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Search for Empirical Evidence of Strange Attractors in Historic Gold Price Data

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Summary

This paper demonstrates empirical techniques which can be used to examine Time Series data to see whether that data forms part of a deterministic system and hence has a strange attractor in phase space. These techniques are then applied to a series of Historic Gold Price Data and show that there is no evidence that the gold price forms part of a deterministic system.

Résumé

Recherche de Preuve Emperique de "Strange Attractors" dans les Données Historiques du Prix de l'Or

Cet article montre des techniques empiriques qui peuvent être utilisées pour examiner des données de série chronologique pour voir si ces données font partie d'un système déterministe et donc ont une attraction étrange en "phase space". Ces techniques sont alors appliquées à une série de données historiques du prix de l'or et montrent qu'il n'est pas prouvé que le prix de l'or dépende d'un système déterministe.

INTRODUCTION

Interest in non-linear dynamical systems has increased in recent years as these theories have been shown to be directly applicable to experimental situations (Roux and Swinney, 1981). With the appearance of popular books on the subject (Gleick, 1987) this area of study is now being applied outside the physical sciences to biology (Schaffer, 1984), medicine and economics (Brock and Malliaris, 1989). The result of this rise in interest in non-linear dynamics or Chaos Theory is that many groups are using Chaos Theory as justification for their previous approach to a problem and Finance is no exception with Chartists appealing to the theory (Griffiths, 1990). This short paper shows how using methods developed in the Physical sciences, data can be examined to see if it forms part of a deterministic system. The data used in this paper is that of historic gold prices.

THEORY

We first specify more clearly what we mean by a Chaotic system. Consider the following set of differential equations in two dimensions (two dimensions are picked for simplicity, the method holds for any number of dimensions and difference rather than differential equations could be used):

$$\frac{dX}{dt} = F(X,Y) \text{ and } \frac{dY}{dt} = G(X,Y) \qquad (1)$$

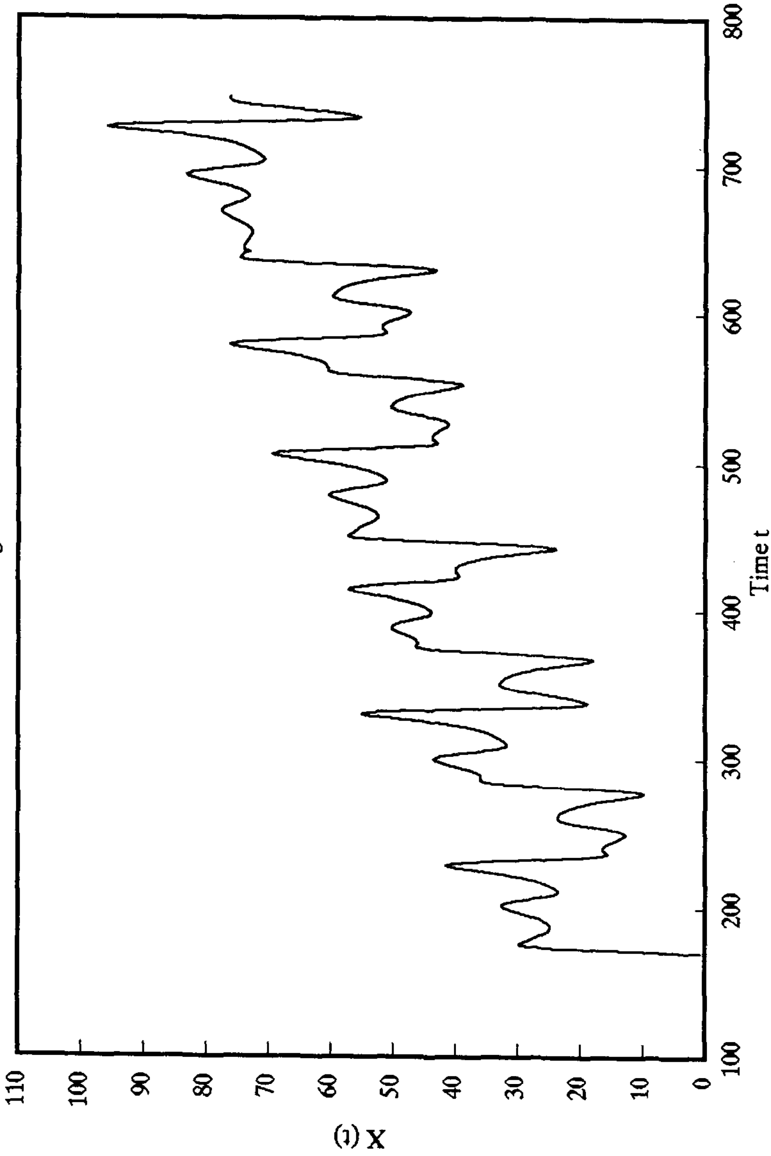
With initial conditions X_0, Y_0 at time t=0.

This system given by equation 1 is said to be chaotic if when X_0 is perturbed by a small amount δX this perturbation grows exponentially ie $\delta X(t) = e^{\mu t}$ where μ is a positive constant. Systems which behave like this display a time evolution which appears very complex and can appear like a stochastic system (Bunow and Weiss, 1979). Figure 1 displays the time evolution of X for a 3-d system (X,Y,Z) given by the Lorenz equations (Sparrow, 1986) with added drift. Whilst not as complicated as for instance a graph of a share price over time it can be seen to be broadly similar and the possibility exists by moving in to higher dimensions to more exactly replicate share price behaviour. The time evolution of Chaotic systems when displayed in a phase space often is confined to a particular locus of points which is termed the strange attractor as opposed to attractors for systems which are not chaotic but converge to a steady state or are periodic.

ANALYTICAL PROCEDURE

The problem now to be posed is how you can discover whether a particular time series X(t) is actually part of a deterministic system. The first difficulty that you

Figure 1


X (t)
Time t
0
10
20
30
40
50
60
70
80
90
100
110
100
200
300
400
500
600
700
800

encounter is, in approaching the problem empirically you do not know what other observables form with X the deterministic system. Shaw et al (1980) managed to obtain a solution to this difficulty. Consider equation 1 the first part can be rearranged to give:

$$Y = H(X, dX/dt) \tag{2}$$

and we thus obtain that:

$$\frac{dH(X, dX/dt)}{dt} = G(X, dX/dt) \tag{3}$$

However dX/dt can be approximated by $[X(t)-X(t-\delta t)]/\delta t$ and therefore we can use $X(t)$ and $X(t-\delta t)$ as our variables and do not require knowledge of Y. Thus in general we can use $X(t)$, $X(t-\delta t)...X(t-n\delta t)$ to represent the variables of a n+1 dimensional deterministic system and thus obtain knowledge of this system from examining only one time series. We can now examine a time series to find if it is deterministic using the method given by Grassberger and Procaccia (1983) which has been used in the Physical Sciences (Nicolis, 1984 and Grassberger, 1986). Consider a sphere in a n dimensional phase space: the volume of this sphere is proportional to L^n where L is the radius. If points in this phase space are distributed uniformly, the number of these points in the sphere should also increase as L^n. Thus a graph of log(nos of points) against log L should have gradient n. This is a rather crude description of the method of Grassberger and Procaccia. If a strange attractor is embedded in a n dimensional phase space then if a n+1 or greater phase space is formed the gradient of the log graph will still be n. Using the method above, from a time series we construct points $X_1 \ldots X_N$ in a n dimensional phase space $(X(t), X(t-\delta t), \ldots X(t-(n-1)\delta t))$ and from these construct a correlation coefficient given by:

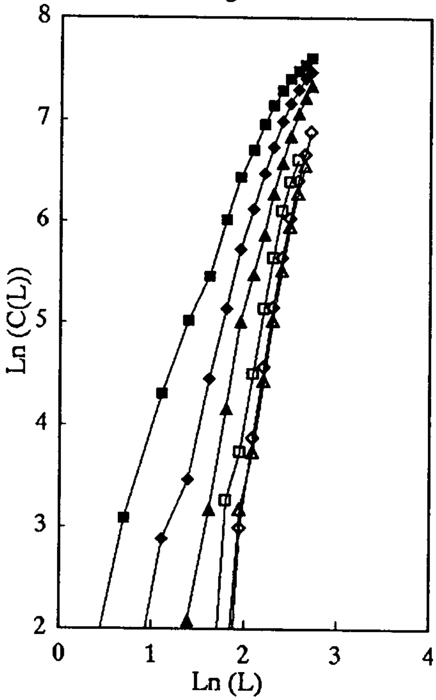
$$C(L) = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \theta(L - |X_i - X_j|) \qquad (4)$$

where $\theta(y)$ is the Heaviside function which takes the value of 0 if $y<0$ and 1 if $y>0$, N is the number of points formed from the time series (the time series will actually consist of N+n-1 points which allows N n dimensional points to be formed) and $|x-y|$ represents the magnitude of the distance between the two points x and y. That is we select each of the N points in turn and count the number of points other than itself which lie a distance less than L from it. The procedure is repeated for each point, and the totals given by each point are then summed and divided by N^2 to give the correlation coefficient. Experimentally then from our time series we construct points in a n dimensional phase space, calculate C(L) for a number of values L and plot log(C(L) v log(L). This graph should be a straight line and from it the gradient d is found. This procedure is repeated for n=2 upwards until d is found to level off as n is increased. If the time series does not form part of a deterministic system then d will continue to increase with n.

APPLICATION TO HISTORIC GOLD PRICE DATA

The above procedure was used to examine a time series of Gold Price data to see if there was any evidence that it formed part of a deterministic system. There are two reasons for using the Gold Price data: they tend to be available continuously for relatively long periods and some authors claim to have observed trends in the gold price (Jastram, 1977). From yearly data on the purchasing power of Gold for the periods 1851-1910 given by Vilar (1984) the real return on holding gold was found for each year and this data was then used to construct points in a phase space of dimensions 2 up to 8 with δt taken as one year. The real return on Gold was used rather than the Gold data itself because returns can be seen as more fundamental; thus if a return on a share price is constant the share price will still change every year. For each of these dimensions a graph of ln(C(L)) v ln(L) was constructed. These graphs are shown in Figure 2. Using a regression program the gradients of these plots d were found and the results are shown in Table 1. The R^2 coefficient indicates the goodness of fit of the straight line. As there has been some discussion on the importance of the time delay δt used, the same procedure was followed for

Figure 2



a delay of two years and the same trends were found as for the one year delay.

Table 1		
n	d	R^2
3	2.08	.979
4	2.86	.982
5	3.54	.987
6	4.2	.999
7	4.81	.992
8	5.55	.996

CONCLUSION

As can be observed from Table 1 there is no evidence that the Gold Price data forms part of a deterministic system or at least one of small dimension. The value of d increases with n and there is no evidence of it levelling off. The evidence would indicate that the gold price is stochastic and responds to fluctuations in supply and demand. The small number of data points used, about 50, must also be admitted, other work in Economics having been done with about 150 points (Brock, 1986) and the physical sciences using many thousands. However this would appear to only limit the accuracy with which we can determine d and not the overall conclusions. The fact however that this procedure leads to a negative conclusion does allow us to place more confidence in any positive results should they be found in the future. Finally it is an open question as to what help it gives us even if a time series should be found to be part of a deterministic system.

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