Corporate Pension Plan Design in a Mean-Variance Framework

Jihad S. Nader

Department of Finance, Concordia University, 1455 de Maisonneuve Blvd West, Montréal, Québec H3G 1M8, Canada

Summary

This paper develops a mean-variance (MV) framework for integrating pension plan design into corporate finance. Firm preference for defined benefit (DB) vs. defined contribution (DC) plans is analyzed by reference to MV dominance conditions. These conditions are shown to hinge upon the age structure of plan members, but they only yield dominance if mandatory flat-premium benefit insurance is incorporated into the analysis. In the latter case, "hybrid" DB-DC plans are found to dominate pure DB plans. The analysis is broadly supported by recently observed firm behaviour towards pension plan design.

Résumé

Modèle de Plan de Retraites d'Entreprise dans un Cadre d'Ecart de Moyenne

Cet article développe un cadre de moyenne - variance (MV) pour intégrer le modèle de plan de retraites dans les finances d'entreprise. La préférence des entreprises pour les plans de bénéfices définis (DB) par rapport aux plans de contribution définie (DC) est analysée par référence à des conditions de dominance MV. Ces conditions dépendant de la structure d'âge des membres du plan font l'objet de démonstrations, mais elles ne produisent une dominance que si l'assurance de bénéfices de prime uniforme réglementaire est incorporée dans l'analyse. Dans le dernier cas, des plans DB-DC "hybrides" dominent les plans purement DB. L'analyse est largement confirmée par l'attitude récemment observée chez les entreprises à l'égard de la conception des plans de retraite.
I. INTRODUCTION

The contractual literature of labor market economics, pioneered by Baily (1974), Gordon (1974), and Azariadis (1975), has provided numerous insights into the essence of corporate pension plans. One such insight is particularly profound and has special relevance for financial decision making in the pension area. This is that pension plan design, which encompasses the entire spectrum of benefit formulas, accrual methods, funding patterns, and a host of other provisions, is a mechanism through which firms effect a restructuring of the temporal relationship between workers' total compensation, and the value of their marginal product (VMP). This restructuring is a response to demand and price uncertainty in the product market, which translates through the workers' VMP into wage uncertainty in the labor market. For various reasons, workers (through their personal portfolios) are less able than firms to efficiently diversify this compensation risk in the capital markets.

Through compensation restructuring, firms express a commitment to absorb some portion of the workers' compensation risk into the corporate earnings stream, and thus internalize part of this risk into the capital market risk-reward tradeoff of the shareholders (Azariadis (1975), p. 1184).

The nature (implicit or explicit), extent, and time frame of the firm's commitment to play this insurance role are expressed through the specific pension plan design which the firm selects for effecting this compensation restructuring. To illustrate, it is useful to distinguish two "polar" cases. In the first case, the firm views the labor contract strictly in a spot (short-term, period-by-period) framework. Accordingly, the pension plan is designed such that the workers' total compensation equals their VMP in each and every period. In the second case, by contrast, the labor contract is viewed in a lifetime framework. Enforcement of such a contract dictates that the pension plan be designed

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1 Notes appear at the end.
to effect a total compensation path which is below the VMP path in the early part of the workers' career, and above it late in their career, so that total compensation and VMP are equated over the workers' entire worklife, rather than in each period.\textsuperscript{4}

The importance of the preceding insight lies in that it reveals a fundamental linkage among the product, labor and capital markets, through the vehicle of the pension plan. This linkage could bring significant conceptual enrichment to existing treatments of pension plan design in the corporate finance literature, which are mostly descriptive in nature.\textsuperscript{5} The purpose of this paper is to formally integrate pension plan design into the corporate finance framework, by exploiting the linkage just mentioned. Section II incorporates pension compensation, together with wage compensation, into the income statement setting of corporate earnings determination. Through the use of simplifying assumptions, this "augmented" income statement allows a formulation of the linkage among the uncertain product price, the workers' total compensation, and the shareholders' earnings stream. Section III applies this formulation to a defined contribution (DC) and a defined benefit (DB) plan,\textsuperscript{6} and develops the conditions for plan design preference by a shareholder-wealth maximizing firm, using the mean-variance (MV) dominance framework. These conditions are analyzed by referring to the labor contract underpinnings of each plan design. The analysis reveals that these dominance conditions hinge upon the age structure of the pension plan population. However, for any given age structure, it is shown that MV dominance does not exist between DC and DB plans, in the absence of benefit insurance. In this light, Section IV discusses factors which might explain the observed preference (and changes in preference) by firms for one plan design over another. The analysis indicates that the composition of mandatory flat-premium benefit insurance through the Pension Benefit Guaranty Corporation (PBGC) has caused "hybrid" (DB-DC) plans to dominate pure DB plans, and that, through successive and substantial increases in the insurance premium, the PBGC appears to have reinforced this dominance. Section V provides the conclusion.

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II. PENSION COSTS AND THE CORPORATE EARNINGS STREAM

Consider an "augmented" income statement\(^7\) for a single-product company which employs a homogeneous labor force. Let

\[ \begin{align*}
Q &= f(L, K) = \text{units of output produced and sold;} \\
L &= \text{number of workers employed to produce } Q; \\
K &= \text{capital employed to produce } Q; \\
\Pi &= \text{price per unit of } Q; \\
v &= \text{variable cost (excluding labor's compensation) per unit of } Q; \\
w &= \text{wage compensation per worker; } \\
W &= \text{total wage compensation; } \\
c &= \text{pension compensation per worker; } \\
C &= \text{total pension compensation;} \\
F &= \text{fixed operating costs; } \\
I &= \text{interest expense; and } \\
Y &= \text{net earnings to the shareholders}. \quad \text{8}
\end{align*} \]

Thus,

\[ Y = (\Pi - v)Q - F - I - (w + c)L = (\Pi - v)Q - F - I - (W + C). \]

The \( W + C \) represents the workers' total (wage plus pension) compensation. Thus, under a \textit{spot} labor contract, \( W + C = \text{VMP}_L \) in each and every period, while under a \textit{lifetime} contract \( W + C < \text{VMP}_L \) in early periods and \( W + C > \text{VMP}_L \) in late periods. Under both types of labor contract, \( \tilde{\text{VMP}}_Lt \) captures the product price uncertainty at any point in time \( t \).

That is:

\[ \tilde{\text{VMP}}_Lt = \tilde{\Pi}_t \frac{\partial Q_t}{\partial L_t} L_t. \]

The extent to which this uncertainty is absorbed into \( \tilde{Y}_t \) depends on how total compensation is structured (partly through the pension plan design) in relation to \( \tilde{\text{VMP}}_Lt \). To analyze this risk-absorption process, three simplifying assumptions are made. First, it is assumed that
a random, once-only change in the current $\pi$ will occur at the end of one period. The new price then will be $\bar{\pi}_1$, and no further price changes are foreseen in all subsequent periods. Second, it is assumed that $Q$, $L$, $K$, $v$, $F$ and $I$ will remain unchanged at time $1$. Third, at time $1$ workers are assumed to negotiate a salary adjustment at the rate $\bar{q}_1 = \bar{\pi}_1/\pi - 1$. In other words, in the absence of real productivity changes (as is implicit in the two preceding assumptions), the salary adjustment will merely maintain the "purchasing power" of the workers' salary, or (more precisely) the workers' "share" of the firm's real output. Thus,

\begin{equation}
\tilde{W}_1 = W(1 + \bar{q}_1) = W \frac{\bar{\pi}_1}{\pi}.
\end{equation}

It follows from equation (1) that the firm's net earnings at time $1$ are

\begin{equation}
\tilde{Y}_1 = (\tilde{\pi}_1 - v)Q - F - I - \left(\frac{\bar{\pi}_1}{\pi} + \bar{C}_1\right).
\end{equation}

Note that $\bar{C}_1$ is generally a function of $\tilde{W}_1$, i.e. $\bar{C}_1 = h(\tilde{W}_1)$, and that the parameters of $h$ are in fact parameters of pension plan design. By setting these parameters, the firm restructures total compensation in a manner consistent with its interpretation of the labor contract. Thus, each plan design may imply a different degree of compensation risk absorption into $\tilde{Y}_1$. In the next section, the absorption mechanism – the shareholders' risk-reward tradeoff – is compared for a DC and a DB plan.

III. CORPORATE PENSION PLAN DESIGN PREFERENCE
A. The Shareholders' Risk-Reward Tradeoff

Consider first a DC plan. The firm's pension obligation is discharged in each period by contributing a specified percentage, $a$, of that period's wages into the pension fund. It follows from equation (4) that
Using $E$ and $\sigma$ to denote the expectations operator and the standard deviation operator respectively, the shareholders' risk-reward tradeoff is

\begin{equation}
E(\tilde{Y}_1) = (Q - A)E(\tilde{\Pi}_1) - vQ - F - I
\end{equation}

and

\begin{equation}
\sigma(\tilde{Y}_1) = (Q - A)\sigma(\tilde{\Pi}_1),
\end{equation}

where

\begin{equation}
A = \frac{(1 + a)W}{\Pi}.
\end{equation}

The shareholders' risk-reward tradeoff under a DB plan can be derived in like manner. Assuming for simplicity that the DB plan promises pension benefits equal to $b\%$ of the final salary, for each year of service,\(^{11}\) the pension component of total compensation at time $t$ is obtained by discounting the benefits accrued for service in the first period, from the retirement date to time $t$.\(^{12}\) Thus

\begin{equation}
\tilde{C}_1^* = \left(\frac{b W^*}{\Pi}\right) \frac{\tilde{\Pi}_1}{(1 + r)^{N-1}},
\end{equation}

where $r$ = the riskless interest rate\(^{13}\) and $N$ = the number of periods from time $0$ to retirement. Substituting the value of $\tilde{C}_1^*$ for $\tilde{C}_1$ in equation (4) and simplifying, we have

\begin{equation}
\tilde{Y}_1^* = (Q - A^*)\tilde{\Pi}_1 - vQ - F - I.
\end{equation}
Applying the $E$ and $\sigma$ operators, the shareholders' risk-reward tradeoff is

\begin{equation}
E(\tilde{Y}^*_1) = (Q - A^*)E(\tilde{\Pi}_1) - vQ - F - I
\end{equation}

and

\begin{equation}
\sigma(\tilde{Y}^*_1) = (Q - A^*)\sigma(\tilde{\Pi}_1),
\end{equation}

where

\begin{equation}
A^* = \left[ 1 + \frac{b}{(1 + r)^{N-1}} \right] \frac{W^*}{I}.
\end{equation}

The meaning of $A$ and $A^*$ is noteworthy. As indicated by equation (8) and equation (13), these terms express the workers' total compensation at time 1 as the workers' share of the firm's real output, $Q$, under a DC and a DB plan respectively. Under both plan designs, the firm may seek to attain a higher (lower) risk-reward combination for the shareholders, by specifying a low (high) level of pension plan "generosity" through $a$ and $b$. In this way, the firm may attempt to control the degree to which product price risk (and hence compensation risk) to the workers is absorbed into the shareholders' risk-reward tradeoff, pursuant to equations (6) and (7), and (11) and (12). Note, however, that under both plan designs the workers can offset such control by the firm, through compensating wage differentials,\textsuperscript{14} which would make the initial $W$ and $W^*$ higher (lower) as $a$ and $b$ decreased (increased), such that the $A$ and $A^*$ demanded by the workers were left intact.

B. Plan Design and MV Dominance

The preceding result suggests an absence of a clear preference for a particular pension plan design by firms. To further explore this question, the MV dominance criterion is used. Under this criterion, rational, risk adverse firm managers will prefer to offer workers a DB plan if
with at least one strict inequality. A comparison of equations (11) and (12) with (6) and (7) respectively, reveals that (14) cannot be satisfied, since

\[
E(\tilde{Y}_1^*) \geq E(\tilde{Y}_1) \quad \text{and} \quad \sigma(\tilde{Y}_1^*) \leq \sigma(\tilde{Y}_1)
\]

That is, neither DB nor DC plans dominate in an MV sense, as depicted (for \(A^* < A\)) by the plan design opportunity set in Figure 1.

This result raises questions about observed firm behavior, which strongly suggests that firms may in fact have clear preferences regarding pension plan design. Why, for example, have DB plans been much more prevalent than DC plans, both in number and in membership? Why have so many firms, in the early 1980s, gradually transformed their DB plans into "hybrid" plans that contain many DC elements? Why have a growing number of newly formed corporate pension plans, as well as most plans replacing terminated DB plans in recent years, been designed as DC plans? The following section attempts to advance plausible answers to these questions.

IV. EXPLAINING OBSERVED FIRM BEHAVIOR TOWARDS PENSION PLAN DESIGN

Clues to answering the preceding questions can be sought in two distinct sources. One source is the contractual foundations of pension plan design, and their relationship to workforce age. The other is the legislative environment governing pension plans, particularly with regard to benefit insurance.

A. Plan Design, Workforce Age, and the Labor Contract

The contractual foundations of a given pension plan design can be discerned from the time path of pension compensation relative to total compensation. Under a DC plan, workers receive a constant fraction, \(a/(1 + a)\), of their total compensation, in the form of pension accruals in each and every period. This suggests that the firm views the labor
FIGURE 1

Pension Plan Design Efficient Frontier for A Young Workforce: No Benefit Insurance
contract in a spot framework, since the DC pension plan does not provide a mechanism for effecting deviations between the workers' periodic total compensation and $VMP_L$.

In contrast, the periodic pension accrual under a DB plan, expressed as a fraction $z_t$ of total compensation, is service-dependent, since (from equation (13))

$$z_t = \frac{b/(1+r)^{N-t}}{1 + b/(1+r)^{N-t}}$$

where $t$ is the number of periods of service by the workers. To determine the time path of $z_t$, it is convenient to use the continuous-time version of equation (16), from which it follows that

$$\frac{dz_t}{dt} = \left[ \frac{r be^r(t-N)}{1 + be^r(t-N)} \right] \left[ 1 - \frac{be^r(t-N)}{1 + be^r(t-N)} \right] > 0.$$

Thus, $z_t$ increases with length of service. This suggests that the firm holds a lifetime view of the labor contract, since the DB plan appears to serve as a mechanism whereby total compensation starts initially below $VMP_{Lt}$ and increases with service until it eventually exceeds $VMP_{Lt}$, as posited by the lifetime contract model.°

The preceding analysis gives a perspective on $A$ and $A^*$, and thus on the corporate risk-reward relationship under each plan design, as indicated by (15). For a young workforce, (i.e. for a long $N-t$ in equation (16)), total compensation at time 1 will be $A^* < VMP_{L1}$ under a DB plan, whereas $A = VMP$ under a DC plan. Thus $A^* < A$ and, pursuant to (15) $E(\tilde{Y}_1) > E(\tilde{Y}_1)$ and $\sigma(\tilde{Y}_1) > \sigma(\tilde{Y}_1)$. That is, offering a DB plan to a young workforce will entail a higher risk-reward combination for the firm than offering a DC plan, as illustrated in Figure 1. In this case,
despite the lack of MV dominance, firms may exhibit a preference for DB plans, which may be attributed to the managers' and stockholders' low degree of risk aversion. This preference would exist as long as the firm maintains a young workforce through continuous hiring of young workers, and through offering early retirement options and incentives for older workers. The observed prevalence of DB plans may be interpreted in this light.

In the same vein, observations of an evolving shift in pension plan design from DB plans towards DC plans, may to some extent be attributed to population aging, and the concomitant shift in the age structure of the workforce. For a DB plan whose membership is concentrated in the higher age-and-service cohorts, the preceding analysis of equation (17) indicates that \( A^* \) will be larger than for a DB plan with a younger membership. According to (15), a larger \( A^* \) will result in a lower corporate risk-reward combination. Thus, with workforce aging the upper end of the plan design opportunity set will not be at DB but at DB', as shown in Figure 1. Clearly, the plan design distinction between DB' and DC will not be as sharp as between DB and DC. Thus, workforce aging may be an important factor in explaining the observed "metamorphosis" of DB plans into DB-DC "hybrids" (e.g. DB' in Figure 1), recently reported by Pesando (1982).

B. Plan Design and Mandatory Benefit Insurance

Since the passage, in 1974, of the Employee Retirement Income Security Act (ERISA), U.S. firms with DB plans have been required to buy mandatory benefit insurance from the Pension Benefit Guaranty Corporation (PBGC) at a flat per-head premium. Firms with DC (and other non-DB) plans are not subject to this requirement. Denoting the insurance premium by \( \lambda \), and using double asterisks to distinguish the insured DB plan, it follows from equation (10) that

\[
\tilde{y}^{**}_1 = \tilde{y}^*_1 - \lambda L.
\]

Thus, equation (11) becomes

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while equation (12) remains unchanged. That is, $E(\tilde{Y}_{1}^{**}) < E(\tilde{Y}_{1}^{*})$ where as $\sigma(\tilde{Y}_{1}^{**}) = \sigma(\tilde{Y}_{1}^{*})$, as shown at DB** in Figure 2.

If firms paying the insurance premium were to raise the shareholders' earnings expectations back to $E(\tilde{Y}_{1}^{*})$, then they must attempt also to increase the corporate earnings risk beyond its initial level. In effect, this would involve shifting the insurance costs to the workers. For example, firms may design less "generous" DB pension plans by lowering $b$ in equation (13), until $E(\tilde{Y}_{1}^{*})$ and $\sigma(\tilde{Y}_{1}^{**})'$ were attained (DB' in Figure 2). However, as stated earlier, this would merely prompt the workers to negotiate an offsetting increase in $W^*$, such that their total time 1 compensation of $A^*$ is left intact. Thus, DB' would be infeasible as a response by firms to benefit insurance. Firms' response may instead take the form of adopting a "hybrid" plan design, H', which dominates the insured DB** as shown in Figure 2.\textsuperscript{24} The larger is $\lambda$, the greater is the extent to which "hybrid" plan designs will reflect DC aspects. This is depicted by H'' and H''' in Figure 2, for the case where $\lambda$ is doubled and tripled respectively.

The preceding analysis is broadly supported by actual plan design developments in recent years. For example, shortly after the introduction of mandatory benefit insurance in Ontario,\textsuperscript{25} Pesando (1982) reported an apparent "metamorphosis" of many DB plans in Canada into DB-DC "hybrids." A similar and more pronounced trend has been observed in the U.S., coincident with the substantial increases (both actual and planned) in the PBGC insurance premiums since the early 1980s.\textsuperscript{26} Many firms with ongoing DB plans in that period have considered a switch to a DC plan design.\textsuperscript{27} Moreover, most firms which terminated their overfunded DB plans in that period have replaced them with DC plans.\textsuperscript{28}

V. SUMMARY

Treatments of pension funds in the corporate finance literature
FIGURE 2
Pension Plan Design "Metamorphosis" Under Flat-Premium Benefit Insurance
have generally lacked a formal theoretical framework for analyzing plan design preference by shareholder-wealth maximizing firms. This paper has presented such a framework, which uses the "augmented" income statement as a point of departure, and develops MV dominance conditions for analyzing firm preference for DB vs. DC pension plans. The conceptual richness of this framework lies in that it employs an insight from the contractual literature, regarding the existence of a linkage among the product, labor and capital markets, through pension plan design. Using this analytical framework, this paper has shown that the conditions for for MV dominance between DB and DC plans hinge upon the age structure of the workforce. However, for any given age structure, MV dominance does not exist in the absence of pension benefit insurance. Thus, in such (pre-ERISA) conditions, the prevalence of DB plans may not be attributed to their MV dominance over DC plans, but (for a young workforce) possibly to the prevalence of high risk tolerance among corporate managers and shareholders. Similarly, the reduction in prevalence of DB plans may be attributed to workforce aging, rather than to MV dominance of DC plans.

Under mandatory flat-premium benefit insurance, the above framework indicates that for a young workforce, DB plans are dominated by "hybrid" (DB-DC) plan designs. The larger is the insurance premium, the more efficient it is for firms to incorporate DC features into their pension plans and, ultimately, to switch entirely to a DC plan design.

These analytical results are highly representative of recently observed firm behavior towards pension plan design. Thus, the conceptual framework developed in this paper may serve more than the limited purpose of incorporating pension plan design into corporate finance. Expanded properly, this framework may also prove useful for analyzing the impact, on firms, of alternative insurance premium structures presently under consideration by the PBGC.29
NOTES

1. For a detailed technical discussion of pension plan design, see McGill (1984).

2. The most commonly mentioned reasons are the difficulties in diversifying human capital, the limited access of workers to capital markets, and the workers' high degree of risk aversion. See Baily (1974), Azariadis (1975), and Polemarchakis (1979).


4. Lazear (1979) shows that such a lifetime compensation path is optimal for the firm, due to the presence of a lag before the firm can detect worker "cheating" (shirking or underperformance). Bulow (1982) points out that such a compensation path reflects implicit pension obligations for the firm, beyond what is explicitly specified under the terms of the pension plan.


6. These are by far the two most important types of plan designs, both in terms of prevalence and membership. See McGill (1984).

7. Treynor, Regan and Priest (1976) were among the first to introduce the notion of "augmented" financial statements.

8. For simplicity, and without any loss of generality, the analysis abstracts from taxes. The incorporation of a tax rate $T$ would merely cause both sides of equation (1) below to be multiplied by $(1 - T)$.

9. The shorter is the period from time 0 to time 1, the more reasonable is this assumption.

10. We return to this point later, in the analysis of equation (8) and equation (13).
It can readily be shown that the results of the analysis also hold for other benefit formulas, such as career average, final average, and so on.

The asterisk distinguishes the DB case. Note that for each plan design the workers may factor a different "compensating differential" into their current wage settlement. Thus, in general $W^* \neq W$ and, as a consequence, $\hat{C}_1^* \neq \hat{C}_1$. For a theoretical exposition of compensating differentials, see Rosen (1974). For empirical evidence on compensating wage differentials for pensions, see Schiller and Weiss (1980) and Smith (1981).

The appropriate discount rate for valuing the firm's pension obligation is the riskless rate. As pointed out by Treynor, Regan and Priest (1976), this rate reflects the highest degree of certainty that the firm will discharge its contractual pension obligation.

See the references cited in footnote 12.

The lifetime contract model assumes that firm managers and shareholders have low risk aversion. See for example Akerlof and Miyazaki (1980). Some authors, such as Azariadis (1975) and Polemarchakis (1979), even assume that managers and shareholders are risk neutral.

Lazear (1979) shows that DB plans and mandatory retirement are complementary mechanisms for enforcement of lifetime labor contracts. Evidence on early retirement options is documented in Clark and McDermed (1982).

Such a "metamorphosis" would of course leave the workers with a larger share of compensation risk. For a discussion of this issue,
24 H' is some combination of DB and DC. The workers' total time 1 compensation at H' is $\bar{A}$, which is some average of $A^*$ and $A$. For a young workforce, $A > \bar{A} > A^*$ as stated above. Thus, to attain H' the firm must increase the workers' total time 1 compensation from $A^*$ to $\bar{A}$.

25 In December 1980, Bill 214 introduced a Pension Benefit Guarantee Fund in Ontario, to provide mandatory benefit insurance for DB plans.

26 See Allen, Melone and Rosenbloom (1984), p. 381, for a discussion of these increases.


29 For a discussion of these alternatives, see McGill (1984), pp. 598-602.
REFERENCES


