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Financial Approach to Actuarial Risks?

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Summary

The present paper questions the application of modern financial theory based on continuous markets and the no-arbitrage condition to value actuarial risks. Insurance markets are shown to be very far from financial markets in organizational structure. Especially arbitrage opportunities are quite natural in insurance markets.

Résumé

Une Approche Financiere aux Risques Actuariels?

Cet article met en question l'application de la théorie financière moderne basée sur les marchés continus et la condition de non-arbitrage pour évaluer les risques actuariels. L'article montre que les marchés d'assurance sont très différents des marchés financiers du point de vue de la structure organisationnelle. En particulier, les opportunités d'arbitrage sont assez naturelles dans les marchés d'assurance.

1. INTRODUCTION

There is an increasing actuarial interest in models and methods from the toolkit of modern theory of finance. This is most clearly demonstrated by the recent creation of an AFIR (Actuarial Approach for Financial Risks)-section within the IAA (International Actuarial Association). Modern financial theory has developed methods to value (price) various financial instruments and to control portfolios of securities. So it would seem natural to use these results to value and to control the *assets* of an insurance company. However, there are also a number of (mostly recent) papers with apply methods from financial economics, especially from that part which is concerned with preference-free pricing of derivative financial assets, e.g. options on stocks, to price insurance contracts, cf. e.g. *Artzner/Delbaen* (1990), *Delbaen/Haezendonck* (1989), and *Schöbel* (1985), respectively reinsurance contracts, cf. e.g. *Chang/ Cheung/Krinsky* (1989), *Föllmer/Schweizer* (1988, Chapter 6), *Sanders* (1990) and *Sondermann* (1988). This approach implies that principles for the valuation of special goods (derivative assets) in financial markets are used in another framework, namely for valuing special goods (risks) in insurance markets. Implicit to this kind of approach is the assumption that both financial and (re-) insurance markets are organized in a similar way.

From a risk theoretical point of view the results of the above mentioned papers are unfamiliar if not to say unreasonable. What could be the reason for this? Supposing that the mathematical derivations are correct, the reason must be that the assumptions on the market and on the pricing mechanisms, which may be very relevant for financial markets, may not be adequate for insurance markets.

It is the purpose of this paper to take a closer look at the assumptions of the theory of preference-free pricing of derivative assets and to question their adequacy within the framework of insurance markets. Especially the central assumption of arbitrage free markets, i.e. the existence of (equilibrium) market prices that do not admit riskless arbitrage opportunities, is questioned for the insurance case.

2. ASSUMPTIONS FOR PREFERENCE-FREE PRICING OF DERIVATIVE ASSETS

Option pricing studies the problem of valuing *derivative* financial assets only. A derivative asset is a security whose value is explicitely dependent on the *exogeneously given value* of some underlying primary asset on which the option is written.

As a starting point we list some of the crucial assumptions of the classical Black-Scholes-model, cf. e.g. *Black/ Scholes* (1973, p. 640), as some of the above mentioned papers remain within the framework of this model.

(1) *Continuous markets*, i.e. the trading of the assets takes place continuously in time.

(2) Securities are *perfectly divisible*.

(3) *Frictionless markets*, i.e. there do not exist transaction costs.

(4) Existence of a *riskless security*, i.e. there is the possibility of unrestricted borrowing and lending money at an *identical* risk free interest rate.

(5) The price process $S(t)$ of the primary asset (stock) follows a *geometrical Brownian motion*, i.e. the rate of return process is given by

$$\frac{dS(t)}{S(t)} = \mu\, dt + \sigma\, dW(t)$$

where $W(t)$ is the standard (zero drift and unit variance) *Wiener* process.

The problem is to determine the market value $V(0)$ of an European call option on the stock with *exercise price* c and *expiration date* T. The market value at expiration date clearly is $V(T) = \max(S(T)-c, 0)$. Now the central idea (*pricing by duplication*) of valuing the option is to construct a position in the stock and the risk free security which perfectly duplicates, i.e. produces exactly the same pattern of cash flows as the call option. In brief, the option is *attainable* in the given market. An equivalent alternative method is to create a hedge portfolio that is riskfree by combining a long position in the stock and a short position in the call option in the appropriate proportion. However, as the stock price and the time to expiration of the option changes, it becomes necessary (for both methods) to *continuously readjust* the weights of the assets to perfectly duplicate the call option respectively to maintain the portfolio's risk-free status. The possibility to be able to continuously rebalance one's position is *crucial* for obtaining a preference-free valuation formula. Except for the trivial case when the value of the underlying asset follows a two state jump process, cf. *Cox/Ross/Rubinstein* (1979), in case of *discrete trading* the preferences of the investors have to be revealed, i.e. certain assumptions on the utiliy functions of the investors have to be made, to be able to obtain a valuation formula. Cf. e.g. *Egle/Trautmann* (1981) or *Lee/Rao/Auchmuty* (1981), who make this point perfectly clear. However, under special assumptions on the preferences of the investors (constant proportional risk aversion) and the underlying asset return (distributed bivariate lognormal with the market return) a Black-Scholes-type valuation formula can be re-obtained, cf. *Rubinstein* (1976) and *Brennan* (1979).

As the values of the riskless security and the stock are given at any point of time the corresponding value of the call option can be derived at any point in time by simply comparing the call option with the value of the duplication portfolio or equivalently the risk-free hedge portfolio with the value of the riskless security. This can be done, however, only on the basis of a further assumption:

(6) *Arbitrage free market*, i.e. no arbitrage opportunities exist internal to this market for making riskless returns, i.e. a "free lunch".

The condition of an arbitrage-free market in modern financial theory is considered to be crucial for a market in equilibrium, i.e. all securities are traded at equilibrium prices only . Or, taken the other way round, if securities would be traded at prices which admit opportunities for riskless arbitrage, there would be investors to realize these profits and by this process market prices would converge to equilibrium prices at which riskless arbitrage profits can no longer be realized.

More precisely the no-arbitrage condition can be stated mathematically the following way. Two identical random variables $X \equiv Y$ must have identical prices $\pi_X = \pi_Y$. In the context of a discrete one-period capital market model this means that two securities (or two portfolios of securities) with an identical random value (pay-off) X(1) and Y(1) at the end of the period must have the same prices $\pi_X(0)$ and $\pi_Y(0)$ at the beginning of the period,

$$X(1) \equiv Y(1) \Rightarrow \pi_X(0) = \pi_Y(0) \quad . \tag{1}$$

In the case of continuous trading the condition can be stated as follows. Two securities (or portfolios of securities) with identical market values $X(t)$ and $Y(t)$ at any time $t > t_0$ must have identical prices $\pi_X(t_o)$ and $\pi_Y(t_o)$ at time t_o,

$$X(t) \equiv Y(t) \text{ for all } t > t_o \Rightarrow \pi_X(t_o) = \pi_Y(t_o) \ .$$

In the insurance framework the X, Y are (individual or collective) *risks*, the $X(t)$, $Y(t)$ are (individual or collective) *risk processes* and π_X, π_Y are the corresponding *risk premiums*.

Now what do these conditions, which are considered to be adequate for capital markets, mean in the context of insurance markets?

A first consequence can already be derived from the valuation problem, the valuation of *derivative* assets. This means that valuation is only done *relative* to the *given price* of the primary asset. One can only value options on stocks or bonds but not the stocks or bonds itself. Or as *Huang/Litzenberger* (1988) state it on p. 224 in their chapter on valuationly by arbitrage: ... *only relative prices are determined in an economic equilibrium* ... .

An insurance contract or a portfolio (collective) of insurance contracts is a primary asset, which is only traded in a primary market, the insurance market. There is no secondary market for insurance contracts, because the risks insured are not tradeable assets themselves. *Schöbel*

(1985, p. 1449) puts this point very clearly: *A perfect secondary market ... is a necessary condition for a valuation along the original Black/Scholes lines. In fact, the Chicago Board of Options Exchange is such a market, at least at principle. But insurance markets certainly are not!*

To avoid this difficulty and to apply a Black/Scholes-type valuation nevertheless, *Schöbel* (1985, p. 1451) introduces a basic principle, which he calls *insurance principle* and which amounts to the condition, that the "actuarial risk" approaches zero (cancels out) for a large collective of risks. As also *Kraus/Ross* (1982, p. 1021) use such a principle to value insurance contracts by arbitrage arguments *(by assumption, the firm we consider is large enough for individual actuarial risks to even out in the aggregate)* and as *Sondermann* (1988, p. 27) states that: ... *by the Law of Large Numbers, individual risks average out and can be neglected in a large economy ...,* we will take here the opportunity to clarify that financial theorists may have a wrong idea of how insurance does work and that this is not a proper way to rescue a Black/Scholes-type or arbitrage-type valuation of insurance contracts. Doing this, we will follow the lines of argument of *Albrecht* (1982, 1987), cf. also recently *Zigenhorn* (1990).

The law of large numbers only says that under certain conditions the *average risk* $(X_1 + ... + X_n)/n$ converges almost surely to a constant, especially non-random, term. This fact is, however, at best relevant for estimation problems and not for the problem of how insurance companies carry risks or portfolios of risks. An insurance company has *not* to carry the *relative* risk $(\Sigma X_i/n)$ but the *absolute* risk $S_n = X_1 + ... + X_n$. Stabilization of S_n/n, as given by the Law of Large Numbers, does *not* imply stabilization of S_n !

On the contrary: $Var(S_n)$ goes to infinity (in case of independent risks, which is the standard insurance case) as n goes to infinity, i.e. the collective of risks gets larger and larger, and S_n does not converge to a constant but approximates a normally distributed random variable. This is given by the Central Limit Theorem. The effect of growing collectives of (independent) risks, which one may call "insurance principle", is to be found within another framework, namely that for a growing collective the relative (i.e. divided by the number of risks) safety loading respectively the relative security capital required to maintain a certain security level of the insurance company is *decreasing*. We will take a closer look at this assertion, following arguments of *Albrecht* (1982, 1987).

We begin with the analysis of the case of a growing collective $X_1, X_2, \ldots, X_n \ldots$ of *independent, identically distributed* risks with common mean μ and standard deviation σ. Let $S_n = X_1 + \ldots + X_n$ denote the *accumulated claim amount* of a collective of size n for a fixed period and $\pi(S_n) = E(S_n) + Z_n$ the *collective risk premium*, where Z_n denotes the *collective safety loading*. Disregarding for the moment the security capital, the required amount for the collective premium to maintain a fixed security level ϵ (implicitely) is given by the condition

$$P(S_n > \pi(S_n)) = \epsilon \ . \qquad (2)$$

Approximating S_n by a normal distribution, standardizing S_n and denoting the $(1-\epsilon)$-quantile of the *standard* normal distribution by N_ϵ, we derive the following results:

$$\epsilon = P(S_n > \pi(S_n)) = P\left(\frac{S_n - E(S_n)}{\sigma(S_n)} > \frac{\pi(S_n) - E(S_n)}{\sigma(S_n)}\right) \qquad (3)$$

$$\Longleftrightarrow Z_n = N_\epsilon \, \sigma(S_n) \Longleftrightarrow Z_n = \sqrt{n} \, N_\epsilon \, \sigma \ .$$

This implies, that the *relative* safety loading (per risk) Z_n/n is given by $N_\epsilon \, \sigma/\sqrt{n}$ which is strictly *decreasing* and converging in the limit to zero. The required collective safety loading does not have to grow proportional with the size of the collective, only proportional to the square root of this size. This means, that although the risk of the accumulated claim amount, as measured by its variance $Var(S_n)$ or its standard deviation $\sigma(S_n)$ gets larger and larger with a growing size of the collective, this growing risk can be carried in such a way, that the necessary safety loading per risk is getting smaller and smaller and the necessary individual risk premium is decreasing to the expected value of the risk. *This* result is the core of the "principle of insurance".

The result remains valid in case S_n is not approximated by a normal distribution (which can be justified for very large collectives by the Central Limit Theorem) but by a Normal-Power (NP)-distribution, cf. *Beard/Pentikäinen/Pesonen* (1984, p. 111, p. 116), which due to the inclusion of the skewness of the distribution results in an improved approximation. According to Sundt (1984, p. 120) the collective risk premium derived from principle (2) is given by:

$$\pi(S_n) = E(S_n) + N_\epsilon \, \sigma(S_n) + \frac{1}{6} (N_\epsilon^2 - 1) \frac{M_3(S_n)}{Var(S_n)} , \qquad (4)$$

where $M_3(X) = E[(X - E(X))^3]$, the third central moment of a random variable.

The collective safety loading Z_n, as a function of the standard deviation σ and the third moment M of the individual risk is given by ($M_3(S_n) = nM$)

$$Z_n = \sqrt{n} \, N_\epsilon \, \sigma + \frac{1}{6} (N_\epsilon^2 - 1) \frac{M}{\sigma^2} \qquad (5)$$

Again, the relative safety loading Z_n/n is strictly decreasing and converging to zero.

The preceding analysis was for the case of independent and identically distributed risks. Similar results can be derived for independent risks, which are not necessarily identically distributed. This can be done best by the analysis of a merger of two independent portfolios of risks whose accumulated claim amounts follow a Normal or a Normal Power distribution. It can be shown in this case, cf. *Albrecht* (1987, p. 106, pp. 112 - 113), *Beard* et al. (1984, p. 144), that the risk premium resp. the reserves needed by the merged portfolio are always less than that of the separate portfolios together, if the security level ϵ is unchanged.

After this detour about insurance principles, which we, however, will recur to when discussing the relevance of the condition of no-arbitrage for insurance markets, we have to state as a supplement that while insurance contracts are *not* reasonably considered as derivative assets, this is not necessarily the case for *re*-insurance contracts. *Given* the insurance premium one can under certain conditions, cf. e.g. *Föllmer/Schweizer* (1989) or *Sondermann* (1988), duplicate the position of e.g. a stop loss contract by a suitable mixture of cash and fractions of the re-insured risk (which can be conceived as a proportional reinsurance contract), so that the stop loss contract can be considered as a derivative asset. This, however, does not mean that a Black/Scholes type of analysis is necessarily valid for re-insurance contracts, because we have not yet discussed the consequences of the assumptions (1) - (6) for the assumed structure of insurance or re-insurance markets. We will come to this now.

Ad (1): Continuous Trading

Clearly continuous trading does not take place at insurance or re-insurance markets. Anyhow, it does not take place continuously at markets for primary and derivate assets either. It is just an approximation to the real world. Nevertheless, as already mentioned, the possibility for continuously readjusting one's risk position (continuous hedging) is central for obtaining a preference-free valuation formula. So the decisive question is, how close the approximation is to the real world. And here clearly are important differences between capital markets and insurance markets. The trading of financial assets of a certain category in the real world is by far more frequent than the trading of an insurable risk of a certain category. In fact, in addition, the *trading structure* is totally different for both markets. In financial markets typically every buyer of a certain asset has also the possibility to sell this asset again at the market. In insurance markets the trading only takes place in one direction, from the insured (who never buys his risks back) to an insurance company. The direct insurer himself will eventually sell parts of the taken risks again to another insurance company, typically a reinsurance company. If the insurance company considered first is not engaged in active reinsurance it will not buy back transferred risks, too. Moreover, the transferred risks stay in the portfolio of the insurance company typically as long as the insurance period for the given contract is, which may be at least one year for a property-liability insurance contract or e.g. 30 years in the case of a life insurance contract. In short, there are considerable differences in the *trading structure* of financial and insurance markets. Especially insurance markets are much more distant from the fiction of a continuously trading market, so that this assumption is violated to an extent, which may be tolerated for capital markets, but surely not for insurance or reinsurance markets.

Ad (2): Perfect Divisibility

Perfect divisibility is a requirement for perfect hedging. Only that portion of a certain asset is bought and sold which is needed for maintaining a hegde portfolio or for duplicating a certain position. Again this requirement is violated for financial markets too, but again much more severely for insurance markets. The ability to trade small fractions of risks in insurance (by means of proportional deductibles, coinsurance or insurance pools) or reinsurance (proportional reinsurance) markets is relatively low. Assumptions like: *We furthermore assume that the insurance market is competitive, i.e. there are many insurance companies trading only small fractions of the total risk - or: At any time, an insurance company can decide to buy or sell an arbitrary fraction of the risk -* , which SONDERMANN (1988, p. 9) makes explicit or "... *in a liquid insurance market where products can be bought and sold very frequently and in different quantities, models of financial markets avoiding arbitrage opportunities may well be applied", of DELBAEN/HAEZENDONCK* (1989, p. 269), are very far from reality. This point is closely connected with the next assumption.

Ad (3): No Transaction Costs

Again this assumption is violated for financial markets too and in reality continuously re-adjusting a portfolio would result in infinite transaction costs, which clearly cannot imply a feasible strategy. But, as already seen, continuous trading is just an approximation to reality and the crucial question again is how close this approximation is to the real world. In case of insurance or reinsurance markets the transaction costs, which in this context correspond to that part of the gross premium, which is required for the cover of operating expenses, achieve a much higher volume compared to transaction costs of capital markets. This in fact makes the existence of very liquid insurance markets with frequent trading virtually impossible as well as strategies of continuously or even frequently readjusting the insurance company's collective of risks. In addition the existence of IBNR (Incurred But Not Reported) - and IBNER (Incurred But Not Enough Reserved) - claims, which is a very important subject in property-liability insurance, is another real-life reason why insurance or re-insurance markets can not have the structure assumed by financial theory.

Ad (4): Price Process

A geometric Brownian motion process is a rather unfamiliar assumption for an insurance risk process, which from its structure is a jump process (with random jump heights), not a diffusion process. However, this again could be seen as an approximation and in addition other types of processes S(t) can be priced too, which however changes the valuation formula. There remains, however, what HAKANSSON (1979, p. 722) called The *Catch 22 of Option Pricing: A security can unambiguously valued by reference of the other securities in a perfect market if and only if the security being valued is redundant in that market*. Indeed all preference-independent valuation formulas have to assume that the asset to be valued is *attainable*, i.e. that it can be perfectly duplicated by a dynamically adjusted portfolio of the existing assets. *But if this is the case, the option adds nothing new to the market... - the option is perfectly redundant. So we find ourselves in the akward position of* *being able to derive unambiguous values only for redundant assets and unable to value options which do have social welfare,* cf. HAKANSSON (1979, p. 723). In the insurance context this means e.g.that it depends on the assumptions on the risks process S(t) whether e.g. a stop loss contract is attainable and therefore can be unambiguously valued. HARRISON/KREPS (1979) and HARRISON/PLISKA (1981, 1983), based on a technique developed by COX/ROSS (1976),give precise conditions for the discounted price process of the primary asset so that every (integrable) contingent claim is attainable, i.e. the market is complete, in terms of martingale theory and thus clarified the mathematical structure of the problem. Results for non-redundant contigent claims are given e.g. by FÖLLMER/SONDERMANN (1986) and FÖLLMER/SCHWEIZER (1989), but they derive results only for the problem of hedging the contingent claim, not for its valuation.

Just as a supplement we want to remark, that - as only relative prices may be obtained by arbitrage valuation arguments - the valuation of reinsurance contracts entirely is based on the assumption, that the direct insurance market has priced the relevant risks correctly and unambiguously, which should not be taken for granted.

To sum up, of a number of requirements for the market (continuous trading, perfect divisibility, no transaction costs) have to be valid in order that a preference-free valuation formula can be obtained. Those requirements are violated both by financial and insurance markets, but for insurance markets to such a great extent, that the results are unreliable and useless for insurance applications in the author's opinion. But even if insurance and reinsurance markets were very close to such idealistic markets, there still remains one assumption, which we don't consider to be adequate for insurance markets - the assumption of arbitrage - free markets. This will be the point of the next chapter.

3. ARE INSURANCE MARKETS REASONABLY ARBITRAGE-FREE ?

The condition of no (riskless) arbitrage possibilities is a central assumption in modern financial theory for financial markets in equilibrium, cf. e.g. VARIAN (1987). However, the relevant question for the present case is, if it is a valid assumption that *insurance* markets are arbitrage free (or close to being it). In our opinion the answer is: No! What are the reasons for this assertion?

In the most simple setting the no-arbitrage condition (1) in an insurance framework requires that two identical (insurance or reinsurance) risks must have the same price, the same risk premium. However, as explained in section 2 the risk premium required from different insurance companies for a certain fixed risk will depend on the security level and the "size" (measured by the number of insureds and by the security capital present) of the company.

Let us fix the security level (only this assumption makes the situation reasonably comparable and assures that an identical product is sold which gives the same level of protection to the insured), neglect the security capital (its inclusion to the analysis is simple) and look at the idealistic situation that all risks are identically normal distributed and independent (all other situations will be worse). Then the adequate risk premium for an additional risk X required by a company with a collective of n risks $X_1, \ldots, X_n$ already being insured at a collective risk premium given by (4) has to satisfy the condition

$$P\ (S_n + X > \pi(S_n) + \pi(X)) = \epsilon \tag{6}$$

From (3) we obtain under the given assumptions:

$$\left.\begin{aligned}\pi(S_n) + \pi(X) &= E(S_n + X) + N_\epsilon\ \sigma(S_n + X)\\ &= (n+1)\ \mu + N_\epsilon\ \sqrt{n+1}\ \sigma\ .\end{aligned}\right| \tag{7}$$

As $\pi(S_n)$ is given by $n\mu + N_\epsilon \sqrt{n}\ \sigma$ we finally obtain:

$$\pi(X) = \mu + N_\epsilon\ (\sqrt{n+1} - \sqrt{n})\ \sigma\ . \tag{8}$$

As $f(n) = \sqrt{n+1} - \sqrt{n}$ is easily seen to be a strictly decreasing function in n this confirms the assertion, that a bigger insurance company will require for the same risk a lower price.

As in reality insurance companies differ in size to a considerable extent this implies that in insurance markets arbitrage possibilities would be a rather *natural* thing!

In real financial markets arbitrage possibilities will exist, too, but it is argued that these possibilities will vanish or tend to zero, as certain investors (arbitrageurs) will realize the possibilities of a riskless profit. So, finally we have to analyze whether the arbitrage possibilities demonstrated above can be realized in insurance markets and whether equilibrium prices would result by doing so.

Consider an insurance (or reinsurance) market with companies operating at an identical security level and with different sizes. As bigger companies require lesser premiums, a profit will only be made (without regarding transaction costs, the problem of this assumption was already criticized in section 2) by a transaction of the risk from a smaller to a bigger company. At the end of the process of realizing all arbitrage opportunities, all risks will be at the company being the biggest at the start of the process. The vanishing of all arbitrage possibilities has the price that all but one insurance company will withdraw from the market. As long as empirical markets are far from this situation, arbitrage opportunities exist to a large extent (and will typically not be realized) and no-arbitrage premiums (which would be the risk premium required by the fictitious company insuring all risks at the market) will tell *nothing* about the premium required by real insurance companies. In fact, real premiums will always be above the hypothetical no-arbitrage premium!

4. CONCLUSION

The present paper is critical on the usefulness of applying modern financial theories based on continuous markets and the no-arbitrage condition for valuing *actuarial* risks. The arguments given can summarized in that insurance markets have a highly different *organizational structure* compared with financial markets and because of that an approximation of insurance markets based on conditions of a financial market will be invalid and the conclusions drawn will be irrelevant.

In particular

- insurance markets are very far from the fiction of continuous trading
- insurance contracts are very far from being perfectly divisible
- transaction costs (loadings to the risk premium) are of a substantial amount

- arbitrage opportunities are quite natural in insurance markets
- insurance markets are very far from a fictitious arbitrage free market.

To put the things into perspective the author of the present paper is convinced of the usefulness

- of the financial approach to financial risks (FAFIR)
- of the actuarial approach to financial risks (AFIR), e.g. within the framework of asset-liability-management of an insurance company,

but he thinks that there are good reasons for rejecting the "financial approach for actuarial risks"!

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