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Determinism and Chaos in Long Financial Time Series

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Summary

The academic finance literature offers broad support for the view that stock price movements are a random process in which prices respond to randomly arriving information. Recent developments in nonlinear dynamics open the possibility of reconciling this view with the "chartist" perspective that patterns of price fluctuations are, at least partly, deterministic. This reconciliation is possible if the price fluctuations are "chaotic".

We offer a detailed analysis of "correlation dimension "for long series of index data and find firm evidence for deterministic behaviour. We offer tentative evidence for a qualitative change in observed dynamics in the 1980's. Finally we discuss alternative methods of analysis that may offer further insight on the problem of distinguishing chaos from noise.

Résumé

Determinisme et Chaos dans des Séries Chronologiques Financières Longues

La documentation financière académique soutient l'idée que les mouvements des prix des actions sont un processus aléatoire dans lequel les prix réagissent à des informations survenant de manière aléatoire. Des progrès récents dans la dynamique non linéaire ouvrent la possibilité de réconcilier ce point de vue avec la perspective "graphique" (chartist) selon laquelle les schémas de fluctuation des prix sont au moins en partie, déterministes. Cette réconciliation est possible si les fluctuations de prix sont "chaotiques".

Nous offrons une analyse détaillée de "dimension de corrélation" pour de longues séries de données indexées et nous trouvons des preuves solides de comportement déterministe. Nous offrons des preuves non définitives d'un changement qualitatif dans la dynamique observée dans les années 80. Enfin, nous discutons d'autres méthodes d'analyses qui peuvent offrir une meilleure compréhension du problème qui consiste à distinguer le chaos du bruit.

I. INTRODUCTION

Since Kendall's (1953) seminal paper on the movement of stock market prices, researchers in finance have attempted to identify exploitable patterns in stock return series (see e.g.Fama (1970) and references therein). The balance of evidence from studies using standard autoregression techniques supports market efficiency, although anomalies such as the weekend and year-end effects (French (1980)) have been identified, and recent papers by Jegadeesh (1990), Fama and French (1988)and Lo and MacKinley (1988) have succeeded in identifying statistically significant serial correlations in stock returns. The status of these recent studies is still open to debate; for example the apparent anomalous returns of Jegadeesh rely on a risk adjustment based on a specific, and possibly inappropriate, asset pricing model. Within the academic finance literature there remains broad support for weak form market efficiency, and the apparent random nature of stock price changes is rationalised as a process whereby share prices immediately and unbiasedly impound randomly arriving information.

In spite of this broad consensus in the academic literature, there is a substantial market for "chartist services" within the financial services industry. Chartists search for, and claim to find, patterns of price fluctuations which repeat themselves, and can therefore be used to forecast price changes and make abnormal returns. Until now the two points of view, represented by the chartists on the one hand and the academic financial economists on the other, were irreconcilable but recent developments in non-linear dynamics, in particular those topics related to "deterministic chaos" provide a possible way of accommodating both points of view within a single theory. The insight from this work is that the fluctuations in any variable may be described by simple deterministic rules and yet over all but the shortest time scales appear random and aperiodic.Appreciation of this has led to considerable effort being devoted to the statistical problem of distinguishing chaos from random noise.

In its most general form this problem is probably insoluble - for example the pseudo-random number generators in computers are generated by deterministic rules, yet are designed to produce sequences of numbers effectively indistinguishable from a "true" random sequence. For practical purposes it is appropriate to tackle the more restrictive problem of distinguishing random noise from "low dimensional chaos" (defined rigorously below, but loosely characterised as chaos in a system whose deterministic dynamics only involve a small number of variables). Linear, autoregressive models, such as those used by Jegadeesh (1990) cannot of themselves generate chaotic fluctuations, and tests for chaos attempt to identify relevant features induced by nonlinearities. Previous workers have used three types of test:

1. Calculations of "correlation dimension".

2. Calculations of "Lyapunov exponents" - numbers which characterise any tendency of trajectories starting from similar values to diverge rapidly.

3. Tests related to nonlinear forecasting.

Of these, the first has proved by far the most popular to workers in finance, with one particular algorithm - that of Grassberger and Proccacia (1983a, 1983b, 1983c) - established as the trade standard. In this paper we join this particular bandwagon for a study of four indices. We have work in progress using the other two approaches. These will be reported in a subsequent paper.

The Grassberger-Proccacia algorithm was originally used on computer-generated data from iterated mappings and on laboratory experiments; in both cases tens of thousands of data points were typically available and noise contamination was small. However, in financial time series the raw data is rarely of sufficient quality or quantity to enable the direct computation of correlation dimension, due to the low effective signal to noise ratio. To combat this, the data is usually manipulated to remove several extrinsic effects, e.g. the Monday, Wednesday, Friday, and January effects (French (1980), Ariel (1987)). This approach requires that one distinguish between effects driven by forces outside the system, which should be removed, and effects which are intrinsic, and therefore contain information about the underlying dynamics.

Previous applications of this method to financial index data have produced interestingly low values for the dimension of the underlying attractor, the most notable being the analysis of CRSP data by Scheinkman and LeBaron (1989) which produced evidence for a six dimensional attractor (see also Brock (1988)). This result contrasts with analysis of German blue chip stocks by Booth et al (1990) which, although showing some nonlinear behaviour, apparently exhibit no low dimensional attractor. Frank and Stengos (1986) also give a value of around six for gold and silver returns, while T-bill returns (Brock (1988)) and Barnett's Divisia index (Barnett and Chen (1988)) produce much lower values of around two and three. The possible effect of extrinsic noise on such results has been investigated by Lines (1989), who concludes that the addition of noise to a system of given dimension will increase the value of its observed correlation dimension.(see also Mayer-Kress (1984))

In this paper we attempt a detailed analysis of correlation dimension for four long series of index data. The work is distinctive in two ways. First, by a variety of transformations of the data we are able to tease out some regularities that have been missed in previous studies. Second, our choice of indices for analysis permits international comparisons: similar results for all four indices would suggest that any observed determinism is related to the (unknown) economic variables for which the indices are surrogates, while major differences would point to effects of local features (e.g. trading rules).

II. DATA AND METHODOLOGY

The analysis described below was carried out on four major financial indices. The data was sampled on a weekly basis using Wednesday closing values in order to remove effect of the weekend trading gap and so enable the time series to be pseudo-continuous. This however greatly reduced the amount of data available. The indices used were :-

Financial Times All Share Index	- 1965-1989
Dow Jones Index	- 1969-1986
Standard and Poor Composite Index	- 1965-1986
Nikkei Index	- 1966-1989

Implicit in the idea of dimensionality is the notion that index movements may be represented by the motion of a representative point in some "state space"[1]. Random noise has, by definition an infinite number of degrees of freedom, that is, it will normally fill some region of the state space completely, irrespective of the dimension of that state space. This contrasts with motion in accordance with some non-linear system of equations with a finite number of degrees of freedom, for which at some value of state space dimension, the motion of the representative point will fail to fill any hypervolume of state space *irrespective of how long we wait.* For systems which have *attractors*, i.e. where the long term evolution of the system is bound to a hyper-surface in state space, then obviously when the dimension of the state space is greater than the dimension of the attractor, increasing it further will not change the distribution of points in the space.

In order to measure the distribution of points in state space we calculate the *correlation dimension,* using the algorithm of Grassberger and Procaccia. We first select a state space by the procedure of footnote 1; its dimension is known as the *embedding dimension, E.* For a given embedding dimension, the correlation dimension, $d(E)$, is defined via the number, $C(r)$, of hyper-spheres of radius r needed to cover the surface of the attractor as the size of these spheres tends to zero. Formally

1. Plotting in state space is analogous to plotting functions using x,y,z coordinates, but replacing the axes with physical states of the system. For example - plotting profit vs. cash vs. assets. Here we plot x_t vs. x_{t+1} vs. x_{t+2}, where x_t is the value of the index at time t. See Takens (1981)

$$d(E) = \lim_{r \to 0} \{\log C(r)/\log r\}$$

There are practical problems in evaluating this limit. One well-tried approach is to identify an approximately linear region in the graph of log $C(r)$ versus log r. To do this objectively, we first computed numerical derivatives to locate a roughly linear region, then performed a linear regression within that region. If this proved impossible, an estimate of the maximum correlation dimension was sought.

The above procedure can be repeated for different choices of embedding dimension, and it is the form of dependence of $d(E)$ on E that we use to distinguish deterministic from random motion. For random data where the 'attractor' is in effect embedded within itself, the correlation dimension will obviously just reflect the embedding dimension used. Therefore, as we can never embed random data in a sufficiently high state space, the value of $d(E)$ will rise to infinity with increasing E. For an attractor, however, once the embedding dimension is substantially[2] greater than the dimension of the attractor the value of d should remain constant, by the above arguments. Figure 1 shows three graphs which illustrate the different possibilities.

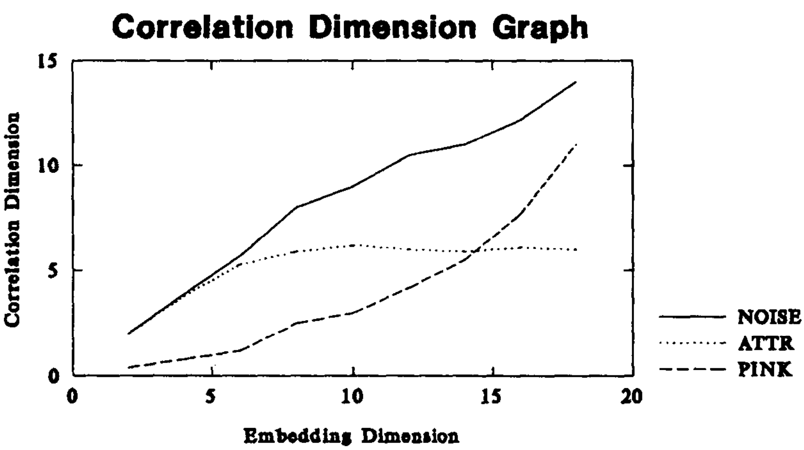

Figure 1 Variation of the correlation dimension with embedding dimension for 3 systems: (A) White noise (Noise). (B) An attractor system (Attr). (C) A random system with power-law spectra (Pink).

2. For a system of dimension D, the minimum state space dimension required for a complete representation of the system is 2D+1 (Takens, 1981).

There remains the (likely) possibility that our time series is a superposition of some exogenous noise on to a finite attractor; in this case the calculated limiting correlation dimension will exceed that of the underlying attractor (Lines (1989)).

All the above analysis is compounded by slow, aperiodic secular trends which can dominate analysis of the data so that the fine structure of the deep dynamics is lost. The traditional way round this problem is to transform the data (e.g. by taking logs and differencing). To aid our choice of a suitable differencing scheme, we made use of a non-linear forecasting method due to Sugihara and May (1990). This is the subject of the next section.

III. TRANSFORMING THE DATA

Blythe and Stokes (1988) have shown that secular trends in data can seriously bias correlation dimension estimates. Our ability to find evidence of a deterministic attractor can thus be improved by taking logarithms and differencing, thereby removing any exponential trend. In this context a suitable choice of difference can increase the apparent determinism[1]. The difference selected must be large enough to increase the effective signal, yet small enough so that it does not remove small scale structure from the signal. The optimum value was found using a non-linear forecasting method recently popularised by Sugihara and May (1990), and was found to be the 8th difference. This method looks at how similar circumstances in the past evolved, and takes the average of these evolved futures to be an approximation to the future of the present system. In practice, the time series is divided into 2 parts, one library of past points and one of predictee points. If the data is embedded in E dimensional state space, then for each predictee point, the $E+1$ nearest neighbour points from the library of past behaviour are found. These points form a simple shape around the predictee point called a simplex. Each point forms a vertex of the simplex and these vertices are moved forward in time - by replacing the nearest neighbour point with the next point along in the original time series.

Thus the simplex evolves along the trajectories of its vertices to form a new simplex shape. The centroid of this new simplex is taken to be an approximation to the next point along from the original predictee point. This procedure is repeated for a number of predictee points and a number of different evolution times. The resulting approximations are then compared to the actual next in line points and the linear correlation coefficient of the two data sets is calculated.

The graph of correlation coefficient versus prediction time allows distinction between 3 classes of system. Random data will naturally give a zero correlation for any value of prediction time, while an integrable system should produce a coefficient close to one for all values of time. Chaotic systems, on the other hand, are characterised by their exponential divergence of nearby trajectories in state space so, for small enough prediction time, they should be predictable. As the trajectories diverge the observed

1.By 8th differences, we mean a time series of points of which the first is (log(8th point)-log(1st point)), the second is (log(9th point)-log(2nd point)) etc.

predictability should therefore drop, the rate at which it drops being proportional to the rate of divergence of the trajectories.

Figure 2 shows the graph of correlation coefficient versus difference used for varying values of prediction time, for the F.T.A.S. data. For low-order differences (first or second), the predictions are very poor, implying that the forecasts are little better than random. The performance then improves rapidly until around the 8th-10th difference, after which there is a slow approach to a plateau. This in itself is not indicative of determinism, as differencing a random walk produces a moving average process with significant serial correlation. Recognising this, later results obtained from differenced financial indices were contrasted with those of differenced random walks.

As our prime interest is in returns, we would prefer to work with the lowest practical difference. However, Fig. 2 points to substantially enhanced determinism with the higher differences. As a crude compromise, we decided to concentrate our analysis on 8th differences - close to the "knee" of the curves in Fig. 2. The region incorporating the 52nd difference is also of interest, The large effective sampling rate used seems to miss regions of fine structure which the 8th difference detects, and could well coincide with some periodic behaviour of the attractor. This region proved harder to analyse numerically so only tentative speculations were possible.

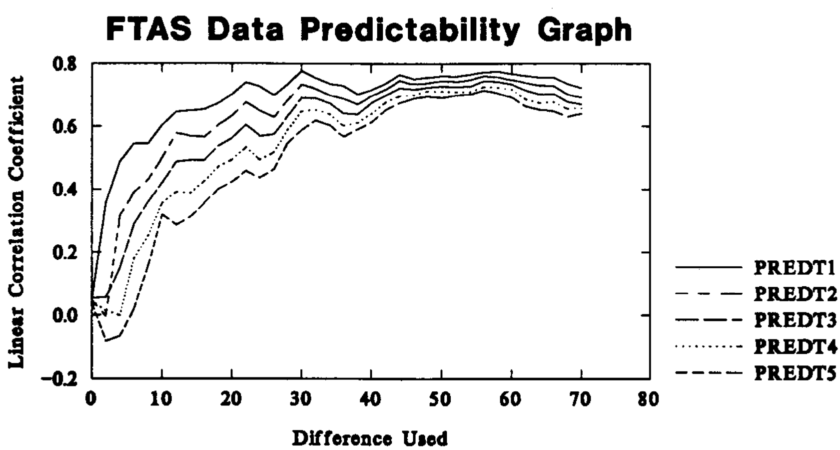

Figure 2 Predictability (linear correlation coefficient) versus differencing used for different values of prediction time. The values of prediction time used are from 1 to 5 weekly time steps.

IV. CORRELATION DIMENSION ESTIMATES

The correlation dimension was calculated for the 8th difference of the log-transformed data, as previously detailed. To confirm that the dimension estimates obtained were not due to a distributive property of the data set, the same analysis was performed on 'mixed' data. That is, the same data was reordered randomly. If the correlation dimensions obtained in the first tests were to have been produced by an attractor, then the mixed data should exhibit markedly higher dimensions and be close to the values produced by random data (Ramsay *et al* (1990)).

The range of embedding dimensions used was generally from 4 to 18, and for each embedding dimension the correlation dimension was calculated. Figure 3 shows the resulting graphs for all four indices. The asymptotic values are very well defined, the graphs rising quickly and remaining constant for very large values of embedding dimension, emphasising that these values are not temporary plateaus. The results for all the indices, and their randomised counterparts, are given in Table 1.

Data Set	Correlation Dim.	Mixed Data Cor. Dim.
F.T.A.S.	5.9 ± 0.3	> 14
Nikkei	6.9 ± 0.4	> 13
Dow Jones	7.0 ± 0.5	> 11
S&P Comp.	6.2 ± 0.3	> 12

Table 1: Correlation dimension results.
Asymptotic correlation dimensions are given for each index. The values given for the mixed data are not asymptotic values, but are the last values observed at the maximum embedding dimension.

These results are remarkably similar, especially considering the differing lengths of time series used. They suggest that all four indices were produced by attractors of dimension around 6 or 7, consistent with the possibility that they all reflect the same underlying attractor. The dimension of 6 or 7 is the same result as obtained for CRSP data (Scheinkman and LeBaron (1989)), again consistent with the possibility that we are seeing some manifestation of macroeconomic dynamics.

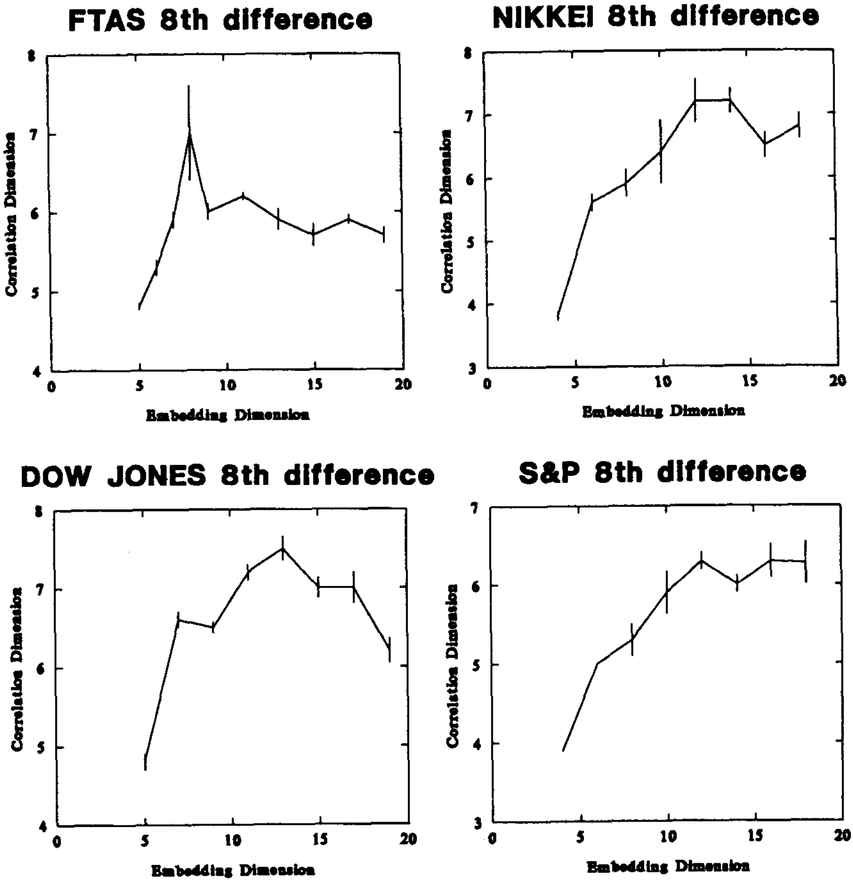

Figure 3 Graphs of correlation dimension versus embedding dimension for the 8th difference of the logarithms of all four indices, illustrating the asymptotic approach to the final values.

V. TEMPORAL INHOMOGENEITY

Having found apparently strong evidence for an attractor, we performed some further analyses in which we varied the number of data points used for a given index and a given embedding dimension. We did this for two reasons. The first was essentially defensive: a check that our results were not some artefact associated with a short series. The second was as an exploratory study related to future work which will use nonlinear forecasting methods: we wanted to see if the apparent structure was associated with any one part of the time history.

The 8th difference data was used as it has the most sparse distribution of points and hence is potentially the most susceptible to differing data lengths. It thus provides an upper bound for the minimum number of points required to produce stable results. The data set sizes started at 400 points, as this was the minimum number required when analysing simple systems of lower dimension such as the Henon attractor (Cvitanovíc (1984)).

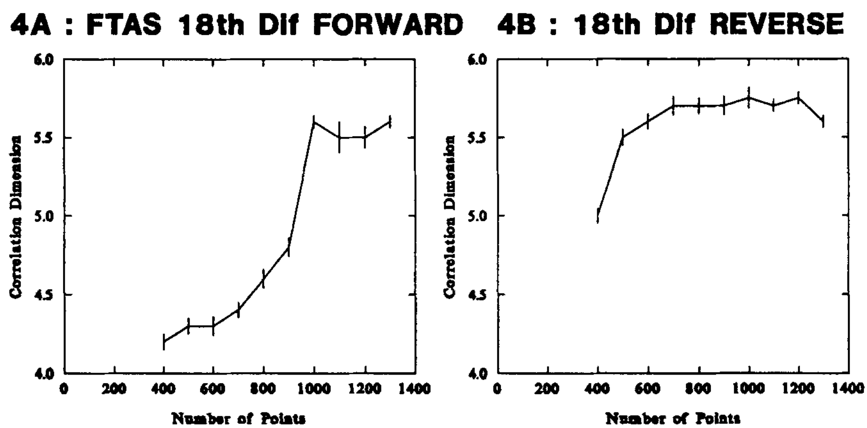

Figure 4 Correlation dimension graphs for increasing numbers of points. Fig. 4A is of increasing points from the start of the time series, while 4B increases from the end of the time series.

The results, as shown in Figure 4A for the FTAS data, show a marked increase at around 1000 points and thereafter simply fluctuate about the final value. Similar patterns were seen in the other indices, but the position of the increase varied with each. To test whether these observations were statistical in nature or were in fact properties of the data sets involved, we repeated the tests with the data ordering reversed. Instead of taking the first 400 then 500 etc., we first took the last 400, 500 etc., but without changing the chronological order of the time series.

Figure 4B shows the difference found, the dimension estimate shows no large increase as before, and is markedly higher and closer to the final value much earlier. From these results we conclude that the points which exhibit higher dimensional behaviour are grouped towards the end to the time series. Relating the position of this change to the temporal span of the time series reveals that the change happened at roughly the same time for all the indices, in the early eighties.

As for some indices this change was hard to pin-point, dimension estimates for 400 points were made throughout the time series. Although these yield values typically 30% smaller than final values, they are more affected by sudden changes in the data as the difference is bigger relative to the overall sample size. We find, therefore, that this increase, typically of 1 to 2 dimensions, occurred in the region of 1980 ± 2 years for all the indices.

As stated previously, the 52nd difference time series is difficult to analyse numerically, however where analysis was possible, the correlation dimension is markedly lower than for the 8th difference, being typically around 4. This result also held for the pre-1980 data, where the 8th difference had the abrupt change. So for pre-1980 both the 8th and 52nd differenced data give a dimension of 4, and for the entire time series the 8th difference gives a value of around 6, and the 52nd gives a value still approximately 4. Examination of the two sets of differenced data using recurrence plots (Eckmann *et al* (1988)), which indicate the distribution of a set in a state space of given dimension, reveals a marked difference between the two. The 8th difference is homogenously distributed through the space, while the 52nd difference has homogenous regions interspersed with areas where certain points appear to be clustered together. A possible explanation of all these differences is that the 52nd difference is commeasurate with some periodicity of the attractor. Thus, by sampling data on or near this period, the finer structure of the attractor would be missed.

VI. CONCLUDING REMARKS

The primary conclusion from this work is that there is evidence of the existence of an attractor for all four indices. The dimension of this attractor is around six, consistent with previous analyses of the CRSP data (Brock (1988); Scheinkmann and Le Baron (1989)). The similarity of values for attractor dimension in the four indices is consistent with the possibility that all are reflecting some common economic variables, and that the attractor is not related to local trading conditions.

The only work on index data of which we are aware that points to a different conclusion is that of Booth *et al.* (1990) who were unable to find any asymptotic attractor dimension in a study of German Blue Chip stocks. We believe this failure may be for technical reasons - in particular their use of first differences. A similar analysis on our four indices also fails to tease out the attractor dimension.

We cannot rigorously exclude the possibility that our various data transformations have introduced spurrious correlations, as there is little statistical theory underpinning the methods used (Brock and Dechert (1987)). We are therefore investigating more thoroughly the results obtained when our methods are applied to a variety of standard linear and non-linear models.

Our second conclusion is that there is tentative evidence of a substantial qualitative change in observed dynamics in the early 1980s. The methods used in this work are inappropriate for a more detailed investigation of this point. We have started an investigation that uses *local* methods based on nonlinear forecasting. These will be reported in a later publication.

Finally, we emphasise that our identification of an attractor does *not* imply that the fluctuations in index value are of necessity chaotic. To establish this requires demonstration that in some region of the state space, trajectories starting from nearby initial values should diverge. The appropriate statistic here is the largest Lyapunov exponent (Eckmann and Ruelle (1985)), which must be positive for chaos. The calculations are, however, very delicate, and great care is required in their interpretation. These will also be reported in a later publication.

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