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Another Proof that the Proper Rate for Discounting Insurance Loss Reserves is Less than the Risk Free Rate

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Summary

In this paper it is proved that a linear capital market theory implies that whenever the equilibrium return on equity of an insurer exceeds the equilibrium investment rate for return, regardless of investment strategy, then the discount rate for discounting insurance loss reserves is necessarily less than the risk free rate.

Résumé

Une preuve supplémentaire que le taux approprié d'escompte des réserves de perte d'assurance est inférieur au taux sans risque

Cet article démontre qu'une théorie linéaire du marché financier implique que chaque fois que le rendement des fonds propres d'équilibre pour un assureur excède le taux de rendement d'investissement d'équilibre, quelle que soit la stratégie d'investissement, le taux d'escompte pour escompter les réserves de perte d'assurance est nécessairement inférieur au taux sans risque.

Introduction

Very little effort has been directed toward determining the appropriate rate for discounting insurance loss reserves. Certainly this is an important issue in the United States since the Internal Revenue Service discounts loss reserves for income tax purposes. Further, although property-casualty loss reserves are generally carried at an undiscounted level for both regulatory and shareholder reporting purposes, there are exceptions for certain lines of insurance in certain states. Moreover, life insurers discount their reserves for both regulatory and shareholder reporting.

When reserves are discounted, the typical practice is to select a discount rate that is equal to the yield on some type of investment or portfolio of investments. The Internal Revenue Service, for example, uses the yield on a mid maturity Treasury bond to discount property-casualty loss reserves. Others use the embedded or portfolio yield of the insurance company. These discount rates, however, may not be consistent with financial theory.

Financial theory asserts that the appropriate discount rate is a function of the risk of the future loss payments. It is not a function of the investment strategy that is followed when investing the assets that back the reserves. And so, an inquiry into the appropriate discount rate must consider the risk of the future loss payments. Nevertheless, it is possible to make some inferences of that risk, and thus the appropriate discount rate, without directly measuring the risk of future loss payments.

Review of the Literature

Although Fairley (4) did not determine the discount rate for loss reserves, he did estimate the risk of an insurer's total liabilities, which consist predominately of loss reserves. His estimate depends on the Capital Asset Pricing Model (5) as well as a number of other assumptions. He concluded that the liability beta for property-casualty insurers is -.21.

D'Arcy (3) attempted to determine the discount rate for a property-casuaty insurer's loss reserves. He also relied on the Capital Asset Pricing Model (CAPM) as well as Fairley's empirical work. He attempted to estimate the beta for loss reserves from Fairley's estimate of the total liability beta. D'Arcy concluded that the discount rate for loss reserves is 1.7 percentage points lower than the risk free rate.

Butsic (2) proved that the appropriate discount rate for the loss reserves of a reinsurer with a risk free investment strategy is necessarily less than the risk free rate. Although he did not state it in his paper, his argument implicitly assumes that the equilibrium return on equity exceeds the risk free rate.

This paper generalizes Butsic's result to any insurer with any investment strategy. It is shown that a linear capital market theory implies that whenever the equilibrium return on equity exceeds the equilibrium investment rate of return (which depends on the investment strategy), then the discount rate for loss reserves is necessarily less than the risk free rate.

The Model

The model presented here assumes a world without taxes.

The underwriting cash flows of an insurer consist of premium receipts, expense payments and loss payments. These cash flows occur at various points in time. They can be modeled most simply by assuming that they occur once a year at the mid point or at the end of the year. Greater accuracy can be achieved at the cost of greater detail by assuming that the cash flows occur monthly, weekly or daily. However, the notation in such a model is cumbersome. For purposes of this paper, continuous cash flows are assumed. This assumption has no affect on the results, but does simplify the notation. Accordingly:

Let, t = number of years from time $t_0 = 0$

P_t = premium payments received at time t

e_t = expenses paid at time t

L_t = losses paid at time t

d_p = annual discount rate for premiums

d_e = annual discount rate for expense payments

d = annual discount rate for loss payments

Define P to be the present value (at time t_0) of the net premium (i.e. net of expenses).

$$P = \int_0^\infty P_t \, dt \,/\, (1+d_p)^t - \int_0^\infty e_t \, dt \,/\, (1+d_e)^t$$

Define D to be the present value of the future loss payments.

$$D = \int_0^\infty L_t \, dt \,/\, (1+d)^t$$

Let, K = economic value (as distinguished from book value) of the equity at time t_0

r_e = equilibrium return on equity

r_s = equilibrium investment rate of return

V_t = equilibrium value of firm at time t

The equilibrium value of the firm at time t = 1 is given by,

$$V_1 = (K+P)(1+r_s) - D(1+d)$$

The present value of the firm can be obtained by discounting V_1 by the equilibrium return on equity. Thus,

$$V_0 = V_1/(1+r_e) = ((K+P)(1+r_s) - D(1+d))/(1+r_e)$$

The present value of the firm is also equal to the sum of the present values of the future cash flows. Thus,

$$V_0 = K + P - D$$

At equilibrium P must equal D since the present value of the firm at equilibrium is simply K.

Thus, at equilibrium,

$$V_0 = K = ((K+D)(1+r_s) - D(1+d))/(1+r_e)$$

Hence,

$$K(1+r_e) = (K+D)(1+r_s) - D(1+d)$$

if and only if,

$$d = (r_s - r_e)K/D + r_s \quad \text{and,} \tag{1}$$

$$r_e = ((K+D)r_s - dD)/K \tag{2}$$

These equations are valid for any investment strategy. In particular they are valid for a risk free investment strategy.

Let r_e' be the return on equity that corresponds to a risk free investment strategy.

Then,

$$d = (r_f - r_e')K/D + r_f \quad \text{and,} \tag{3}$$

$$r_e' = ((K+D)r_f - dD)/K \tag{4}$$

Hence,

$$r_e' - r_e = ((K+D)r_f - dD)/K - ((K+D)r_s - dD)/K$$

$$r_e' - r_e = (K+D)(r_f - r_s)/K$$

Thus,

$$r_e' = r_e - (r_s - r_f)(1 + D/K) \tag{5}$$

Assume a linear capital market theory. That is, assume that equilibrium returns are a linear function of risk. (CAPM is an example of such a theory. However, it is not necessary to assume CAPM. Any linear theory will do. It does not matter how risk is measured as long as it is linearly related to equilibrium returns.) It follows then, that the equilibrium return on any security is given by the following;

$$r = r_f + Br_p$$

where,

B = units of risk

r_p = risk premium per unit of risk (i.e. the additional return necessary to compensate the investor for assuming the risk)

Then,

$$r_s = r_f + B_s r_p$$

where,

B_s = units of risk corresponding to the investment strategy

Then, from equation (5),

$$r_e' = r_e - B_s r_p(1+D/K)$$

If $r_e > r_s$ then,

$$r_e' > r_s - B_s r_p(1+D/K) = r_f + B_s r_p - B_s r_p - B_s r_p D/K$$

$$r_e' > r_f - B_s r_p D/K$$

Case 1: $B_s \geq 0 \quad \Rightarrow \quad r_e' > r_f$ since r_p,D and K are positive
From eqation (3),

$$d = (r_f - r_e')K/D + r_f$$

And so, $r_e' > r_f \quad \Rightarrow \quad d < r_f$

Although most risky investments have positive risk, and consequently, equilibrium returns that are greater than risk free rates, there is no theoretical reason why negative risk securities can not exist. Gold, oil and diamonds are possible examples. In any event, the possibility of negative risk investments must also be considered.

Case 2: $B_s < 0 \quad \Rightarrow$

$$r_e' > r_f - B_s r_p D/K > r_f$$

Hence, $d = (r_f - r_e')K/D + r_f < r_f$

Thus, whenever the equilibrium return on equity exceeds the equilibrium investment rate of return, regardless of investment strategy, then the discount rate for discounting future loss payments is necessarily less than the risk free rate. Conversely, a discount rate that is greater than the risk free rate is appropriate only if the equilibrium return on equity is less than the equilibrium investment rate of return.

Equation (2) together with a linear capital market theory implies that;

$$B_e = ((K+D)B_s - DB_L)/K = B_s + (B_s - B_L)D/K \quad (6)$$

where,

B_e = risk of the return on equity

B_L = risk of the loss reserves

From equation (1),

$$d = (r_s - r_e)K/D + r_s$$

$$d = (r_f + B_s r_p - r_f - B_e r_p)K/D + r_f + B_s r_p$$

$$d = (B_s - B_e)r_p K/D + r_f + B_s r_p$$

And from equation (6),

$$d = (B_s - B_s + (B_L - B_s)D/K)r_p K/D + r_f + B_s r_p$$

$$d = r_f + B_L r_p$$

Thus, the discount rate is a function of B_L, the risk of the loss reserves, as expected. And if $d < r_f$, then B_L must be negative.

There is no evidence that the equilibrium return on equity for the average-property casualty or life insurer in the United States does not exceed the equilibrium investment rate of return. The investment portfolios for both industries are displayed below in Table 1. Bonds and other debt instruments predominate. Equity investments constitute only a small portion of the portfolios. These portfolios can be characterized as having relatively low investment risk.

Table 1

1988 Distribution of Invested Assets

	U.S. Insurance Industry	
Asset	Property-Casualty	Life
Bonds	75.0%	57.3%
Preferred stock	2.4	1.0
Common stock	10.9	4.5
Mortgage loans	1.4	23.1
Real estate	.3	2.8
Cash and deposits	1.4	.4
Short term	7.4	3.2
Policy loans	0.0	5.5
Other	1.1	2.3

Source: Best's Aggregates & Averages

Although the investment portfolios of insurers have relatively low risk, investing in an insurance company is widely believed to be risky. Certainly this conclusion is supported by historic measures of risk. Table 2 displays the beta risk measures of the CAPM as published in the Value Line Investment Survey (an investment advisory firm) for the insurers included in the Value Line reports. These betas indicate that investing in insurance companies is approximately as risky as investing in the average common stock.

Table 2

Estimates of Betas

Property-Casualty Insurers		Diversified Insurers		Life Insurers	
Firm	Beta	Firm	Beta	Firm	Beta
AmBase	1.05	Aetna	.95	Am. Family	1.00
Chubb	1.00	Am. Bankers	.90	Aon	.95
Cinc. Fincl.	.80	Am. General	1.00	Capital Hldg.	1.15
Continental	1.05	AIG	1.15	First Exec.	1.55
Fremont Genl.	1.05	CIGNA	.95	Jefferson-Pilot	1.05
Frontier	.85	CNA	1.15	Liberty	.85
Fund American	1.00	General Re	1.00	Monarch Capital	1.20
Geico	.85	Kemper	1.25	NWNL	.85
Hartford Steam	.95	Lincoln National	1.05	Provident Life	.95
Ohio Casualty	.90	Reliance	1.40	Torchmark	1.05
Orion Capital	1.15	Transamerica	1.10	UNUM	.95
Progressive	1.00	Travelers	.90	USLIFE	1.05
Safeco	1.10			Washington Natl.	1.05
St. Paul	1.05				
Seibels Bruce	.85				
Selective	.95				
USF&G	.95				
Average	.97		1.07		1.05

Source: Value Line reports dated 6/15/90, 7/13/90 and8/10/90

Moreover, insurers have historically earned rates of return on equity that have exceeded investment rates of return by wide margins. So, it is not unreasonable to expect the equilibrium return on equity to exceed the equilibrium investment rate of return.

Summary and Implications

In the absence of evidence to the contrary, the only sensible conclusion is that the equilibrium return on equity exceeds the equilibrium investment rate of return, and hence, the appropriate discount rate for discounting aggregate loss reserves is less than the risk free rate. This in turn implies that insurance reserves have negative risk.

Since the discount rate depends on the risk of the reserves, and that risk is likely to vary by line of insurance, then discount rates are also likely to vary by line of insurance. And for some of the riskier lines, such as medical malpractice, it is theoretically possible that the discount rate is negative. That is, if B_L is negative but sufficiently large in absolute value, then;

$$r_f + B_L r_p < 0$$

This, of course, would imply the counterintuitive result that discounted reserves are larger than undiscounted reserves.

Future research efforts should be directed at estimating the risk of the loss reserves by line of insurance.

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