted for a threshold T, is either very negative or positive, see figure 2.

The competitor strategy is modeled by taking the maximum value of the crediting rate and the market yield for a selected maturity n, adjusted for a spread. The rationale for this is that competitors generally may have the same crediting rates when market yields decline, but that when market yields rise the relevant competitor rates are initial crediting rates at market yields.

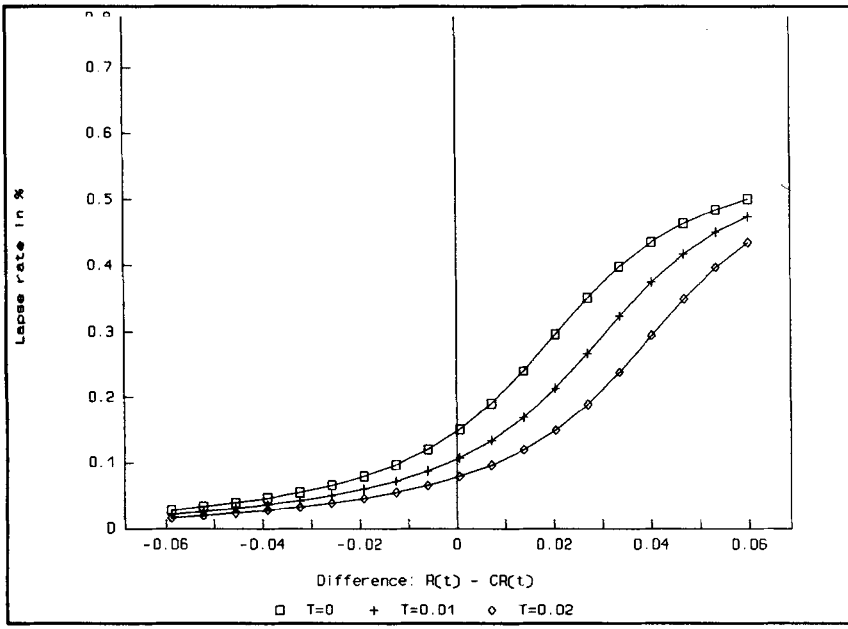

Figure 2. Lapse rates, for different threshold values.

In formal notation this is:

$$R(t) = \max[CR(t), Y(t,n) - S_1]$$

From the model above it is clear that there are a number of variables under control of the insurer which determine the profitability of the SPDA's. Also, there is a number of variables not under insurer control but directly related to the term structure of interest rates, which have significant impact on the valuation of the SPDA's. In order to establish the yield of the SPDA's it is necessary to project the expected values of cashflow's, the liability value of the SPDA and it's corresponding cash fund. This is done using the simulated term structures and their development over time.

4 Risk/return framework and risk measures

In analyzing the different product and investment alternatives the insurer has, a consistent analytical framework is required. From portfolio theory the risk/return analysis based on the ideas of Markowitz (Markowitz, 1952) is available. However, there are no absolute yields analyzed here, as is usually the case in portfolio analysis, but the spread of investment yield over liability yield. This, because it is the spread that determines the profitability of the underlying liability.

The basic notion of applying the portfolio theory to an insurance company is that in fact the life insurance products may be analyzed as borrowed assets, i.e. assets with negative portfolio coefficients. Therefore, the total asset and liability portfolio can be optimized under the constraint that the sum of all liability holdings equals the sum of asset holdings minus a surplus. For any particular product the optimal alternative has the highest profitability for a particular degree of risk. In order to do this, it is required to have estimates of the liability and asset yields and their respective covariances. What results is the optimization of the expected spread between asset and liability return, subject to minimization of the variability in this spread.
That is:

Max: $E\{YS\} = E\{Y(Assets)\} - E\{Y(Liabilities)\}$,

Min: $R\{YS\} = R\{Y(Assets) - Y(Liabilities)\}$,

where R{.} stands for the measure of variability, e.g. standard deviation or below-target standard deviation. As usual this results in an efficient set of many alternative portfolios. Implicitly, in this formulation it is assumed that there is no surplus. Even though from an insurance point of view a surplus may be highly desirable, it does not result in an economically sound analysis to incorporate a surplus here.

The two risk measures in this analysis are standard (SD) and below target standard deviation (BTSD). The formal definitions of these statistics are given by:

$$S.D. = \sqrt{\frac{1}{T}\sum_{t=1}^{T}[y(t)-E(y)]^2}$$

$$B.T.S.D. = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\min\ [0,y(t)-T(y)])^2}$$

where E(y) is the expected value of y(t) and T(y) the target value for y(t). It is easy to see that the BTSD is always smaller then the SD except when the target return T(y) is very high. From rewriting the BTSD equation into:

$$BTSD = \sqrt{\frac{1}{T}\sum_{t=1}^{n}(\min\ [0,(y(t)-E(y))+(E(y)-T(y))])^2}$$

it is clear that when the target return equals the expected value, then the true - or when t becomes very large the sample- BTSD for symmetric distributions is -

equal to $1/\sqrt{2}$ times SD. When T(y) becomes smaller, BTSD declines until it is minimally zero and when T(y) becomes larger, BTSD increases and gets a value more and more depending on T(y)-E(y).

The argument for using BTSD instead of SD is that SD does not adequately reflect the differences between a return higher and a return lower then required. For an investor, like an insurance company, a below target return has a far more dramatic impact then an above target return. Moreover, more variability in above target returns increases the probability of highly positive spreads, and is therefore favourable over less variability, while below target variability is always unfavourable. This is reflected by the fact that the BTSD is greater then SD when the target is well above the expected return, i.e. about 1 standard deviation greater than E(y).

From the characteristics of the BTSD and the SD given above, it is clear that the two have a different impact on the portfolio selection when used in a risk/return analysis. This can easily be seen by looking at the ratio between the expected value and the BTSD or SD, i.e. E(y)/BTSD and E(y)/SD. For any asset the first ratio is larger then the latter (assuming that T(y) is not much larger then E(y)). Also, when we look at two different assets A and B, with the following characteristics:

$$\frac{E(y[A])}{E(y[B])} < \frac{SD(y[A])}{SD(y[B])} \wedge E(y[A]) > E(y[B])$$

then we have that the ratio E(y[A])/BTSD(y[A]) has improved more relative to E(y[A])/SD(y[A]) then the improvement in the corresponding ratios for asset B, assuming that both expected returns are above the target. Therefore, the higher risk/higher return asset A has become more attractive relative to asset B.

In general, we may expect that using BTSD favours the more risky assets, given that these assets also have a higher expected value. This will result in higher average returns on the portfolio. On the other hand, using SD as the risk measure, will result in portfolios with a lower average return and therefore, the difference between these average returns may be regarded as opportunity costs for not using the appropriate risk measure.

5 Simulation results

The model was run, using Lotus 123 (C) and the Lotus Add-In Risk (C). The key parameter assumptions are summarized in exhibit 1.

Exhibit 1

Key parameters

Base case:

- Termstructure parameter movements	Autoregressive, lag 2.
- Expected parameter values	
- β_0	8.40
- β_1	-0.23
- β_2	3.00
- β_3	100
- Credited rate spread	100 basis points
- Competitor rate spread	50 b.p.
- Credited rate basis yields	50% 1 year, 50% 3 year
- Guarantee period	1 year
- Lapse rates	
- minimum	4.7 %
- maximum	55.5 %

The simulation consisted of 1000 iterations for 20 periods. The number of 20 periods was chosen because most of the fund value had lapsed after this period of time. Calculated were the average yields on the SPDA and the various analysed bonds. These bonds were all zero coupon bonds with 5 and 7 years to maturity but different risk and return characteristics. These maturities were chosen because of the average duration of 5.91 years.

The risk characteristics were reflected by a stochastic default margin, which, in the case of the high yield bonds, was made interest rate sensitive. The returns varied by a spread to the term structure, details are given in exhibit 2.

Exhibit 2

Bond yields:

- CY, Corporate yield	Termstructure yield + 100 bp. - default CY
- HY, High yield	Term structure yield + 250 bp. - default HY

- Defaults
  Truncated normally distributed:

	CY	HY
average:	0,	max ((Yield - treshold) and 0)
standard deviation:	0.0025,	0.01,
minimum:	0,	0,
max:	1,	1.

After calculating yields, spreads between the SPDA and the bonds were calculated and a covariance matrix was derived for the normal and the below target standard deviation. The yields are summarized in exhibit 3.

Exhibit 3

Yield	Average:	Standard Deviation:
Yield on SPDA:	7.504 %	0.627 %
Bonds:		
Corporate		
- 5 year	9.917 %	6.414 %
- 7 year	10.333 %	9.090 %
High Yield		
- 5 year	10.801 %	6.432 %
- 7 year	11.145 %	9.076 %

The final step in each simulation consisted of calculating the efficient frontier using the Haugen diskette (Haugen, 1990) for both risk measures.

The two typical portfolios, minimum variance and maximum return for the standard deviation based port-folio and the below target portfolio are presented in exhibit 4.

The main differences between the standard deviation based portfolios and the below target portfolios are related to the size of the efficient set. Looking at the efficient sets of the below target portfolios, it is clear that these sets have return ranges which constitute subsets from the standard deviation return range. However, the minimal variance portfolio in the below target case has a higher return than the standard deviation based minimal variance portfolio. Clearly,the minimization of risk in the standard deviation framework

Exhibit 4

Portfolios on the efficient frontier

Expected spread/Risk in expected spread

	minimum standard deviation:	minimum below-target std.dev.:	maximal return:
[illegible]	[illegible]	[illegible]	[illegible]
Std. deviation:	6.07 %	6,08 %	3.50 %
Below Target Std.dev.:	2.51 %	2.22 %	8.75 %
Portfolio:			
Corporate			
- 5 yr	78.69 %	100.00 %	0.00 %
- 7 yr	0.00 %	0.00 %	0.00 %
High yield			
- 5 yr	21.31 %	0.00 %	0.00 %
- 7 yr	0.00 %	0.00 %	100.00 %

leads to unnecessary prudence, which amounts to additional reduction of uncertainty, not risk, and reduces the expected return on the portfolio. This result is in correspondence with results found by Van der Meer e.a. (Meer van der, e.a., 1990)

Also, as measure of the opportunity costs associated with using the standard deviation as a risk measure, may be defined the distance between expected returns on the minimal variance portfolios. The estimated opportunity costs are 0.71 % for the target level of 0%. Obviously, there is no difference between the maximal attainable returns under the different risk measures, for this depends only on the expected returns of the underlying assets.

6 Conclusions

The simulation results indicate that this formulation of the portfolio approach to asset/liability matching leads to a consistent evaluation of different investment alternatives. However, care should be taken in defining the appropriate risk measure. Although portfolio analysis essentially is based on perfect foresight, which is usually not available, simulation can be useful in valuing assets and liabilities, and thereby increase the applicability of the analysis.

It is clear that the choice of risk measure can have a significant impact on portfolio choice. Choosing the below target standard deviation as the relevant measure will in general favour more risky assets, given that these also have higher returns. Moreover, choosing the standard deviation as measure for profitability risk, may lead to a significant opportunity loss.

References

Asay M.R., Bouyoucos P.J. and Marciano A.M., "An economic approach to valuation of single premium deferred annuities", Goldman Sachs & Co, 1989.

Brockwell P.J. and Davis R.A., "Time series: theory and methods", Springer Verlag, New York, 1987.

Clarkson R.S., "The measurement of investment risk", 1989.

Fishburn P.C., "Mean risk analysis with risk associated with below-target returns", American Economic Review, Vol. 67, no.2, March 1977.

Griffin M.W., "An excess spread approach to non-participating insurance products", Proceedings of the 1st AFIR International Colloquium, Paris, 1989.

Harlow W.V. and Rao R.K., "Asset pricing in a generalized mean-lower moment framework: theory and evidence", Journal of Financial and Quantitative Analysis, Vol. 24, No.3, September 1989.

Haugen R.A., Modern Investment theory, Prentice Hall, Englewood Cliffs, 1990

Khang C., "Bond immunization when short-term interest rates fluctuate more than long-term rates", Journal of Financial and Quantitative Analysis, december 1979, Vol.XIV, no.5.

Markowitz H.M., "Portfolio selection", Journal of Finance, 1952, no.7.

Meer, R.A.H. van der, Sortino F. and Shu Kwai Liu, "Why most investors cannot win with tactical asset allocation", Investing Magazine, november 1990.

Nelson, C.R. and Siegel, A.F., "Parsimonious modelling of yield curves", Journal of Business, 1987, vol.60, no.4.

Tilley, J.A., "A stochastic yield curve model for asset/liability simulations", Proceedings of the 1st AFIR International colloquium, Paris, 1989.

Wilkie A.D., "Portfolio selection in the presence of fixed liabilities: a comment on "the matching of assets and liabilities"", Journal of the Institute of Actuaries, no. 112, 1985.

Wise A.J.,"The matching of assets and liabilities", Journal of the Institute of Actuaries, no. 111, 1984.

Wise A.J., "Matching and portfolio selection", part 1 and 2, Journal of the Institute of Actuaries, no. 114, 1987.