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Pension Funding Methods and Autoregressive Interest Rate Models

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Summary

A mathematical model is set up which can be used to compare different pension funding methods. Rates of return for the pension fund are assumed to be represented by a first order autoregressive model for the corresponding force of interest. Expressions for the variability of contributions and fund levels have been derived in an earlier AFIR paper and these are used to discuss methods of funding which are "optimal" in the sense of the period for spreading supluses and deficiencies which should be chosen in order to reduce the variability of both contributions and fund levels.

Résumé

Méthodes de Financement de Retraite et Modèles de Taux d'Intérêt Autoregressifs

Un modèle mathématique est mis en place afin de comparer différentes méthodes de financement de retraites. Les taux de rendement pour le fonds de retraite sont supposés comme étant représentés par un modèle autorégressif de premier ordre pour la force d'intérêt correspondante. Des expressions de variabilité des contributions et des niveaux de fonds ont été tirées d'un article précédent d'AFIR et sont utilisées pour discuter des méthodes de financement qui sont "optimales" du point de vue de la période qui devrait être choisie pour étaler les excédents et les déficits afin de réduire la variabilité des contributions et des niveaux du fonds.

INTRODUCTION

For a pension fund, let C(t) and F(t) respectively be the overall contribution and fund level at time t, and assume that we adopt a discrete time formulation.

This paper considers the behaviour of C(t) and F(t) in the presence of stochastic investment returns of a particular type.

At any time t, a valuation is carried out to estimate C(t) and F(t), based only on the scheme membership at time t. However, as t changes, we do allow for new entrants to the membership so that the population remains stationary - see assumptions below.

In the foregoing discussion, we make the following assumptions.

1. All actuarial assumptions are consistently borne out by experience, except for investment returns.

2. The population is stationary from the start.

3. There is no inflation on salaries, and no promotional salary scale. For simplicity, each active member's annual salary is set at 1 unit.

4. The interest rate assumption for valuation purposes is fixed.

5. The real interest rate earned during the period, $(t,t+1)$ is $i(t+1)$. The corresponding force of interest is assumed here to be constant over the interval $(t,t+1)$ and is written as $\delta(t+1)$. Thus, $1 + i(t+1) = \exp(\delta(t+1))$.

6. $E[1+i(t)] = E[\exp \delta(t)] = 1+i$, where i is the valuation rate of interest. This means that the valuation rate is correct "on average". This assumption is not essential mathematically but it is in agreement with classical ideas on pension fund valuation.

Assumptions 1, 2, 3, and 4, imply that the following parameters are constant with respect to time, t:

NC the total normal contribution
AL the total actuarial liability
B the overall benefit outgo (per unit of time).

Further, assumptions 1, 2, and 6 imply that the following equation of equilibrium holds:

$$AL = (1 + i)(AL + NC - B). \qquad (1)$$

We shall consider funding methods where

$$C(t) = NC(t) + ADJ(t). \qquad (2)$$

Here, NC(t) is the total normal cost at time t and ADJ(t) is adjustment to the contribution rate at time t represented by the liquidation of the unfunded liability at time t, UL(t).

$$UL(t) = AL(t) - F(t) \quad (3)$$

and AL(t) is the total actuarial liability at time t in respect of all members. We consider methods where ADJ(t) is given by the following "spread" equation:

$$ADJ(t) = \frac{UL(t)}{\ddot{a}_{\overline{m}|}} \quad (4)$$

i.e the adjustment to the normal cost is equal to the overall unfunded liability divided by the present value of an annuity for a term of M years.

It should be noted that this definition of ADJ(t) uses the same fraction of the unfunded liability regardless of the latter's sign. So, surpluses and deficiences are handled in exactly the same fashion - this would not always be the case in practice.

Then, with $k = \frac{1}{\ddot{a}_{\overline{m}|}}$ representing the fraction of UL(t)

which makes up ADJ(t), we have

$$C(t) = NC + \frac{(AL - F(t))}{\ddot{a}_{\overline{m}|}} = NC + k(AL - F(t)). \quad (5)$$

k can also be thought of as a rate of interest being charged on UL(t).

MOMENTS OF C(t) AND F(t): FIRST ORDER AUTOREGRESSIVE MODELS

Now it is assumed that the (earned real) force of interest is then given by the following stationary autoregressive process in discrete time of order 1 (AR(1)):

$$\delta(t) = \theta + \varphi\, [\delta(t-1) - \theta\,] + e(t) \tag{6}$$

where e(t) for t=1,2, ... are independent and identically distributed normal random variables each with mean 0 and variance γ^2. This model suggests that interest rates earned in any year depend upon interest rates earned in the previous year and some constant level. Box and Jenkins[1] have shown that, under the model represented by equation (6),

$$E\,[\delta(t)] = \theta$$

$$\text{Var}\,[\delta(t)] = \frac{\gamma^2}{1-\varphi^2}$$

$$\text{Cov}\,[\delta(t),\ \delta(s)] = \frac{\gamma^2}{1-\varphi^2}\ \varphi^{|t-s|}$$

The condition for this process to be stationary is that $|\varphi|<1$.

Given that

$$F(t+1) = (1+i(t+1))(F(t) + C(t) - B)$$

we obtain $$F(t+1) = (1+i(t+1))(QF(t) + R) \qquad (7)$$

where $Q = 1-k$ and $R = NC - B + k.\ AL$.

Haberman[2,3] has discussed in detail the derivation of formulae for E F(t), E C(t), Var F(t) and Var C(t) for finite t and, in the limit, as $t \to \infty$.

RELATIONSHIP BETWEEN VAR F AND VAR C

In this section we consider the behaviour of the relative limiting values (as $t \to \infty$) of Var F(t) and Var C(t) as functions of M (or equivalently as functions of k).

Let $A(m) = \dfrac{\text{Var } F(\infty)}{(E\ F(\infty))^2}$ and $B(m) = \dfrac{\text{Var } C(\infty)}{(E\ C(\infty))^2}$.

From the results of Haberman[2,3] we have that

$$A(m) = \frac{e^{-z}\ 2Qw(1-Qc)}{(1-Q^2cw)} + \frac{e^{-2z}w(1-Qc)^2}{c(1-Q^2cw)} - 1 \qquad (8)$$

where $Q = 1-k$,

$$k = \frac{1}{\ddot{a}_{\overline{m}|}}, \quad z = \nu^2\varphi\ (1-\varphi)^{-2}, \quad c = \exp\left[\theta+\tfrac{1}{2}\ \frac{1+\varphi}{1-\varphi}\ \nu^2\right]$$

and $w = c.\ \exp\left[\dfrac{1+\varphi}{1-\varphi}\ \nu^2\right]$.

It is convenient to view A( ) as a function of k rather than M, where d < k ≤ 1 and, for convergence (see Haberman[2,3]) we require $(1-k)c < 1$ and $(1-k)^2 cw < 1$.

Then

$$\frac{dA}{dk} = \frac{2e^{-z}w(1-Qc)}{(1-Q^2cw)^2}\left[Qc\,\frac{(1-Qw)}{1-Qc} + e^{-z}\,(1-Qw)\,-1\right] \quad (9)$$

If $\frac{dA}{dk} < 0$, A( ) is a decreasing function of k and so an increasing function of M, as M varies from 1 to ∞.

Given that $w > 0$, $Qc < 1$, $Q^2cw < 1$, we see that the condition for $\frac{dA}{dk} < 0$ is that

$$Qc\,\frac{(1-Qw)}{1-Qc} + e^{-z}\,(1-Qw) - 1 < 0.$$

If we regard $e^{-z} \simeq 1$, then, with $\mu = \frac{w}{c} = \exp\left(\frac{1+\varphi}{1-\varphi}\,\nu^2\right) > 1$ for notational convenience, this condition would be equivalent to

$$Qc\,(1-Qw) + (1-Qw)(1-Qc) - (1-Qc) < 0$$

$$\text{i.e. } Qc(1-\mu) < 0$$

which holds for all permissible values of the parameters concerned.

A more precise analysis, with e^{-z} replaced by $1+\varepsilon$ (where ε has the opposite to sign φ) leads to a quadratic equation for k viz

$$f(k) = \varepsilon c^2\mu\,(1-k)^2 - (1-k)\,c\,(\varepsilon(\mu+1) + \mu-1) + \varepsilon.$$

If the quadratic f(k) has no real roots and if $\varphi > 0$, then $f(k) < 0$ for all k; if $\varphi < 0$ then $f(k) > 0$ for all k.

If the quadratic f(k) has real roots (k_1 and k_2 say) and if $\varphi < 0$ then $f(k) < 0$ for k in the interval $[k_1, k_2]$, assuming that these are permissible values. If $\varphi \geqslant 0$ then $f(k) < 0$ in the intervals $[d, k_1]$ and $[k_2, 1]$ with the same provisos.

Numerical investigations indicate that $f(k) < 0$ generally for all permissible values of k so that

$$\frac{dA}{dk} < 0, \text{ or } \frac{dA}{dM} > 0.$$

The only exceptions found in extensive numerical experiments have been in cases where φ is close to -1.

$$\text{Now } B(k) = \frac{k^2 \ \mathrm{Var}\ F(\infty)}{(E\ C(\infty))^2}$$

$$= \frac{k^2 (k-d)^2 AL^2 e^{-2z} c \left[\dfrac{e^{-z} 2Qcw}{(1-Qc)(1-Q^2cw)} + \dfrac{e^{-2z}w}{(1-Q^2cw)} - \dfrac{c}{(1-Qc)^2} \right]}{\left[NC + k \left(AL - \dfrac{e^{-z}AL(k-d)c}{1-Qc} \right) \right]^2} \qquad (10)$$

where $Q = 1-k$ and $d < k \leq 1$, $Qc<1$ and $Q^2cw<1$.

Thus B(k) is a ratio of polynomials which we can write in the following simplified form.

$$B(k) = \frac{C.k^2(k-d)^2 \; (a_2(1-k)^2 + a_1(1-k) + a_0)}{(1-cw(1-k)^2)\; (b_2k^2 + b_1k + b_0)^2} \tag{11}$$

where $C = AL^2e^{-2z}c$, $a_2 = c^2w\;(1-e^{-z})^2$

$$a_1 = 2cw\; e^{-z}(1-e^{-z}), \quad a_0 = w\; e^{-2z}-c$$

$b_2 = c\; AL(1-e^{-z}\;)$, $b_1 = AL + c\; NC - c\; AL + e^{-z}\; dc\; AL$

and $b_0 = NC(1-c)$.

If we regard $e^{-z} \propto 1$, then the situation simplifies with

$a_2 = a_1 = b_2 = 0$, $a_0 = w - c$

and $b_1 = AL\;(1 - c + dc) + c.NC$.

If A(k) is monotonically decreasing, as indicated by the previous discussion, then optimal values of M (or k) will depend on the turning points of B(k). In the general case, the search for turning points of B(k) reduces to equating to zero the product of a polynomial of degree 7 and the quadratic $b_2k^2 + b_1k + b_0$. Unlike the case for random independent identically distributed rates of return (discussed in the early sections of Haberman[2,3]) there will not be a single turning point necessarily (at $k = k^*$ corresponding to M^*).

Rather than considering the general case, we pursue further the simplification arising from $e^{-z} \simeq 1$, so $a_2 = a_1 = b_2 = 0$.

We then have that, for the turning points of B(k), either $0 = b_1 k + b_0$

$$\text{or} \qquad 0 = k^3 \, cw \, (b_1(1-d) - b_0)$$

$$+ k^2(b_1(1 - cw(1-d)) + 3b_0 cw)$$

$$+ k \, b_0(2(1-cw) - cwd) + b_0 d(cw-1).$$

Hence, either

$$k = - \frac{b_0}{b_1} = \frac{NC(c-1)}{AL(1-c + dc) + NC} \qquad (= d \text{ if } \varphi = 0)$$

or we are required to solve a cubic equation, which, of course, may have between 0 and 3 roots in the permissible range of k. Much depends on the particular values of the key parameters and the coefficients b_0 and b_1. Interestingly, a_0 has cancelled out.

In the next section, a numerical example will be used to indicate cases where there are different numbers of turning points of B(k) and hence different profiles thereof.

NUMERICAL EXAMPLES

A numerical example is now introduced to explore further the properties of the limiting values of the moments of F(t) and C(t). The assumptions made are:

Population: English Life Table No.13 (Males) stationary

Entry Age: 30 (only)

Retirement Age: 65

No salary scale, or inflation on salaries

Benefits: Level life annuity ($\frac{2}{3}$ of salary).

The following sets of parameter values are used:

i: -.01, +.005, +.01, +.03, +.05

ν: .05, .10, .15, .20, .25

φ: ±.01, ±.03, ±.05, ±.07, ±.09

M: integer values between 1 and 500, subject to $Qc < 1$ and $Q^2cw < 1$ for convergence.

Tables 1 and 2 show values of α and β where $\alpha^2 = A$ and $\beta^2 = B$ i.e.

$$\alpha = \frac{(\text{Var } F(\infty))^{\frac{1}{2}}}{E\ F(\infty)} \quad \text{and } \beta = \frac{(\text{Var } C(\infty))^{\frac{1}{2}}}{E\ C(\infty)} \quad \text{for a}$$

range of values of M and φ with i = .01 and ν = .05 held fixed. Values for φ = 0.9 are not given because of the presence of negative values.

Tables 1 and 2 indicate the following general features viz

(i) $\alpha(\varphi,M)$ increases with M (for fixed φ) and with φ (for fixed M)

(ii) $\beta(\varphi,M)$ increases with φ (for fixed M) and decreases with increasing M (for fixed φ) except that for some values of φ (e.g. φ=0.1) there appears to be a minimum at some M^*.

The corresponding values of α and β for different i and ν yield the same general features (details not shown). However, there are some exceptions which we discuss further in the paragraphs below - the exceptions refer to the turning points of $\beta(\varphi,M)$ for fixed φ.

Following on from the previous section, we have also investigated numerically the trade-off between Var F(t) and Var C(t) as represented by their limiting values as $t \to \infty$, i.e. as represented by α and β. When values of α and β are plotted for combinations of i, ν and φ , we find that a number of distinct patterns emerge. When the rates of return are independent, identically distributed random variables (i.e. when $\varphi = 0$ as in Haberman[2,3]), the graphs of $\beta(M)$ v. $\alpha(M)$ (for given i and σ) follow the pattern shown as Type A in Figure 1. Then, as already noted, there is a trade off between Var F and Var C but only up to the value M^* where $\beta(M)$ reaches a minimum. If we desire to minimize variances then any $M > M^*$ is to be rejected for clearly some other $M \leq M^*$ reduces both Var F and Var C. Thus, we have described the interval $1 \leq M \leq M^*$ as an "optimal" region.

With $\varphi \neq 0$, inspection of the results from the numerical example indicates that there are at least seven patterns in terms of profiles of $\beta(M)$ v $\alpha(M)$ (see Figure 1):

TYPE A: graph has a minimum at M^* so $1 \le M \le M^*$ is "optimal".

TYPE B: graph has maximum at M_1^* and global minimum at $M_2^* (>M_1^*)$. Then if M_0 is the minimum value feasible, usually 1, then $M_0 \le M \le M_1^*$ and $M_3^* \le M \le M_2^*$ constitute the "optimal" region where M_3^* is defined as $M_3^* = \min \{M \ne M_0$ such that $\beta(M) = \beta(M_0)\}$.

TYPE C: graph has a maximum at M_1^* and local minimum at M_2^* $(>M_1^*)$ so $M = 1$ is optimal.

TYPE D: graph has minima at M_1^* and M_3^* and a maximum at M_2^* with M_1^* the global minimum and $M_3^* > M_2^* > M_1^*$. Then $1 \le M \le M_1^*$ is "optimal".

TYPE E: as for D with M_3^* the global minimum. Then $1 \le M \le M_1^*$ and $M_4^* \le M \le M_3^*$ constitute the "optimal" region where M_4^* is defined as $M_4^* = \min \{M \ne M_1^*$ such that $\beta(M) = \beta(M_1^*)\}$.

TYPE F: graph is monotonically decreasing so there is no "optimal" region.

TYPE G: graph is monotonically increasing so $M = 1$ is "optimal".

These empirical results support the theoretical discussion of the previous section, where it was shown that, if $z \sim 0$, then the profile of $\alpha(M)$ v. $B(M)$ would have up to three turning points;

specifically, the profile would have 0 turning points for types F and G, 1 turning point for type A, 2 turning points for types B and C, 3 turning points for types D and E.

When φ takes values -0.9, -0.7, and -0.5, the patterns are of Type F and there is no optimal region. When φ takes values close to +1.0, either the patterns are of type F or there are inadmissable values for Var F, E C or Var C.

Table 3 corresponds to φ = -0.3, and shows the classification of α - β profiles and, where appropriate, the optimal regions for M. Tables 4 - 8 similarly refer to φ = -0.1, 0.1, 0.3, 0.5 and 0.7. Given the set of values of M for which calculations have been performed, the optimal regions for M reported in Tables 3 - 8 are only approximations. Because we are interested only in general features, no attempts have been made at this stage to estimate more precisely the turning points in the α - β graphs (using, for example, numerical methods).

[From Tables 4 and 5, corresponding to $\varphi = \pm 0.1$, we note that the implied optimal values of M are consistent with those shown in Haberman[2,3] which correspond approximately to the case φ = 0].

The pattern of optimal M values across Tables 3 - 8 mirrors that in Haberman [2,3] with some exceptions. In general, the optimal region decreases as i increases (for fixed ν and φ) and as ν increases (for fixed i and φ). Also, the optimal region decreases as φ increases (for fixed i and ν); thus, an increase in the autoregressive parameter φ appears to have a similar effect on the optimal region as an increase in the variance parameter ν.

The predominant patterns in Tables 3 - 8 are those of Type A or Type F. However, at some of the margins, the pattern changes with the appearence of multiple turning points in the α - β profile e.g.
in Table 3 for φ = -0.3 and ν = .05
in Table 7 for φ = 0.5 and ν = .05 and .10
in Table 8 for φ = 0.7 and ν = .05.

These exceptions correspond to "larger" values of φ and "smaller" values of ν. These phenomena are being investigated further.

CONCLUSIONS

Varying levels of inflation and fluctuations in investment returns are problems with which the actuary must contend on an almost daily basis. Unlike mortality and other decrements or movements, for which deterministic and stochastic models are readily available, the movements of these economic factors are more difficult to model. Autoregressive methods appear to be very appropriate for this purpose. An objective of this paper has been to show that formulae are available for studying mathematically the variability of contributions and fund levels for a pension scheme. Practical implications for the choice of funding method are then considered as a consequence.

The funding methods which are considered are the Aggregate Method and those methods that prescribe the normal cost to be adjusted by the difference between the actuarial liability and the current fund, divided by the present value of an annuity for a term of "M" years. A simple demographic/financial model has been set up which permits the derivation of formulae for the first two moments of F(t) and C(t). The paper explores the detailed properties of the moments of C(t) and F(t) and their relationship when the earned rates of return are or follow a first order autoregressive process. These properties lead to consideration of the "optimal" range of values of M.

Further work is being carried out to explore the properties of more general autoregressive models for the force of interest.

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Table 1 RELATIVE STANDARD DEVIATIONS OF F(t) AND C(t) AS $t \to \infty$

Assumptions as in Numerical Example

[i=0.01, ν=0.05]

Funding Method	$\frac{(\text{Var } F(\infty))^{\frac{1}{2}}}{E\ F(\infty)}$					$\frac{(\text{Var } C(\infty))^{\frac{1}{2}}}{E\ C(\infty)}$				
	φ=-0.9	-0.7	-.05	-0.3	-0.1	φ=-0.9	-0.7	-0.5	-.03	-0.1
EAN M=1	5.0%	5.0%	5.0%	5.0%	5.0%	170%	170%	170%	170%	170%
5	3.3	4.4	5.5	6.6	7.8	21.6	29.2..	36.7	44.6	53.4
10	3.7	5.5	7.2	8.9	10.7	12.5	18.7	24.6	30.7	37.8
20	4.6	7.4	9.9	12.5	15.4	7.9	13.0	17.6	22.5	28.3
30	5.4	9.1	12.3	15.6	19.3	6.4	10.9	15.0	19.4	24.7
40	6.1	10.5	14.4	18.4	23.0	5.6	9.8	13.6	17.8	23.0
50	6.8	11.9	16.4	21.1	26.5	5.1	9.1	12.8	16.9	22.1
60	7.5	13.3	18.4	23.7	30.0	4.8	8.7	12.3	16.4	21.7
80	8.8	15.9	22.2	28.9	37.2	4.4	8.1	11.7	15.9	21.8

Table 2 RELATIVE STANDARD DEVIATIONS OF F(t) AND C(t) AS $t \to \infty$

Assumptions as in Numerical Example

[i=0.01, ν=0.05]

Funding Method	$\frac{(\text{Var } F(\infty))^{\frac{1}{2}}}{E\ F(\infty)}$				$\frac{(\text{Var } C(\infty))^{\frac{1}{2}}}{E\ C(\infty)}$			
	φ=+0.1	+0.3	+0.5	+0.7	φ=+0.1	+0.3	+0.5	+0.7
EAN M=1	5.0%	5.0%	5.0%	5.0%	170%	170%	170%	170%
5	9.1	10.8	12.7	14.2	63.9	77.0	93.8	108
10	12.9	15.7	19.6	25.9	46.5	58.4	77.0	114
20	18.8	23.4	30.3	44.3	35.7	46.5	65.9	121
30	23.9	30.2	40.3	64.6	31.8	42.8	64.7	148
40	28.7	36.7	50.8	94.3	30.2	42.0	68.3	213
50	33.4	43.7	63.1	160	29.6	42.8	76.3	421
60	38.3	51.1	78.5	*	29.8	45.0	90.2	*
80	48.8	69.2	100	*	31.6	53.5	114	*

* not applicable as $Q^2cw>1$

Table 3: Category of α-β profile and where appropriate optimal region for M, spread period, $\varphi = -0.3$

ν \ i	-.01	.005	.01	.03	.05
.05	F	F	D(1,90)	D(1,30)	E(1,20)&(100,110)
.10	F	A(1,400)	A(1,250)	A(1,120)	A(1,80)
.15	F	A(1,250)	A(1,200)	A(1,90)	A(1,70)
.20	F	A(1,200)	A(1,140)	A(1,80)	A(1,60)
.25	F	A(1,150)	A(1,120)	A(1,70)	A(1,50)

Table 4: Category of α-β profile and where appropriate optimal region for M, spread period, $\varphi = -0.1$

ν \ i	-.01	.005	.01	.03	.05
05	F	A(1,130)	A(1,70)	A(1,25)	A(1,15)
.10	F	A(1,90)	A(1,60)	A(1,25)	A(1,15)
.15	F	A(1,50)	A(1,40)	A(1,25)	A(1,15)
.20	A(1,110)	A(1,40)	A(1,30)	A(1,20)	A(1,15)
.25	A(1,40)	A(1,25)	A(1,20)	A(1,15)	A(1,15)

Table 5: Category of α-β profile and where appropriate optimal region for M, spread period, $\varphi = 0.1$

ν \ i	-.01	.005	.01	.03	.05
.05	F	A(1,90)	A(1,50)	A(1,20)	A(1,15)
.10	F	A(1,40)	A(1,30)	A(1,15)	A(1,10)
.15	A(1,60)	A(1,25)	A(1,20)	A(1,10)	A(1,7)
.20	A(1,20)	A(1,15)	A(1,10)	A(1,8)	A(1,5)
.25	A(1,15)	A(1,9)	A(1,8)	A(1,5)	A(1,3)

Table 6: Category of α-β profile and where appropriate optimal region for M, spread period, φ = 0.3

ν \ i	-.01	.005	.01	.03	.05
.05	F	A(1,60)	A(1,40)	A(1,15)	A(1,10)
.10	A(1,80)	A(1,20)	A(1,20)	A(1,9)	A(1,5)
.15	A(1,20)	A(1,9)	A(1,8)	A(1,2)	A(1,2)
.20	A(1,7)	A(1,2)	A(1,2)	A(1,2)	F
.25	A(1,2)	A(1,2)	A(1,2)	F	F

Table 7: Category of α-β profile and where appropriate optimal region for M, spread period, φ = 0.5

ν \ i	-.01	.005	.01	.03	.05
05	F	E(1,2)&(3,40)	E(1,3)&(3,25)	(E1,2)&(5,10)	D(1,2)
.10	E(1,2)&(6,20)	D(1,2)	A(1,2)	A(1,2)	A(1,2)
.15	A(1,2)	A(1,2)	A(1,2)	A(1,2)	A(1,2)
.20	A(1,2)	A(1,2)	A(1,2)	A(1,2)	A(1,2)
.25	A(1,2)	A(1,2)	A(1,2)	*	A(1,2)

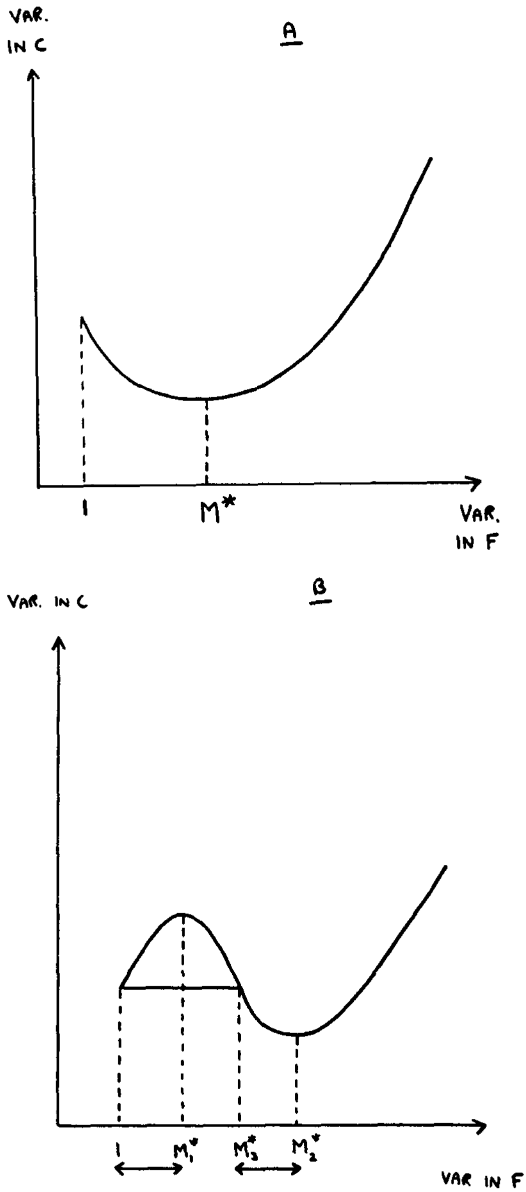
* "negative" values for some variances prevent full assessment of pattern.

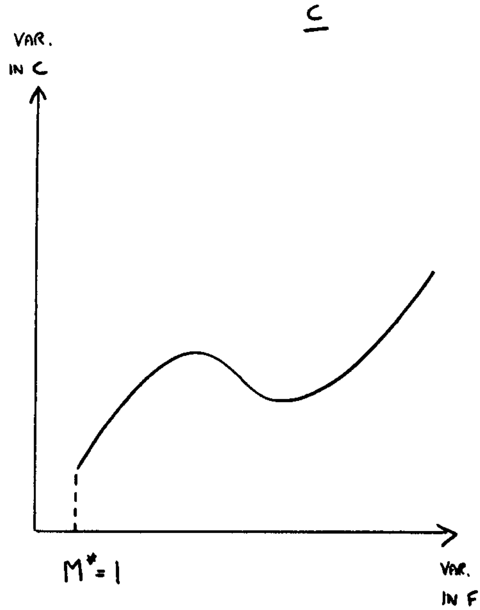
Table 8: Category of α-β profile and where appropriate optimal region for M, spread period, φ = 0.7

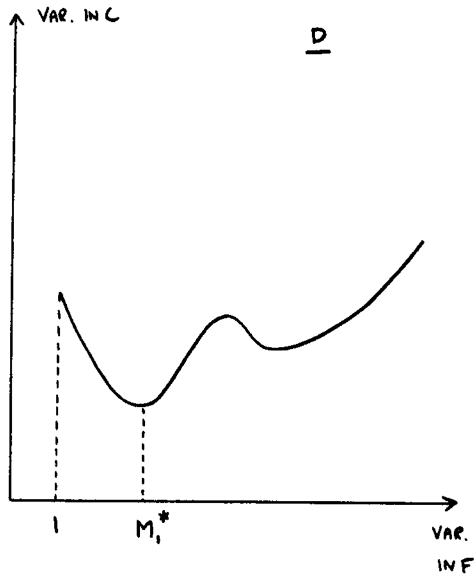
ν \ i	-.01	.005	.01	.03	.05
.05	B(3,6)&(50,80)	C(3,7)	F	F	F
.10	F	F	F	F	F
.15	F	F	F	F	*
.20	F	*	*	*	*
.25	*	A(1,2)	A(1,2)	A(1,2)	A(1,2)

* "negative" values for some of variances prevent full assessment of pattern.

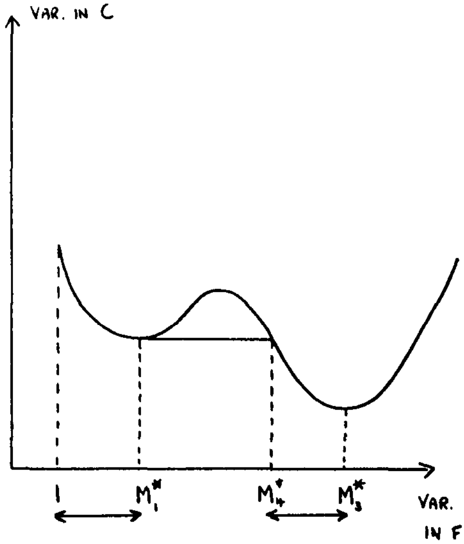
FIGURE 1. Patterns of profiles of relative variability in fund levels compared to relative variability in contributions.




C
VAR.
IN C
M*=1
VAR.
IN F


VAR. IN C
D
1
M_1^*
VAR.
IN F

E



F

G