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On Investment Strategies Using the Wilkie Model

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Summary

Some dynamic asset allocation strategies for a simple life office are examined using the Wilkie stochastic investment model. "Contracyclical" strategies which weight investment towards those sectors which have been least successful in the past are found to be better than strategies which concentrate on investing in assets which have already perfomed favourably. There is a surprising result that the strategies with the highest mean profits also have the lowest standard deviations of profit.

Résumé

Discussions sur les Stratégies de Placement en Utilisant le Modèle Wilkie

Quelques stratégies dynamiques d'optimisation de la répartition des actifs pour un compagnie d'assurance vie sont examinées en utilisant le modèle de placement stochastique Wilkie. Des stratégies "contracycliques" qui orientent les placements vers les secteurs qui ont remporté le moins de succès par le passé s'avèrent meilleures que des stratégies qui privilègient le placement dans des actifs qui ont déjà obtenu des résultats favorables. Un résultat surprenant est que les stratégies qui ont les profits moyens les plus élevés ont également les écarts types de profit les plus faibles.

1. Introduction

The last decade has seen an explosion in personal computing power, to the point where actuaries have on their desks hardware capable of carrying out very sophisticated simulations of a life office's cashflows. In particular, the technique of stochastic simulation has established itself as a powerful tool for approaching questions which are too complex to be analysed by traditional techniques. The use of simulations has thrown up another set of problems, however, and perhaps the most important of these is the need for algorithms which model the reactions of an office's management to the business and investment environment which is created by the underlying stochastic models.

It is clear that the value of the results given by simulation depends on the degree to which the underlying stochastic models reflect reality. These considerations are at their most acute in solvency studies, when it is the tails of the resulting distribution which are of interest, but they can also arise in other fields. For example, if we are trying to develop algorithms which model management's reactions to events, we may discover that an algorithm succeeds because it exploits the structure of the underlying stochastic model, or fails because it does not. The question is then one of belief in the model.

In this paper, we shall study some examples of algorithms which determine the investment strategy of a life office dynamically, and in one case we will have grounds for suspecting that the success of a family of algorithms does seem to follow from the structure of the stochastic investment model. First, we shall review some previous work.

In 1984, the Solvency Working Party of the Faculty of Actuaries in Scotland presented a report to that body in which life office solvency was examined by stochastic simulation methods [1]. Among the important innovations of that report was its use of a stochastic asset model developed by A.D.Wilkie and described in a separate report to the Faculty of Actuaries [2].

In 1986 H.R.Waters [3] used the Wilkie investment model to explore the effect of four different investment strategies on the present value of the profit emerging under a single tranche of non-profit endowment policies. The four strategies were:

(a) To invest each year's cashflow entirely in equities (Strategy EQ)
(b) To invest each year's cashflow entirely in consols (Strategy FI)
(c) To invest each year's cashflow in a mixture of equities and consols, the proportion being weighted towards the better performing sector (Strategy FE).

(d) To reinvest the whole fund each year in a fixed interest security of a term to redemption calculated to immunise to a degree the profit or loss in the next year.

Mortality was assumed to be deterministic, and expenses were ignored, so that the cashflows in each year were deterministic. Further, the cashflows were all positive except in the last year (when the maturity payment fell due) so that the fund was also positive in every year except (possibly) the last.

For various policy terms and premium amounts, the means and standard deviations of the present values of the profit emerging under 10,000 simulations were calculated, where the discount factor for calculating the present value was effectively based on the year-on-year net rate of investment return on the whole fund.

One of the interesting conclusions of [3] was the effect of the mixed investment strategy. If investment entirely in equities may be characterised by high mean profits with a high standard deviation, and if investment entirely in consols may be characterised by low mean profits with a low standard deviation, then the mixed investment strategy achieved a remarkable combination of mean profits only slightly lower than those achieved by investment in equities with a standard deviation only a little higher than that achieved by investment in consols. Professor Waters said of the mixed strategy FE:

> "FE seems to have the best of both worlds in the sense that its means are as high as those of EQ and its standard deviations are (nearly) as low as those of FI. Presumably the explanation for this is that FE is an investment strategy that "backs winners" ... and by doing so it avoids getting caught in a run of poor investment performance for either equities or fixed interest securities."
>
> ([3] Section 3.3.2.)

This paper uses the methods of [3] to explore the effect of a wider range of investment strategies on the mean and standard deviation of the present value of the profits emerging under the same portfolio of non-profit endowment policies. In particular, bearing in mind the autoregressive nature of parts of the Wilkie investment model, we shall look at a number of so-called "contracyclical" investment strategies, under which future investment is weighted towards the sector which has performed least well over some period in the past; in general this is the opposite of the mixed strategy FE described above.

In Section 2 of this paper we give a brief description of the Wilkie investment model. Detailed formulae will be omitted since they are well documented elsewhere.

In Section 3 we describe the calculation of the policy cashflows.

In Section 4 we describe the calculation of the present value of the profit under any one simulation.

In Section 5 we describe the results of three simple investment strategies which we will use as a measure of the success of other strategies.

In Section 6 we describe several contracyclical strategies, and give numerical results of the simulations.

2. The Investment Model

In this section we describe very briefly the components of the Wilkie investment model. A full description is given in [2].

The model consists of four time series, generated by four sequences of independent unit normal random variables QZ(t), YZ(t), DZ(t) and CZ(t).

(1) Q(t) Retail Prices Index

$$\log[Q(t)/Q(t-1)] = 0.05 + 0.6\{\log[Q(t-1)/Q(t-2)] - 0.05\} + 0.05\, QZ(t)$$

(2) D(t) An index of gross equity dividends

$$\log[D(t)/D(t-1)] = 0.8\, DM(t) + 0.2 \log[Q(t)/Q(t-1)] - 0.0525\, YZ(t-1) + 0.1\, DZ(t)$$

$$\text{where } DM(t) = 0.2 \log[Q(t)/Q(t-1)] + 0.8\, DM(t-1)$$

(3) Y(t) The gross dividend yield

$$\log[Y(t)] = 1.35 \log[Q(t)/Q(t-1)] + YN(t)$$

$$\text{where } YN(t) = \log 0.04 + 0.6\{YN(t-1) - \log 0.04\} + 0.175\, YZ(t)$$

(4) C(t) The gross yield on Consols (irredeemable fixed-interest securities)

$$C(t) = CM(t) + CN(t)$$

$$\text{where } CM(t) = 0.05 \log[Q(t)/Q(t-1)] + 0.95\, CM(t-1)$$

$$\text{and } \log[CN(t)] = \log 0.035 + 0.91 \log[CN(t-1)/0.035] + 0.165\, CZ(t)$$

Using these time series we construct three others.

P(t) The equity price index

$$P(t) = D(t)/Y(t)$$

PR(t) An index of rolled up equity prices, including reinvested net dividends.

$$PR(t) = PR(t-1)\{P(t) + D(t)[1-T1]\}/P(t-1)$$

where T1 is the rate of tax on equity dividends.

CR(t) An index for fixed-interest irredeemable securities including reinvested net interest payments.

$$CR(t) = CR(t-1)\{1/C(t) + [1-T2]\}/C(t-1)$$

where T2 is the rate of tax on fixed interest securities.

Following the Faculty of Actuaries Solvency Working Party [1] and Waters [3] we take the rate of tax on equity dividends to be 30% and the rate of tax on fixed interest securities to be 37.5%. It is worth noting that in fact both of these have changed to an effective rate of 25% in recent years, but we have chosen to be consistent with the previous papers.

The parameters used above are the same as those used in [1] and [3], namely the "reduced standard model". For completeness, the following starting values are also given.

Q(0)	= 1
log[Q(0)/Q(-1)]	= 0.05
YN(0)	= log 0.04
D(0)	= Y(0)
DM(0) = CM(0)	= 0.05
YZ(0)	= 0
C(0)	= 0.085
PR(0) = CR(0)	= 1

A full description of the "reduced standard model" is given in [2] Sections 5.17 - 5.19.

3. The Calculation of the Policy Cashflows

Following Waters [3] this paper considers a single cohort of 1,000 policies; the possibility that the office continues to write new business is ignored. All expenses and investment dealing costs are ignored, so that the only items of income are premiums and the only items of outgo are death claims and maturity claims. Each policy has level annual premiums and a sum assured of £10,000. We shall restrict our attention to two of the four types of policy considered by Waters, namely

(a) A policy with term 10 years and annual premium £765.

(b) A policy with term 25 years and annual premium £215.

In each case, we assume that the policies are issued to lives age 35, and that mortality then follows (deterministically) the A1967-70 Ultimate table published by the Faculty of Actuaries and the Institute of Actuaries.

4. The Calculation of the Present Value of the Profit

Given values for the seven time series described in Section 2 (generated by a suitable computer program), the fund is rolled up as follows.

$PRFUND(t^{+})$ = the market value of the amount invested in equities at time t just after payment of premium and claims.

$CRFUND(t^{+})$ = the market value of the amount invested in gilts at time t just after payment of premium and claims.

$FUND(t^{-})$ = the total market value of the fund at time t just before payment of premium and claims.

$FUND(t^{+})$ = the total market value of the fund at time t just after payment of premium and claims.

$CFLOW(t)$ = the cashflow (premiums minus claims) at time t.

At time $t > 1$, we have

$$FUND(t^+) = PRFUND(t-1^+) \frac{PR(t)}{PR(t-1)} + CRFUND(t-1^+) \frac{CR(t)}{CR(t-1)} + CFLOW(t)$$

and

$$FUND(t^+) = PRFUND(t^+) + CRFUND(t^+)$$

The determination of the latter split between equities and gilts depends on the investment strategy, and assumes that equity income is reinvested in equities and gilt income is reinvested in gilts, at least until the year end.

The present value of the profit on a policy of term N years is found from

$$PROFIT = FUND(N^+) / ACC(N)$$

where

$$ACC(t) = ACC(t-1) \frac{FUND(t^-)}{FUND(t-1^+)}$$

This is equivalent to assuming that the life office identifies profits as they emerge against a suitable valuation basis and invests them in the same way as the policy reserves. Other assumptions could be made; for example the life office might adopt an investment strategy for profits which is more or less risky than that which it considers prudent for policy reserves.

5. Investment Strategies - a Baseline

We begin by summarising three of the investment strategies considered by Waters [3]. These will act as a baseline against which we can measure the effects of other strategies.

(1) 100% equity investment.

(2) 100% fixed interest investment.

(3) A mixed investment strategy in which the investment of new money is weighted towards the best performing sector. This is achieved by investing the following proportions of the cashflow at time t in equities and gilts, where "cashflow" means premiums less claims, and excludes investment income.

Equities : PR(t)/[PR(t) + CR(t)]

Gilts : CR(t)/[PR(t) + CR(t)]

The means and standard deviations of the present value of the profit resulting from 10,000 simulations are:

Table 1

Mean and Standard Deviations of present value of profit arising from 10,000 simulations of 1,000 non-profit endowments.

	<-- 10 year term ->		<-- 25 year term ->	
Strategy	Mean	Std. Dev.	Mean	Std. Dev.
	£	£	£	£
(1)	676,229	2,014,749	629,876	1,299,437
(2)	120,482	807,964	194,366	593,230
(3)	632,183	1,051,669	631,108	652,980

These are very similar to the figures given by Waters [3]; the differences are probably attributable to the use of different random number generators.

6. Contracyclical Strategies

The mixed strategy (3) shows that a policy of "backing winners" (as described in the remark quoted in Section 1 above) allows the fund to combine a high expected profit with a moderate standard deviation of profit. It is worthwhile to ask whether other strategies may do as well or better.

At least one large life office is known to espouse what it calls a "contracyclical" investment strategy. Broadly speaking, such a strategy will increase a fund's investments in those areas which have been performing less well, more or less on the grounds that a good thing can only last for so long, so that the top performing assets of the past are less likely to be the top performing assets in the future. It is necessary to suppose that there are several market sectors in which the funds can be invested, and that all these sectors do not move in phase with each other. A contracyclical strategy can be applied where there are any two or more market sectors which do not necessarily move together; for example, the market sectors might be the stock markets in different parts of the world, or the groupings defined by the Financial Times - Actuaries All Share Index, or simply gilts and equities. We will restrict our attention to the last case.

The mixed investment strategy (3) suggests, by contrast, an example of a contracyclical strategy. The measure of past performance is the ratio PR(t)/[PR(t) + CR(t)]. Roughly, if equal amounts had been invested in equities and gilts t years ago, this factor gives the proportion of the market value of the fund now contributed by the equities. A value of between 0.5 and 1 shows that equities have performed better than gilts, and a value of between 0 and 0.5 shows the opposite. Our first contracyclical strategy, called strategy (4), is as follows; instead of investing a proportion PR(t)/[PR(t) + CR(t)] of the cashflow at time t in equities, we invest it in gilts, and put the balance of the cashflow (the proportion CR(t)/[PR(t) + CR(t)]) into equities. In other words, we simply reverse the strategy of (3).

The results are shown in Table 2. For convenience, the table also shows the three strategies (1), (2) and (3) which we are treating as our baseline.

We can pursue this further, by asking if what is good for one year's cashflows is not also good for the whole fund. We modify strategy (4) by reinvesting the whole fund at time t as follows.

Equities in proportion CR(t)/[PR(t) + CR(t)]

Gilts in proportion PR(t)/[PR(t) + CR(t)]

and we call this strategy (5). The results are shown in Table 2.

Table 2

Mean and Standard Deviations of present value of profit arising from 10,000 simulations of 1,000 non-profit endowments.

	<-- 10 year term ->		<-- 25 year term ->	
Strategy	Mean	Std. Dev.	Mean	Std. Dev.
	£	£	£	£
(1)	676,229	2,014,749	629,876	1,299,437
(2)	120,482	807,964	194,366	593,230
(3)	632,183	1,051,669	631,108	652,980
(4)	**702,829**	**1,005,170**	**670,266**	**601,631**
(5)	**759,397**	**944,212**	**728,697**	**521,640**

The contracyclical strategies have a higher expected profit even than 100% equity investment, but a lower standard deviation than the mixed strategy (3). In fact, for the 25 year policy under strategy (5) the standard deviation of the profit is lower than in the case of 100% gilt investment. It is also plain that, in this case at least, what is good for one year's cashflow is indeed good for the whole fund. That this is not invariably true is demonstrated by the next pair of contracyclical strategies.

The criterion of past performance used in strategies (3), (4) and (5) embodies the entire past history of gilt and equity investment. It may therefore be insensitive to more recent price movements which, if the idea of contracyclical investment is sound, may have some value as an indicator of future profits. We therefore define four more strategies as follows, all depending on the rolled-up equity index PR(t).

(6) If the rolled-up equity index PR(t) fell over the last year, invest the cashflow at time t entirely in equities; otherwise invest it in gilts.

(7) If the rolled-up equity index PR(t) fell over the last year, invest the entire fund at time t in equities, otherwise invest it in gilts.

(8) If the rolled-up equity index PR(t) did not fall in either of the last two years, invest the cashflow at time t entirely in gilts; otherwise invest it in equities.

(9) If the rolled-up equity index PR(t) did not fall over either of the last two years, invest the entire fund at time t in gilts, otherwise invest it in equities.

The results are shown in Table 3 below.

Table 3

Mean and Standard Deviations of present value of profit arising from 10,000 simulations of 1,000 non-profit endowments.

	<-- 10 year term ->		<-- 25 year term ->	
Strategy	Mean	Std. Dev.	Mean	Std. Dev.
	£	£	£	£
(1)	676,229	2,014,749	629,876	1,299,437
(2)	120,482	807,964	194,366	593,230
(3)	632,183	1,051,669	631,108	652,980
(6)	**712,990**	**912,794**	**653,547**	**548,019**
(7)	**331,272**	**1,494,095**	**326,167**	**1,152,672**
(8)	**784,550**	**1,173,790**	**709,906**	**667,006**
(9)	**618,061**	**1,663,952**	**575,405**	**1,139,207**

In terms of mean profit, all but (7) can be called successful, and in terms of standard deviation, (6) and (8) are reasonable. It also seems that more extreme strategies, which commit all the funds to one or other of gilts or equities, do not pay. We can investigate the latter feature more closely, and to do so we consider a generalisation of strategies (6) and (7).

Let $PRPROP(t^-)$ be the proportion of the fund invested in equities at time t, just before reinvestment of the funds, and let $PRPROP(t^+)$ be the proportion of the fund invested in equities at time t, just after reinvestment of the funds. Let x lie between 0 and 1. Our first strategy is as follows:

(a) If the rolled-up equity index PR(t) did not fall in the past year, then

$$PRPROP(t^+) = PRPROP(t^-) + x.(1 - PRPROP(t^-))$$

(b) If the rolled-up equity index PR(t) fell in the past year, then

$$PRPROP(t^+) = PRPROP(t^-) - x.PRPROP(t^-)$$

Call this strategy (10). Clearly this is a "backing winners" strategy. By choosing different values of x, we can see the effect of backing winners more or less enthusiastically. Note that x=0 is the same as strategy (3).

By contrast, we define strategy (11) to be the contracyclical counterpart of the above, as follows.

(a) If the rolled-up equity index PR(t) fell in the past year, then

$$PRPROP(t^+) = PRPROP(t^-) + x.(1 - PRPROP(t^-))$$

(b) If the rolled-up equity index PR(t) did not fall in the past year, then

$$PRPROP(t^+) = PRPROP(t^-) - x.PRPROP(t^-)$$

Table 4 shows the results of these strategies for values of x ranging from 0 to 1 in steps of 0.1.

The most successful strategy, for both policy terms, is to move the proportion invested in equities in the opposite direction to the the changes in equity prices, by about 20% - 30% of the difference between the current proportion invested in equities and the extreme values of nil or 100%. Once more, a contracyclical strategy is best, but applied in moderation.

However, one very striking feature of these results is that the lowest standard deviations and the highest means go together, and in the case of the 25 year policy we have the highest mean and the lowest standard deviation of any of the strategies we have looked at. Strategy (3) may have led us to believe that a high mean can be associated with a low standard deviation, but this result seems rather too good to be true.

Table 4

Mean and Standard Deviations of present value of profit arising from 10,000 simulations of 1,000 non-profit endowments.

		<-- 10 year term ->		<-- 25 year term ->	
Strategy	x	Mean	Std. Dev.	Mean	Std. Dev.
		£	£	£	£
(1)		676,229	2,014,749	629,876	1,299,437
(2)		120,482	807,964	194,366	593,230
(3)		632,183	1,051,669	631,108	652,980
(10)	1.0	356,278	1,899,690	375,525	1,430,357
(10)	0.9	405,030	1,748,420	426,729	1,259,883
(10)	0.8	441,156	1,634,403	463,699	1,136,990
(10)	0.7	469,545	1,543,834	492,255	1,042,872
(10)	0.6	493,351	1,468,728	515,856	967,679
(10)	0.5	514,782	1,403,962	536,826	905,570
(10)	0.4	535,512	1,345,629	556,944	852,659
(10)	0.3	556,925	1,289,569	577,726	805,983
(10)	0.2	580,254	1,229,514	600,288	762,482
(10)	0.1	606,268	1,154,861	623,688	716,392
(10)	0.0	632,183	1,051,669	631,108	652,980
(11)	0.0	632,183	1,051,669	631,108	652,980
(11)	0.1	689,239	927,268	686,491	516,558
(11)	0.2	709,082	883,657	704,867	485,186
(11)	0.3	708,051	886,074	704,628	492,709
(11)	0.4	691,352	920,098	689,716	524,089
(11)	0.5	661,286	977,608	662,227	574,667
(11)	0.6	619,070	1,053,350	622,881	643,429
(11)	0.7	565,303	1,143,810	571,362	731,363
(11)	0.8	500,019	1,247,037	506,452	841,455
(11)	0.9	422,565	1,362,964	425,935	979,051
(11)	1.0	331,272	1,494,095	326,167	1,152,672

So far, our criteria for choosing the proportion to invest in equities have depended on the equity price alone. Our next step is to repeat the last experiment, but modify the strategies (10) and (11) by substituting a criterion based on both gilts and equities. Our criterion shall be a simple one, depending on the movement of the ratio PR(t)/CR(t). Again, let x be between 0 and 1.

Strategy (12) is another "backing winners" strategy.

(a) If the ratio PR(t)/CR(t) did not increase in the past year, then

$$PRPROP(t^{+}) = PRPROP(t^{-}) - x.PRPROP(t^{-})$$

(b) If the ratio PR(t)/CR(t) increased in the past year, then

$$PRPROP(t^{+}) = PRPROP(t^{-}) + x.(1 - PRPROP(t^{-}))$$

and strategy (13) is its contracyclical counterpart.

(a) If the ratio PR(t)/CR(t) increased in the past year, then

$$PRPROP(t^{+}) = PRPROP(t^{-}) - x.PRPROP(t^{-})$$

(b) If the ratio PR(t)/CR(t) did not increase in the past year, then

$$PRPROP(t^{+}) = PRPROP(t^{-}) + x.(1 - PRPROP(t^{-}))$$

The results are shown in Table 5.

Tables 4 and 5 are quite similar; Again, the most successful strategy is a moderately contracyclical one, which yields slightly higher means and standard deviations than in the last case. And again, the highest means and the lowest standard deviations go together.

Table 5

Mean and Standard Deviations of present value of profit arising from 10,000 simulations of 1,000 non-profit endowments.

Strategy	x	<-- 10 year term -> Mean	Std. Dev.	<-- 25 year term -> Mean	Std. Dev.
		£	£	£	£
(1)		676,229	2,014,749	629,876	1,299,437
(2)		120,482	807,964	194,366	593,230
(3)		632,183	1,051,669	631,108	652,980
(12)	1.0	410,459	1,772,225	447,020	1,299,631
(12)	0.9	455,583	1,616,778	496,345	1,125,187
(12)	0.8	487,305	1,501,176	529,677	1,005,191
(12)	0.7	510,639	1,410.168	553,195	916,698
(12)	0.6	528,615	1,335,643	570,466	848,367
(12)	0.5	543,292	1,272,931	583,760	793,923
(12)	0.4	556,321	1,219,102	594,646	749,670
(12)	0.3	569,352	1,172,052	604,297	713,556
(12)	0.2	584,396	1,129,941	613,626	694,341
(12)	0.1	604,250	1,090,576	623,229	662,015
(12)	0.0	632,183	1,051,670	631,109	652,980
(13)	0.0	632,183	1,051,670	631,109	652,980
(13)	0.1	697,444	986,880	698,410	562,319
(13)	0.2	719,352	975,897	709,069	554,508
(13)	0.3	715,629	997,154	696,810	584,524
(13)	0.4	693,229	1,042,185	669,175	640,337
(13)	0.5	655,593	1,106,132	628,917	715,581
(13)	0.6	604,620	1,185,866	576,811	807,992
(13)	0.7	541,344	1,279,156	512,489	918,331
(13)	0.8	466,103	1,384,674	434,705	1,049,217
(13)	0.9	378,434	1,502,630	341,263	1,204,822
(13)	1.0	276,710	1,635,832	228,543	1,391,840

7. Conclusion

It would be possible to investigate many other investment strategies and investment criteria. However, it is open to question whether we would be modelling the real world or simply mapping out an inherent feature of the time series model. Given that the model contains autoregressive components, the success of a contracyclical strategy is not too surprising; the question then becomes one of belief in the predictive ability of a fitted time series, and that topic is outside the scope of this paper.

We can draw the conclusion that algorithms for modelling investment strategies using the Wilkie investment model are more likely to be successful if they are moderately, but not extremely, contracyclical.

References

[1] Faculty of Actuaries Solvency Working Party (1986), The Solvency of Life Insurance Companies, *Transactions of the Faculty of Actuaries* Vol 39.

[2] Wilkie, A. D. (1986), A Stochastic Investment Model for Actuarial Use, *Transactions of the Faculty of Actuaries* Vol 39.

[3] Waters, H. R. (1986), Some Aspects of Life Assurance Solvency, in *Financial Models of Insurance Solvency, ed. J.D.Cummins and R.A.Derrig, Kluwer Academic Publishers, 1989.*