Realized Return Optimization
A Targeted Total Return Approach to Funding Liabilities

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Summary

This presentation introduces Realized Return Optimization (RRO), a multi-horizon scenario-based investment strategy for funding liabilities. The strategy defines an investor's targets in terms of total returns. These targets are termed "required return". The return on an asset over a specified time period is known as "realized return" and is measured as the projected total return along a particular interest rate scenario. A risk measure, "downside deviation", is calculated explicitly from the required and realized returns and is used to determine the most efficient means of achieving the investor's goals. This allows total return investors to evaluate risk-return trade-offs and, if they choose, to actively manage their assets by taking positions on interest rates.

Some of the features that make RRO attractive for funding liabilities are:

* Flexibility with respect to objectives and targets
  - Allows for multiple investment horizons
  - Accounts for diverse shifts in interest rates
* Handles cash flow uncertainty due to options in both assets and liabilities
* Enhances the ability to meet cash outflows
* Matches the values of assets and liabilities over time
* Identifies risky positions well in advance
* Enables incorporation of an investor's risk preference profile
* Determines a risk-return frontier based on investor's targets
* Incorporates margins for profit and error.
Résumé

Optimisation de Rendement Réalisée
Une Approche de Rendement Total Fixe pour Financer les Engagements

Cet article présente l'optimisation de rendement réalisé (RRO), une stratégie d'investissement basée sur un scénario multi-horizons pour le financement des engagements. La stratégie définit des objectifs d'investisseur en termes de rendements totaux. Ces objectifs sont appelés "rendement exigé". Le rendement d'actifs sur une période de temps spécifiée s'appelle "rendement réalisé" et est mesuré comme le rendement total projeté selon un scénario de taux d'intérêt particulier. Un calcul de risque "déviation en baisse" est calculé explicitement à partir des rendements exigés et réalisés; il est utilisé pour déterminer les méthodes les plus efficaces pour réaliser les objectifs de l'investisseur. Ceci permet aux investisseurs saisonnent en rendement total d'évaluer les échanges risque-rendement et s'ils le choisissent de gérer activement leurs actifs en prenant position sur les taux d'intérêt.

Quelques-unes des caractéristiques qui rendent RRO attrayante pour le financement des engagements sont:

- Flexibilité vis-à-vis des objectifs et des buts
  - Permet des horizons d'investissement multiple
  - Justifie divers mouvements dans les taux d'intérêt.

- Gère les incertitudes de cash-flow dues aux options à la fois pour les actifs et les engagements.

- Améliore la capacité de faire face aux décaissements.

- Rapproche les valeurs d'actifs et d'engagements dans le temps.

- Identifie les positions risquées bien à l'avance.

- Permet d'incorporer un profil de préférence de risque de l'investisseur.

- Détermine une frontière risque-rendement basée sur les objectifs de l'investisseur.

- Incorpore des marges de bénéfices et d'erreurs.
A. INTRODUCTION

The presentation begins by examining some of the assumptions of immunization theory, which is the basis for conventional liability funding strategies. This is followed by a description of RRO and the risk measure termed "downside deviation." Simple numerical examples will illustrate how the strategy operates.

B. IMMUNIZATION THEORY

Immunization strategies aim to manage the asset portfolio so that the liabilities will always be met when due. There are three conditions necessary for immunization. The first is that the present value of the assets equals the present value of the liabilities. In practice, this condition is met by equating the market values of the assets and liabilities. The second condition is that the duration of the assets equals the duration of the liabilities (where the duration measure is defined in terms of the rate of change of price with respect to the change in interest rates). This makes the immediate price sensitivity to interest rate changes the same for the assets and the liabilities. The third condition states that the convexity of the assets should be greater than the convexity of the liabilities. When this condition is satisfied, a fall in interest rates will result in the asset values increasing by more than the increase in liability values, and a rise in interest rates will result in asset values falling by less than the fall in the value of liabilities.

Using calculus terminology, duration is based on the first order partial derivative and convexity on the second order partial derivative of price with respect to interest rates. Therefore, they are local measures in that they are only representative for very small changes in interest rates. For larger changes in interest rates and for longer time periods, they misestimate the price response. Furthermore, they imply that the price change for a given change in interest rates will be of the same magnitude, irrespective of the direction of the change in interest rates. This feature ignores the "path dependency" of the asset and liability cash flows.
Path dependency means that the value of an asset or liability depends on interest rates at the valuation date and on the rates at each previous point in time. This is particularly important for securities and insurance policies that have embedded options, where the prior exercise of an option may affect all future cash flows.

Immunization generally assumes that changes in interest rates are defined by parallel shifts of the term structure. When the shifts are non-parallel, the changes in asset and liability values may not be as anticipated. (However, considerable work has been done in defining duration-type measures for non-parallel shifts in the term structure.)

The theory requires that the duration and convexity conditions be maintained at all times. Since they are both functions of time, the duration and convexity of the assets and liabilities will change even when there is no shift in the term structure. The only way to maintain the duration and convexity conditions is to have continuous rebalancing of the assets. This is clearly impractical. At best, periodic or triggered rebalancing can be implemented. The general view is that rebalancing every three to six months will meet the immunization objectives. Nevertheless, this still exposes the liabilities to the risk of being underfunded if the asset performance between rebalancing dates is poor.

Duration and convexity do have their uses. They are most effective when comparing securities to be traded, such as callable bonds and mortgage-backed securities which have embedded options. Option pricing theory is used to derive the price sensitivity measures for these securities. Where options may be exercised sub-optimally, for example with mortgage-backed securities, the validity of the pricing models can be tested by comparing the model values to actual prices in the secondary markets.

Immunization assumes that the price behavior of liabilities can be evaluated. The assumption is reasonable for fixed and known liabilities where Macaulay duration can be calculated. Modified duration can then be calculated using the close relationship between the two types of duration. As with path dependent assets, this relationship breaks down with path dependent liabilities leading to a strong temptation to use the option-pricing techniques developed for the assets. Notice,
however, that the luxury of an active secondary market trading insurance liabilities is not there. Therefore, the validity of the same option-pricing techniques applied to insurance liabilities cannot be tested to the same degree. Any duration or convexity numbers derived for the liabilities should be treated with caution.

Even if a reliable liability model can be built incorporating option-pricing methods, there are at least two fundamental problems with immunization theory. First is the inability to recognize a margin for error. If the duration of the liabilities is calculated at 4.0 years, is a margin for error +0.2 or -0.2 years (and why 0.2)? Second is the conclusion implied by the strategy. If duration is considered analogous to the mean of a statistical distribution then immunization is analogous to equating two means and concluding that the underlying statistical distributions are identical.

C. CHANGE IN APPROACH

Immunization is an extremely important and useful strategy, especially in the trading area where decisions have to be made quickly and investment horizons tend to be shorter. But immunization is based on yield and duration measures, neither of which are good measures of longer holding period performance or risk. The limitations of immunization indicate that alternative strategies are required for monitoring and managing longer term asset-liability positions. Many of these alternatives, however, rely in some way or another on the same parameters, primarily duration. Everyone is trying to build a "better" duration. It may be that some other measure is more appropriate.

Start by re-examining the basic problem: Invest the premiums from policyholders so that all benefits and expenses are paid when due. At each point in time, the assets should be sufficient to meet the liability cash flow and reserve requirements. For example, assume $100 million premium is received for an SPDA at the outset. At the end of a given period, say a year, cash flow of $5 million is required and the remaining account value is $105 million. The total requirement at the end of the period is $110 million, implying that the assets should return at least 10%.
The total requirement and the split between cash flow and account value will vary depending on various factors driven primarily by interest rates, e.g. lapse rates, bond calls, and mortgage prepayments. In all cases, however, the assets should earn enough to cover the liability requirements. The investment managers should invest the assets to minimize the risk of underfunding. In other words, the return earned on the assets should be at least that required by the liabilities. This condition should hold over a range of scenarios and investment horizons.

A scenario-based approach which projects the cash flows of assets and liabilities under diverse interest rate scenarios is a reasonable way to proceed. The performance of both the assets and liabilities under these scenarios over a range of horizons will determine the suitability of the asset portfolio for funding the liabilities. (Even immunization has evolved into a scenario-based approach. Immunization models generally use scenario-based methods to generate the duration and convexity measures but, by relying on these two measures, tend to diminish the value of other information provided by such analysis.)

The number of scenarios considered is not an inherent strength or weakness of immunization or any other strategy. The practitioner can use as many scenarios as desired but should evaluate the trade-offs between the number of scenarios used, the incremental information provided by adding another scenario and the time available to do the analysis.

From the viewpoint of a total return investor, conventional portfolio selection strategies based primarily on immunization theory do not seem to provide an answer to the problem of efficiently managing the trade-off between risk and return. The measures used to quantify risk (duration mismatch) and expected return (yield, or spread over the yield curve) have no direct bearing on the one used to evaluate historical performance (total return). The strategy presented here focuses on total return as a measure of performance and risk as well as a basis for selecting a portfolio of assets to fund the liabilities.
D. FRAMEWORK OF THE STRATEGY

The strategy was developed by building a framework of the features required of a good investment strategy. This framework is as follows:

FRAMEWORK FOR MEASURING PORTFOLIO PERFORMANCE AND RISK
(Shimpi-Rajan Framework)

1. Consider Multiple Scenarios Explicitly
2. Consider Multiple Horizons Explicitly
3. Use A Performance Measure Valid For Both Assets And Liabilities
4. Use A Risk Measure Calculated From The Performance Measure
5. Develop A Quantifiable Risk-Return Trade-Off i.e. Efficient Frontier
6. Enable Investor To Impose Own Risk Preference Profile On Solution
7. Allow Flexibility To Incorporate Margins For Profit And Error

The strategy that was developed meets all the elements of the framework. It is interesting to note, however, that immunization does not meet any.
E. REALIZED RETURN OPTIMIZATION (RRO)

E.1. SCENARIOS AND HORIZONS

RRO considers multiple scenarios and horizons explicitly at the portfolio selection stage. The scenarios are primarily interest rate scenarios but other factors can also be used, e.g. policyholder lapse behavior. The scenarios can be generated by some stochastic process or by the investor drawing a set of yield curves on a sheet of paper. Unlike immunization, there is no requirement that a continuum of interest rate changes be used.

Multiple horizons cater to multiple concerns. Investors, particularly insurance companies and pension funds, have short, medium and long term requirements. Conventional strategies target only one horizon which is effectively the duration date. This is not satisfactory since duration is a derived number and not a date that necessarily conforms to an investor's set of horizons. RRO enables an investor to specify investment targets over a range of horizons, e.g., 6 months, 12 months, 5 years, 10 years and 20 years.

There are those who suggest that conventional strategies allow portfolio performance over multiple scenarios and horizons to be evaluated. That is certainly true, but there is a fundamental difference. Conventional strategies only allow such evaluation "after the fact", i.e., after the optimal portfolio has been selected using duration and convexity. RRO incorporates the multi-scenario, multi-horizon targets "before the fact", i.e. RRO uses the targets as constraints in the selection of the optimal portfolio, thereby ensuring that they are met.
E.2. TOTAL RETURNS

The third element of the Framework refers to a performance measure that is valid for both assets and liabilities. RRO uses total return as the critical measure. Simply put, total return measures the cash and value generated by an initial investment over a set period under specified economic conditions, i.e., dollars out compared to dollars in. Such a measure can be calculated for any asset and, as will be shown, for any liability.

Ex post, in judging the performance of an asset portfolio, the most important factor is the return earned. Expected realized return attempts to forecast realistically the earned return. In general, the return on an asset over a specified period of time and a specified scenario depends on:

1. the value of the asset at the beginning and end of the period,
2. the cash flows received during the period, and
3. the reinvestment income earned from those cash flows.

Over any past time period, the realized return on an asset or portfolio can be computed easily. All of the required data is available. In projecting a return over some future time period, however, the only known information is the current value of the security. The other information must be estimated using pricing models which project cash flows and terminal asset values. (These are exactly the same models that would be used in generating duration and convexity measures. RRO simply uses them differently.) The explicit computation of each component of realized return allows an investor to assume realistic or achievable reinvestment rates and to determine the value of the asset at each point in time along a particular scenario, allowing for an explicit judgment about whether the security will be called, put, or in the case of mortgage-backed securities, prepaid.

Required return refers to an investor's target return. Like realized return, required return also depends on a particular scenario and period of time. Model office projections of the liabilities produce cash flow requirements in a similar fashion to asset pricing models. Unlike the assets, however, the concept of market value is not well defined for liabilities.
There is a fundamental difference between RRO and immunization in the estimation of the value of the liabilities. Immunization relies on pricing models to calculate a market value of the liabilities along the same lines as the assets. RRO uses model office analysis to produce estimates of reserves, account values, and cash values at various points in time, which can be used as the market value equivalents for RRO; the choice between the three values depends on the investor's point of view. There is no reliance on hypothetical market values. For example, an insurance company funding an SPDA may feel comfortable if the assets cover at least the cash values in the first few policy years, and the account values in subsequent years.

The initial value of the liabilities does not rely on the measurement of a market value of the liabilities either. The initial value is simply the amount of funds available for investment. RRO does not require the liabilities to be fully funded at the outset; the initial market value of assets could be less than the initial value of liabilities. If the liabilities are underfunded, the assets just have to earn more to fund the liabilities. RRO shows how much more.

Required returns are calculated from the initial funds available and the estimates of cash flows and the terminal value of the liabilities. The required return in a particular scenario over a defined period of time is the return that must be earned on the initial funds available so as to meet the liability cash flows during the period and the terminal value of the liabilities at the end of the period.

E.3. OPTIMIZATION

Once the required returns are determined and the realized returns on each of the available assets have been calculated, the strategy is straightforward: Select a portfolio of assets such that for every scenario and time horizon under consideration, the realized return on the asset portfolio is at least equal to the required return on the liabilities.
In effect, a matrix of required returns is produced for each combination of scenario and time horizon. For example, if ten scenarios ranging from very bullish to very bearish are defined, and three time horizons are selected, say 1, 3, and 10 years, then 30 required returns would be calculated. The asset portfolio selected would have a realized return matrix such that the entries in the matrix are at least equal to the corresponding entries in the required return matrix.

Even when the timing and magnitude of the liabilities are certain, the required returns depend on the course of future interest rates. When the liabilities are uncertain, this dependence is even more pronounced. Similarly, the interest sensitivity of assets varies over time, as they near maturity, or in the case of mortgages, as the factor declines (i.e., as more and more of the principal is paid off). Therefore, the portfolio will need to be rebalanced, or at least re-examined, periodically.

To ensure the feasibility of a successful rebalance, the realized return on the portfolio must exceed the required return over the period until the next rebalance. This is accomplished by selecting that period as one of the time horizons in the analysis. This is similar to the convexity condition of immunization (the immediate return on the assets must exceed the return on the liabilities), with the difference that the strict marginality of the condition has been relaxed to encompass the entire period before the next rebalance, rather than the immediate interval before interest rates change.

It is worth stressing that the ability to consider multiple time horizons simultaneously is a feature of RRO that should be extremely attractive to liability funders. RRO allows the investor to specify both short and longer term investment targets, thereby enabling performance maximization over the short horizon while keeping an eye on the longer term adequacy of the assets.

The portfolio selection process is a straightforward application of mathematical programming. The required returns set minimum bounds for the performance of the assets. The investor can also specify other constraints, such as liquidity, quality, maturity, and coupon. In addition, minimum spreads over the required returns can
be specified. For example, the investor can specify a spread of 100 basis points over some selected required returns and 50 basis points over some others.

The notion of spread over a required return is an important feature of RRO because it allows the investor to define a margin for error and profit, another element of the Framework. A wider margin requires a larger spread. In contrast, immunization does not allow such a margin to be built in because the duration-matching condition is a strict equality.

There may be more than one combination of assets which meets all the constraints. The combination which is selected is based on an investment objective specified by the investor. For example, probabilities could be assigned to the scenarios, and the optimal portfolio would be chosen by maximizing the probability-weighted spread over the required returns over a selected time horizon.

The use of probabilities to weight the scenarios will cater to beliefs about the direction of interest rate changes. The probabilities assigned to a scenario can be generated in a manner consistent with the generation of the scenarios. This is the conventional basis for assigning probabilities and is based on the question: "How likely is it that this scenario will occur?"

RRO makes such a question irrelevant at the optimization phase. The scenarios of concern have already been selected. The required return targets specified as constraints ensure that if a scenario does occur, the assets meet the targets. The likelihood of the scenario is not the issue. Instead, the investor's utility function of portfolio performance across scenarios can be imposed. This is accomplished not by asking the investor to draw the utility curve but instead by asking the question: "Would you rank the scenarios according to the importance of performing well in each scenario?"

For example, consider a portfolio manager who has selected 10 scenarios; 5 specified by the Investment Policy Committee, 3 representing worst case situations and 2 representing most likely scenarios. The constraints ensure that the required returns will be met in all scenarios. The investor can specify zero probabilities to
the 5 scenarios set by the Committee. Some weight can be attached to the 3 worst case scenarios so that some outperformance over the liabilities can be targeted there. For the 2 most likely scenarios, very high probabilities can be assigned thereby producing a portfolio that will perform extremely well if these scenarios occur.

An illustration of the process is provided in Example 1 - Selecting A Portfolio. 5 scenarios are considered over one 12 month horizon. Probabilities are assigned to reflect the investor's risk preference. This investor wants to do best in scenario 3 and wants recognition of the performance in other scenarios to enter into the selection process as well. The liability returns have margins added to give the required returns. Both portfolios A and B have realized returns that outperform the required returns in each of the scenarios. The weighted average return of portfolio B is larger than that of A, so that B is the optimal portfolio.

What if a particular required return cannot be met? In such a case, the possible course of future interest rates that would jeopardize the ability to discharge the liabilities would be explicitly identified. Having been made aware of the risks involved, the investor can hedge against them through the use of options, futures and other instruments, albeit at the cost of lowering the return on the portfolio.

E.4. RISK MEASUREMENT - DOWNSIDE DEVIATION

Typically, the measure of risk used in portfolio management is the standard deviation of the returns about the expected or average returns. Such a risk measure implies that the goal of the investor is to earn a constant return regardless of interest rate scenario. Unfortunately, such an approach does not take into account any of the features of the liabilities.

Risk should refer to the possibility of not achieving a specified target. An improvement on the conventional approach would be to minimize the deviation of the asset returns from the corresponding required returns. This forces the selection of an asset portfolio which has a total return profile close to that of the liabilities. However, this would consider upside deviations from required returns to be just as
risky as downside ones. In other words, if the required return is 8% in a particular scenario and horizon, a portfolio that earns 10% would be considered just as risky as one that earns 6%. This is inappropriate because all risk has been removed once the target is met.

The appropriate measurement of risk for liability funding is downside deviation from the required returns. Alternative asset portfolios are ranked by the extent to which they underperform the required returns. In the event that there is no portfolio which meets the required return conditions in all scenarios, the portfolio selected would be the one that has the minimum downside deviation.

This definition of risk enables the construction of a risk-return frontier. A portfolio is efficient if, for a given level of downside deviation, it produces the maximum level of expected return. The investor can use this frontier to choose between alternative portfolios at different levels of risk and return.

Downside deviation is illustrated in Example 2 - Risk Vs. Return. The same scenarios, horizon and liability are considered as in Example 1. The two portfolios, C and D, cannot outperform the liability targets in all the scenarios. Portfolio C fails in scenario 5 and Portfolio D fails in scenario 2. Which is the riskier portfolio? If the standard deviation (SD) from the portfolio average return is used, D appears riskier. D appears riskier even when total deviation (TD) from required returns is considered. However, when only downside deviation (DD) is considered, both portfolios have the same risk level. Notice that portfolio D has a weighted average return higher than C. This implies that portfolio D is the preferred portfolio since it produces a higher return for a given level of risk. Portfolio D will appear on the risk-return frontier.
EXAMPLE 1
SELECTING A PORTFOLIO

(12-MONTH HORIZON)

<table>
<thead>
<tr>
<th>SCENARIO</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>AVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROBABILITY</td>
<td>10%</td>
<td>20%</td>
<td>30%</td>
<td>20%</td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td>LIABILITY RETURN</td>
<td>7.5%</td>
<td>8.0%</td>
<td>10.0%</td>
<td>10.5%</td>
<td>11.5%</td>
<td>9.75%</td>
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<tr>
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<td>0.0%</td>
<td>0.5%</td>
<td>1.5%</td>
<td>0.65%</td>
</tr>
<tr>
<td>REQUIRED RETURN</td>
<td>8.0%</td>
<td>9.0%</td>
<td>10.0%</td>
<td>11.0%</td>
<td>13.0%</td>
<td>10.40%</td>
</tr>
</tbody>
</table>

REALIZED RETURNS:

| PORTFOLIO A         | 10.0%| 10.5%| 11.0%| 11.5%| 13.0%| 11.30%  |
| PORTFOLIO B         | 8.0% | 9.5% | 12.0%| 12.0%| 14.0%| 11.50%  |
# EXAMPLE 2
## RISK VS. RETURN

(12-MONTH HORIZON)

<table>
<thead>
<tr>
<th>SCENARIO</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>AVERAGE</th>
<th>RISK ( x 1000 )</th>
</tr>
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<tbody>
<tr>
<td>PROBABILITY</td>
<td>10%</td>
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<td>30%</td>
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<tr>
<td>REQUIRED RETURN</td>
<td>8.0%</td>
<td>9.0%</td>
<td>10.0%</td>
<td>11.0%</td>
<td>13.0%</td>
<td>10.40%</td>
<td></td>
</tr>
</tbody>
</table>

**REALIZED RETURNS:**

| PORTFOLIO C   | 9.0%| 9.5%| 10.0%| 11.5%| 12.0%| 10.50%  | 0.2   | 0.4   | 1.2   |
| PORTFOLIO D   | 8.0%| 8.0%| 12.0%| 12.0%| 14.0%| 11.20%  | 0.2   | 1.8   | 5.0   |

DD = DOWNSIDE DEVIATION FROM REQUIRED RETURNS  
TD = TOTAL DEVIATION FROM REQUIRED RETURNS  
SD = DEVIATION FROM PORTFOLIO AVERAGE RETURN
F. CONCLUSION

RRO is a strategy designed to achieve total return performance over any given horizon and under many interest rate scenarios. The key to the strategy is the ability to define the targets in the same terms as the performance measure. The flexibility afforded by the strategy makes it particularly suitable for funding liabilities, especially interest sensitive ones, and for active management.

Some of the applications of RRO have been:

* Cross-market trading
* Funding life insurance liabilities
* Funding pension liabilities
* Meeting mutual fund performance goals
* Maximizing total return performance
* Achieving a floor total return performance across scenarios
* Enabling active management of interest rate risk

I look forward to your comments and to adding to this list of applications.
Mr. Shimpi has co-authored the articles listed below which describe Realized Return Optimization and its applications in greater detail:

