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Unrealized Gains in Stocks from the Viewpoint of Investment Risk Management

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Summary

On the cost basis or "cost or market whichever is lower" basis (adopted in Japan and some European countries), most insurance companies have more or less excess of market over book value of equities.

This paper focuses on those unrealized gains in stocks and those mechanism for controlling evaluation loss in stocks on "cost or market whichever is lower" basis, from the viewpoint of the life insurance company of which portfolio consist of traditional fixed premium products.

First, the anticipated evaluation loss in the stock portfolio is estimated by the level of those unrealized gains, for reservation for future possible evaluation loss in stocks.

Second, the preferable level of unrealized gains which have preferable level of risk of evaluation loss is estimated on buy-and-hold strategy (which is typical in investment in stocks in the life insurance companies above).

And, estimations above are based on some simple stochastic models (the lognormal distribution).

Résumé

Bénéfices d'Actions Non Réalisés du Point de Vue de le Gestion du Risque de Placement

Sur la base des coûts ou sur la base "coûts ou marché selon celui qui est le plus faible" (adoptée au Japon et dans quelques pays européens), la plupart des compagnies d'assurance ont plus ou moins un surplus de marché sur la valeur comptable des actions ordinaires.

Cet article se concentre sur les bénéfices d'actions non réalisés et sur les mécanismes destinés à contrôler la perte d'évaluation dans les actions sur une base "coûts ou marché selon celui qui est le plus faible", du point de vue de la compagnie d'assurance sur la vie dont le portefeuille consiste en produits traditionnels à prime fixe.

Premièrement, la perte d'évaluation anticipée dans le portefeuille d'actions est estimée par le niveau des bénéfices non réalisés, pour réserve de pertes d'évaluation futures éventuelles d'actions.

Deuxièmement, le niveau préférable des bénéfices non réalisés qui ont un niveau préférable de risque de perte d'évaluation est estimé sur une stratégie acheter-et-conserver (qui est caractéristique de l'investissement en actions des compagnies d'assurance sur la vie ci-dessus).

De plus, les estimations ci-dessus sont basées sur quelques modèles stochastiques simples (la distribution lognormale).

1. Introduction

In many countries, legal restrictions require insurance companies to reserve a certain percentage of capital gains in equities. In Japanese insurance companies, listed equities are accounted on "cost or market whichever is lower" basis. Insurance law no.86 requires insurance companies to reserve excess of realized capital gains in stocks over capital losses for particular reserve for insurance law no.86, as far as such surplus is exceptionally used for special dividend in termination. To appropriate evaluation gains in stocks is generally not permited in accounting rules (except for variable life insurances). Approval of the competent authorities is always needed to free from accumulation of surplus to no.86-reserve. In reality, many insurance companies have more or less excess of market over book value of domestic stocks.

Excess of market over book value of domestic stocks of 5 life insurance companies in Japan at the end of the fiscal year 1989

company	(billions of yen)	ratio*
Nippon	8,842	162 %
Daiichi	5,641	140 %
Sumitomo	3,251	96 %
Meiji	4,350	198 %
Asahi	2,609	130 %

* ratio = excess of market over book value / book value

What is the significance of those unrealized gains ?
A function of those unrealized gains is a provision for special dividend in termination. And BIS regulation(the required goal of capital adequacy is 8%) for Japanese banks admit to appropriate 45% of excess of market over book value of stocks in their own funds. And, a part of it may be recognized as a bubble of the stock market.
Are there any other factors including more strategic viewpoints?

2. To control evaluation loss

It is easy to see that excess of market over book value of stocks can work for making up for evaluation loss. From this point of view, it will be useful to anticipate future possible evaluation loss in the stock portfolio.

2-1 Formula of the anticipated evaluation loss

The most popular model for the evolution of stock prices is the lognormal distribution, that is

$$St/S_0 = \exp(\mu t + \sigma\sqrt{t}Z)$$

where

Z:The standard normal random variable with mean 0, standard deviation 1.

μ:The mean logarithmic stock return per unit time.

σ:The standard deviation of logarithmic stock return per unit time.

St/S$_0$:The stock return over the instant 0 to t.

(μ and σ are defined per security.)

Here,we note that B is the book value of a security,and St is the price of a security at time t.

Anticipated evaluation loss is

$$E[\max(0, B-St)] = B \cdot \text{Prob}[St<B] - \int_{St<B} St \cdot \exp(-z^2/2) \cdot 1/\sqrt{(2\pi)}\, dz$$

The first term of the formula above is

$$B \cdot \text{Prob}[Z<-\{\log(S_0/B)+\mu t\}/\sigma\sqrt{t}] = B \cdot \{1-N(\{\log(S_0/B)+\mu t\}/\sigma\sqrt{t})\} = B \cdot N(-h)$$

where $h = \{\log(S_0/B)+\mu t\}/\sigma\sqrt{t}$

N; The cumulative density function of standard normal distribution.

The second term is

$$\int_{-\infty}^{-h} St \cdot \exp(-z^2/2) \cdot 1/\sqrt{(2\pi)}\, dz = \int_{-\infty}^{-h} S_0 \cdot \exp(\mu t + \sigma\sqrt{t}\, z) \cdot \exp(-z^2/2) \cdot 1/\sqrt{(2\pi)}\, dz = S_0 \cdot \exp(\mu t + 0.5\sigma^2 t) \cdot \int_{k}^{\infty} \exp(-x^2/2) \cdot 1/\sqrt{(2\pi)}\, dx$$

where $x = \sigma\sqrt{t} - z$, $k = \sigma\sqrt{t} + h$

$$= S_0 \cdot \exp(\mu t + 0.5\sigma^2 t) \cdot N(-\sigma\sqrt{t} - h)$$

Hence, $E[\max(0, B - St)]$

$$= B \cdot N(-h) - S_0 \cdot \exp(\mu t + 0.5\sigma^2 t) \cdot N(-\sigma\sqrt{t} - h) \quad *(2\text{-}1\ a)*$$

Formula (2-1 a) can be reexpressed by risk neutrality argument.

$$\Delta St/St = \sigma\sqrt{t}Z + (\mu + 0.5\sigma^2 Z^2)\Delta t$$

$$E[\Delta St/St] = (\mu + 0.5\sigma^2)\Delta t$$

Because of the assumption of risk neutrality of the market,

$$E[\Delta St/St] = r\Delta t \quad (r\ ;\ \text{risk free interest rate})$$

then $r = \mu + 0.5\sigma^2$

Hence, $E[\max(0, B - St)]$

$$= B \cdot N(-h) - S_0 \cdot \exp(rt) \cdot N(-\sigma\sqrt{t} - h) \quad *(2\text{-}1\ b)*$$

where $h = \{\log(S_0/B) + (r - 0.5\sigma^2)t\} / \sigma\sqrt{t}$

Now, anticipated evaluation loss in the stock portfolio is

$$\sum_{i \in S^*} E[\max(0, {}_iB - {}_iSt)]$$

S* ; securities on the portfolio

2-2 Preferable level of the reservation for evaluation loss

The level of the reserve for the future possible evaluation loss and the level of the unrealized gains in the portfolio are not to be independent each other.

To make up evaluation loss in stocks, those unrealized gains and the reserve can help each other.

So, to determine the level of the reservation for future possible evaluation loss in stocks, formula (2-1a) and (2-1b) are helpful.

Following are trial calculations on 2 simple assumptions, and they show preferable level of the reservation for evaluation loss of the next year by the ratio to the book value of stocks.

[Assumption 1]

1)We define the model security* of which daily returns have properties that the mean and the standard deviation equal to the average of those means and standard deviations of component securities of Nikkei-225.

2)Here, we use daily logarithmic return on stocks of Nikkei-225 over the term 1983.1~1987.12 (quoted from "The evolution of stock prices in Japan" 1989, Kariya,Tsukuda,Maru).

	mean	standard deviation
Nikkei-225	0.07%	0.94%
Model security*	0.069%	2.25%

Figures above should be transfered to annual basis by multiplying $\times 250$ or $\times\sqrt{250}$ (business day basis).

3)The portfolio considered here consists of model securities, and the ratio(= excess of market over book value / book value) of each of the securities in the portfolio are the same.

4)There are no changes in book value of stocks for one year.

[Assumption 2]

1)We define the model security of which volatility is 2.4 times that of Nikkei -225 option. (2.4 is imported from the ratio of standard deviation in Assumption 1.)

2)Here, we use implicit volatility of Nikkei-225 option and interest rate. We choose that 33% as volatility, 7% as risk free interest rate. They are typical data in the 4th quarter of 1989.

3)The portfolio on this assumption consists of model securities, and the ratio (= excess of market over book value / book value) of each of the securities in the portfolio are the same.

4)There are no changes in book value of stocks for one year.

2-2-1 Trial calculation in Assumption 1

In Assumption 1, anticipated evaluation loss in the stock portfolio (for 1 year) can be calculated by formula (2-1a).

Ratio* at the beginnig of the year	anticipated evaluation loss / book value at the end of the year
0 %	6.0 %
10 %	3.9 %
30 %	1.6 %
50 %	0.7 %
70 %	0.3 %
90 %	0.1 %

* ratio = excess of market over book value / book value

2-2-2 Trial calculation in Assumption 2

Trial calculation in Assumption 1 shows the anticipated level of the

evaluation loss in a moderate situation of the stock market.
In assumption 2, we can calculate it in more volatile situations imported from the implicit volatilities of Nikkei-225 option with formula (2-1b).

Ratio* at the beginnig of the year	anticipated evaluation loss / book value at the end of the year
0 %	28.4 %
100 %	9.9 %
200 %	4.2 %
300 %	2.0 %

* ratio = excess of market over book value / book value

2-3 Preferable level of risk of evaluation loss in stocks

For insurance companies of which portfolios consist of traditional fixed premium products,the typical strategy of investment in stocks is a buy-and-hold strategy.
What is the preferable level of risk of evaluation loss in stocks especially on buy-and-hold strategy?
One idea is that the excess of expected return(market value) over the floor rate (which is required to exceed credited interest rates of insurance products) should be greater than anticipated evaluation loss.
If not so, it would be better to invest in other safety assets.
The idea is expressed that,

$$E\,[\max(0,\ St-F)] \ >\ \sum_{i\,\in\,S^*} E\,[\max(0,\ {}_iB-{}_iSt)]$$

S^* ; securities in the portfolio

where St ;Total price of the stock portfolio.
${}_iSt$;Price of a security in the portfolio.
${}_iB$;Book value of a security in the portfolio.
F ;Floor rate of the stock portfolio($= S_0 \times(1+f)$).

The excess of expected return over the floor rate is,

$$E\,[\max(0,St-F)]$$
$$=-(1+f)S_0 \cdot N\,(h) + S_0 \cdot \exp(\mu t+0.5\sigma^2 t)\cdot N\,(\sigma\sqrt{t}+h) \qquad *(2\text{-}3a)*$$

where $h=\{\log(S_0/B)+\mu t\}\ /\ \sigma\sqrt{t}$

The folmula above can be reexpressed by risk neutrality argument.

$$-(1+f)S_0 \cdot N\,(h) + S_0 \cdot \exp(rt)\cdot N\,(\sigma\sqrt{t}+h) \qquad *(2\text{-}3b)*$$

where $h=\{\log(S_0/F)+(r-0.5\sigma^2)\,t\}\ /\ \sigma\sqrt{t}$

The inequality above will be satisfied by controling the ratio (= excess of market over book value / book value). Following is a trial calculation.

Here, we set that St has the same properties of that of Nikkei-225, and the floor rate is 7%. In Assumption2 (with formula(2-1b),(2-3b)), the preferable ratio above is 35% and more.

3. Conclusion

1) There is ambiguous meaning in holding unrealized gains in stocks. But it is important that a significant function of holding such unrealized gains in stocks is to have a natural (built in) resistibility to its investment risks, as we have seen.

2) Calculations based on even simpler assumptions (as we have seen) will be useful for determination of any regulations or standards on unrealized gains on stocks, since there are few investment regulations that have any mathematical background in our insurance industries.
It will be more interesting for investment risk management to do the above calculations on the real portfolios of stocks with more details.

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