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Simulation Analysis for Evaluating Risk-sharing Pension Plans

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Keywords : Risk-sharing pension plan, Investment Risk, Simulation analysis

Introduction

- ✓ The occupational pensions become more important to complement the public pension.
- ✓ However, the traditional occupational pension plans have some weakness for their sustainability or stability of benefits.
  - ➢ DB (defined benefit) plan: It is difficult for a sponsoring company to make additional payments of the contribution for lack of the plan asset under the worse investment condition.
  - ➢ DC (defined contribution) plan: We have a problem concerning stability of the pension which supports the living expenses in retirement.
- ✓ Recently, occupational pensions with new types of risk-sharing functions have been proposed; DA (defined ambition) plan in the U.K., target benefit plan in Canada, FTK2 in the Netherland, risk-sharing DB plan in Japan

Previous Studies

Hoevenaars & Ponds (2008)	Valuing intergenerational transfers in collective pension plans
Kocken (2012)	Examining two kinds of valuation techniques for pension liabilities in risk-sharing pension plans
Kortleve (2013)	Describing a new Dutch pension contract generically labeled defined ambition(DA) plans
Turner (2014)	Evaluating a number of hybrid pension plans ・Hybrid DB plans in the Netherlands ・Nonfinancial DC plan in Sweden ・Cash balance plans in the United States, Canada and Japan ・Riester plans in Germany
Hardy (2015)	Reviewing target benefit plan, and evaluating the plans through simulations of economic variables to assess risks and benefits

- The plan design in practice is discussed mainly in a qualitative manner, and there are some researches about the plan designs discussed specifically and quantitatively.

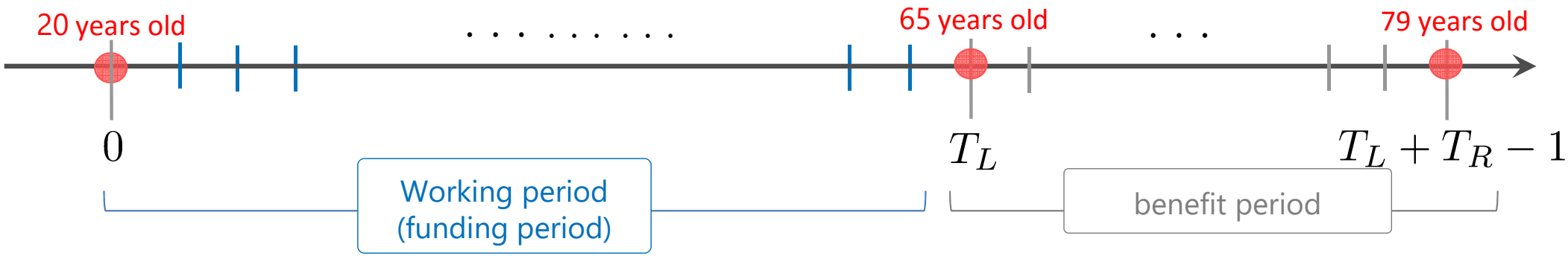
Purpose and contribution of our paper

- ✓ We propose a risk-sharing design, which involves a mechanism of sharing the deficiency and surplus in accordance with the funding ratio, and evaluate it quantitatively using Monte Carlo simulation approach.
- ✓ We formulate the simulation model with five parameters to control the level of risk-sharing. We conduct the sensitivity analysis of the five parameters, and suggest how those parameters affect the plan design.
- ✓ We find the benefits and the contributions of the risk-sharing plan are not only at the level intermediate between the DB and DC plans because of the risk-sharing features, but also superior to them in some cases.
- ✓ We implement the backtest using the historical data, and examine the actual effect on four pension plans for twenty years in Japan.

Models of Pension Plan

- Risks shared by stakeholders
  - investment risk
  - interest rate risk: incorporated in the plan design
  - longevity risk and inflation risk: excluded due to historical background in Japan
- Stakeholders
  - A sponsoring company
  - Active participants
  - Retirees

Problem: Setting



- Four kinds of pension plans: DC, DB, CB, Risk-sharing (RS)
- Match the size of all plans to compare them each other
- Assumption
  - Number of persons of each age = 1
  - Set the same initial actuarial liability of active participants
  - Set the same initial actuarial liability for DC, CB and RS plans
  - Initial plan asset = Initial actuarial liability (Funding ratio = 1)
  - Receive benefit/pay contribution at the beginning of each year

Design of DC, DB and CB plans

	DC plan	DB plan	CB plan
Actuarial liability	Equal to plan asset	Calculate based on expected yield of 10-year government bond	Calculate based on real yield of 10-year government bond
Benefit	Calculate based on actuarial liability	Real benefit = 1 at retirement age, and nominal benefit is fixed in benefit period	Calculate based on actuarial liability
Contribution	Normal contribution	Normal contribution (with 150% rule) + Amortization	Normal contribution (with 150% rule) + Amortization
Plan asset	Calculate based on portfolio return	Calculated based on portfolio return	Calculated based on portfolio return

Risk-sharing Pension Plan (1)

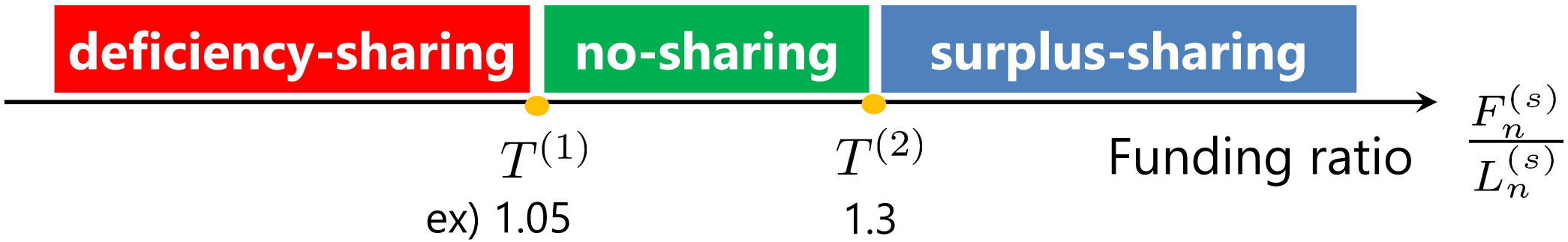
- Sharing investment risk based on the actuarial liability and benefit calculated for the CB plan
- Risk-sharing design, which involves a mechanism of sharing the deficiency and surplus in accordance with the funding ratio
- Five parameters to control the level of risk-sharing
  - Trigger parameters: $T^{(1)}, T^{(2)}$
  - Sharing parameters: $K^{(0)}, K^{(1)}, K^{(2)}$

Risk-sharing Pension Plan (2)

Where are the triggers of funding ratio for risk-sharing?

$T^{(1)}$: minimum funding ratio which triggers the deficiency-sharing ($T^{(1)} \geq 1$)

$T^{(2)}$: maximum funding ratio which triggers the surplus-sharing ($T^{(2)} \geq T^{(1)}$)



Deficiency(+)/Surplus(−)

$$Z_n^{(s)} = \begin{cases} T^{(1)} L_n^{(s)} - F_n^{(s)} & \left(\frac{F_n^{(s)}}{L_n^{(s)}} < T^{(1)}\right) \\ 0 & \left(T^{(1)} \leq \frac{F_n^{(s)}}{L_n^{(s)}} \leq T^{(2)}\right) \\ -\left(F_n^{(s)} - T^{(2)} L_n^{(s)}\right) & \left(\frac{F_n^{(s)}}{L_n^{(s)}} > T^{(2)}\right) \end{cases}$$

$F_n^{(s)}$: plan asset

$L_n^{(s)}$: actuarial liability of CB plan

Risk-sharing Pension Plan (3)

What fractions are shared by stakeholders?

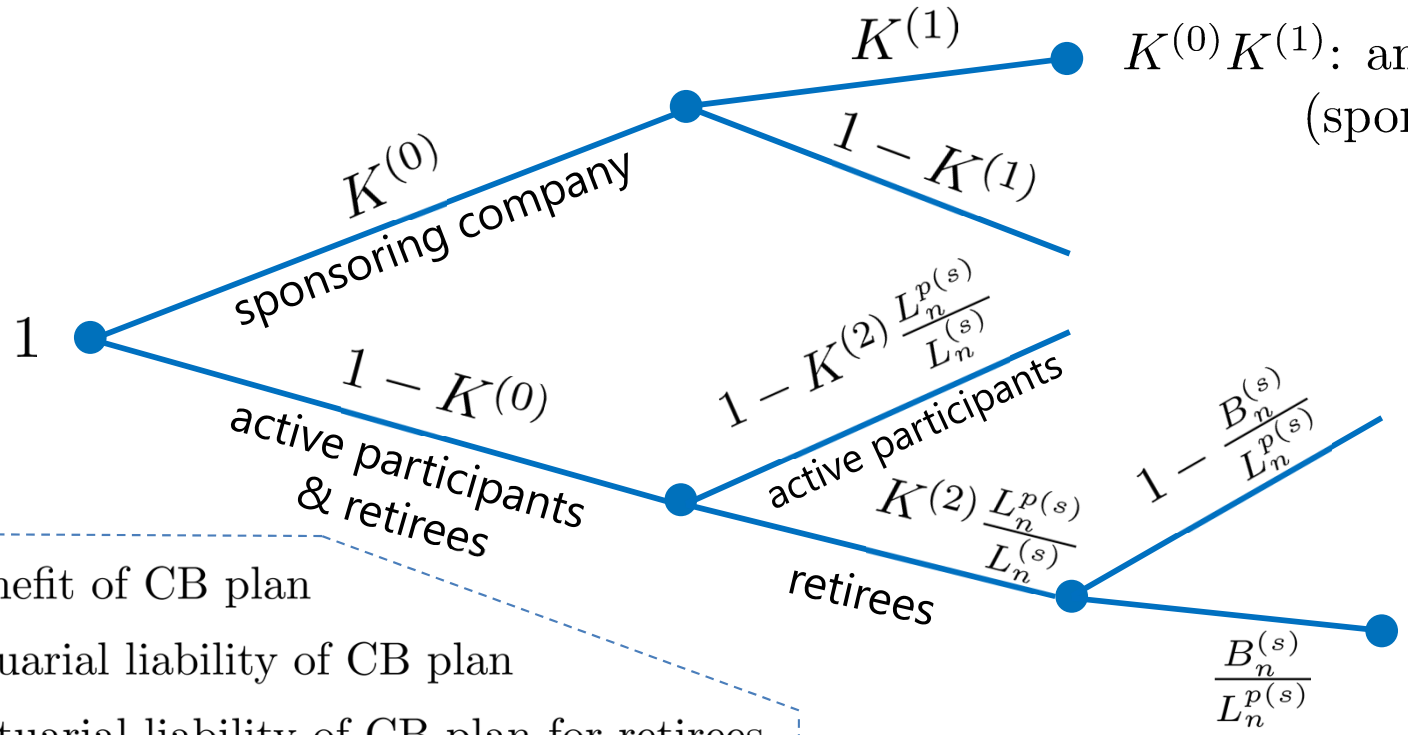
$K^{(0)}$: fraction of the deficiency/surplus the sponsoring company will bear/receive

$K^{(1)}$: annual amortization/decumulation ratio of the deficiency/surplus allocated to the sponsoring company

$K^{(2)}$: fraction of deficiency/surplus the retirees will share with active participants

$(0 \leq K^{(i)} \leq 1,\ (i = 0, 1, 2))$

- **Shared fraction of deficiency(+)/surplus(-) at period n**



$K^{(0)}K^{(1)}$: amortization/decumulation (sponsoring company)

$(1 - K^{(0)})K^{(2)} \left(\frac{B_n^{(s)}}{L_n^{(s)}}\right)$ adjusted benefit (retirees)

$B_n^{(s)}$: benefit of CB plan

$L_n^{(s)}$: actuarial liability of CB plan

$L_n^{p(s)}$: actuarial liability of CB plan for retirees

Risk-sharing Pension Plan (4)

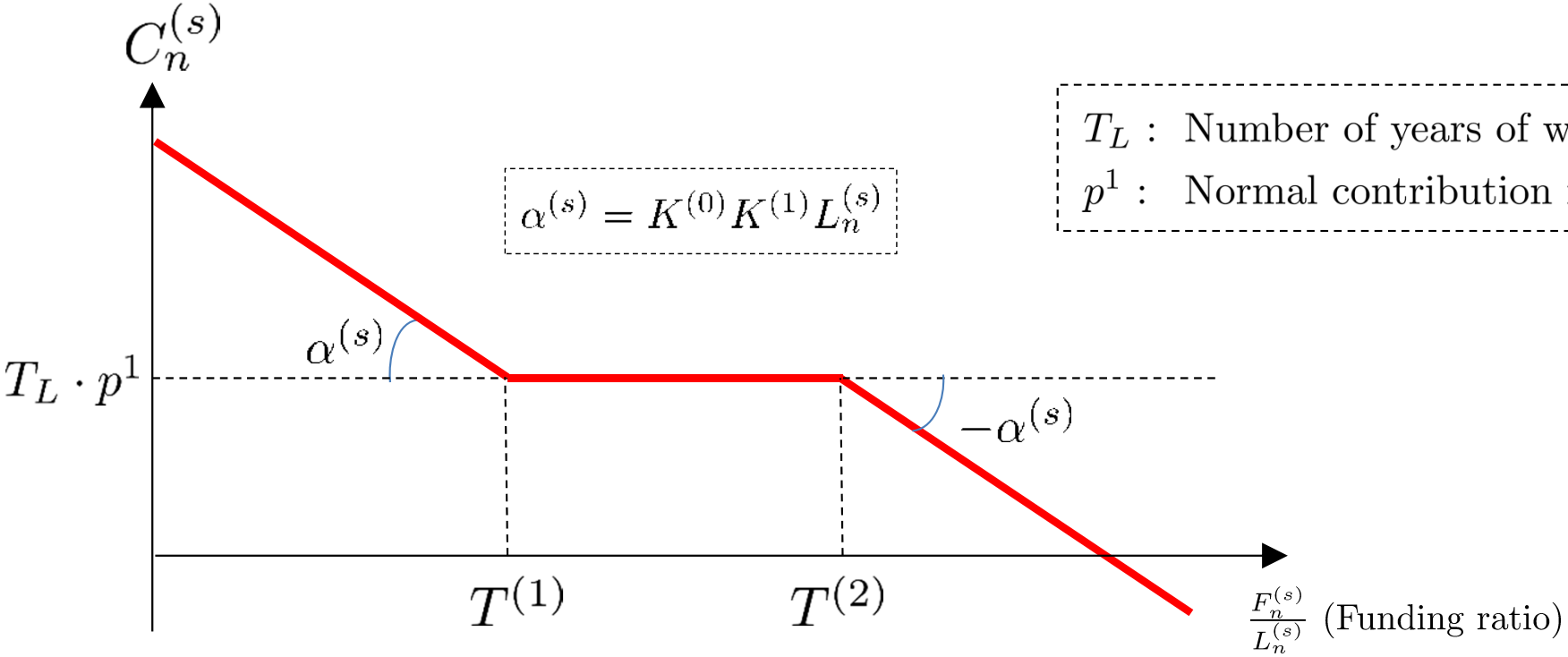
Contribution

$$C_n^{(s)} = \begin{cases} T_L \cdot p^1 + K^{(0)} K^{(1)} L_n^{(s)} \left(T^{(1)} - \frac{F_n^{(s)}}{L_n^{(s)}} \right) & \left(\frac{F_n^{(s)}}{L_n^{(s)}} < T^{(1)} \right) \\ T_L \cdot p^1 & \left(T^{(1)} \leq \frac{F_n^{(s)}}{L_n^{(s)}} < T^{(2)} \right) \\ T_L \cdot p^1 - K^{(0)} K^{(1)} L_n^{(s)} \left(\frac{F_n^{(s)}}{L_n^{(s)}} - T^{(2)} \right) & \left(\frac{F_n^{(s)}}{L_n^{(s)}} \geq T^{(2)} \right) \end{cases}$$

normal contribution

amortization

decumulation



$$\alpha^{(s)} = K^{(0)} K^{(1)} L_n^{(s)}$$

T_L : Number of years of working period

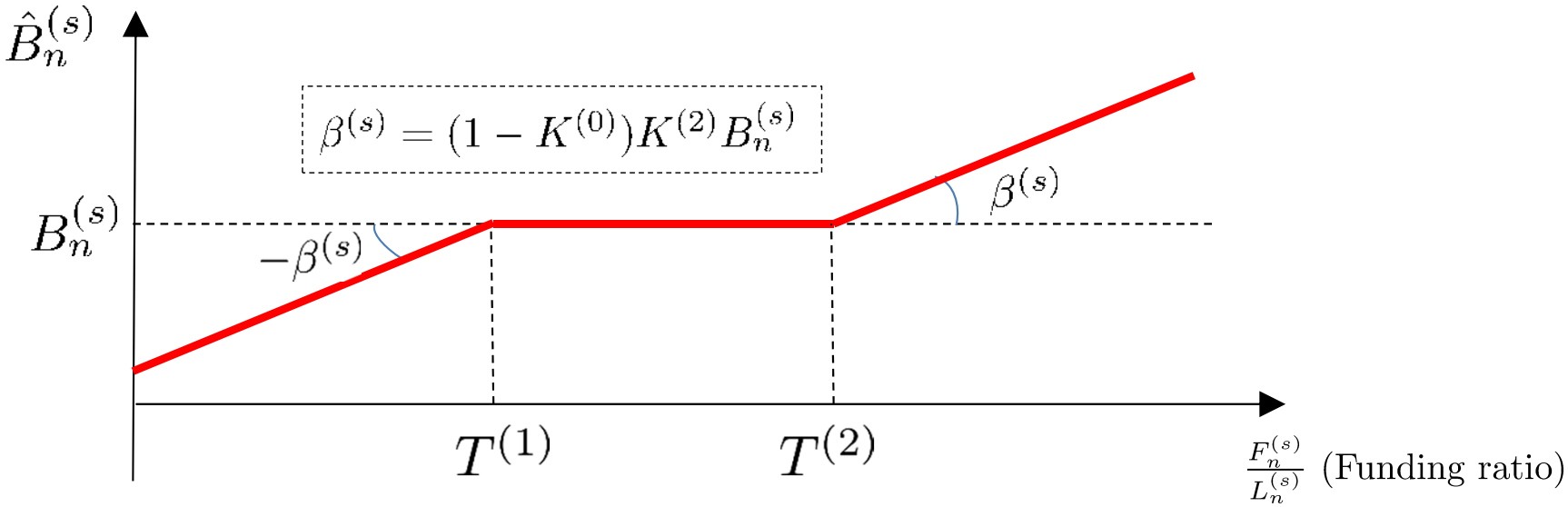
p^1 : Normal contribution rate

Risk-sharing Pension Plan (5)

Benefit

$$\hat{B}_n^{(s)} = \begin{cases} \boxed{B_n^{(s)}}\left\{1-(1-K^{(0)})K^{(2)}\left(T^{(1)}-\frac{F_n^{(s)}}{L_n^{(s)}}\right)\right\} & \left(\frac{F_n^{(s)}}{L_n^{(s)}} < T^{(1)}\right) \\ B_n^{(s)} & \left(T^{(1)} \le \frac{F_n^{(s)}}{L_n^{(s)}} \le T^{(2)}\right) \\ \boxed{B_n^{(s)}}\left\{1+(1-K^{(0)})K^{(2)}\left(\frac{F_n^{(s)}}{L_n^{(s)}}-T^{(2)}\right)\right\} & \left(\frac{F_n^{(s)}}{L_n^{(s)}} > T^{(2)}\right) \end{cases}$$

benefit of CB plan

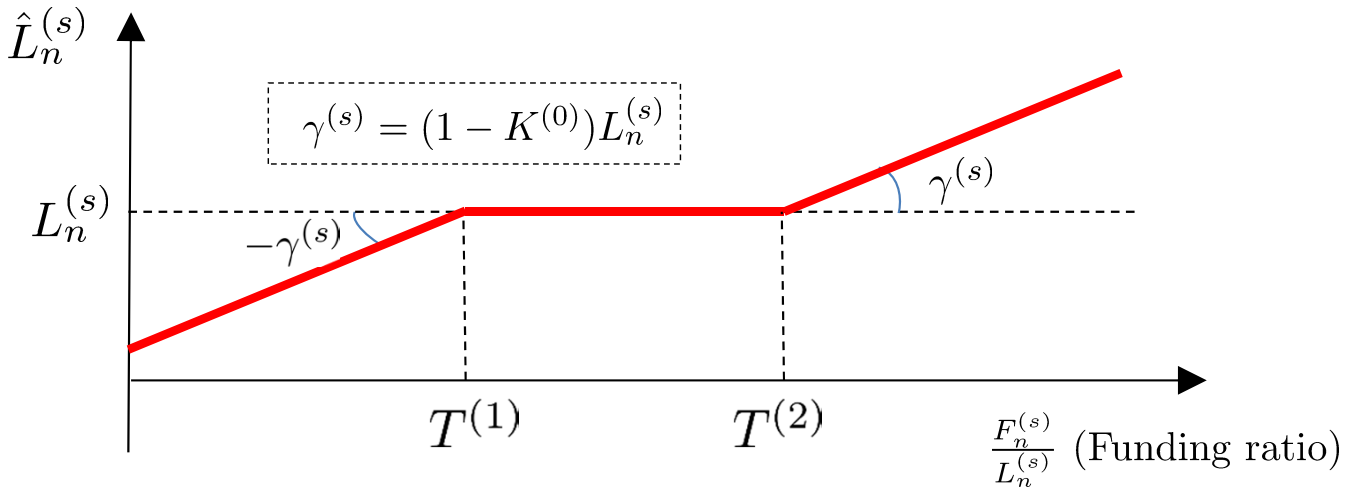


Risk-sharing Pension Plan (6)

Actuarial liability

$$\hat{L}_n^{(s)} = \begin{cases} L_n^{(s)} \left\{1 - (1-K^{(0)})\left(T^{(1)} - \frac{F_n^{(s)}}{L_n^{(s)}}\right)\right\} & \left(\frac{F_n^{(s)}}{L_n^{(s)}} < T^{(1)}\right) \\ L_n^{(s)} & \left(T^{(1)} \le \frac{F_n^{(s)}}{L_n^{(s)}} \le T^{(2)}\right) \\ L_n^{(s)} \left\{1 + (1-K^{(0)})\left(\frac{F_n^{(s)}}{L_n^{(s)}} - T^{(2)}\right)\right\} & \left(\frac{F_n^{(s)}}{L_n^{(s)}} > T^{(2)}\right) \end{cases}$$

Actuarial liability of CB plan



m : Management fee

$r_{M,n-1}^{(s)}$: Portfolio rate of return

Plan asset

$$F_n^{(s)} = \begin{cases} L_1^{(s)} & (n=1) \\ \left(F_{n-1}^{(s)} + C_{n-1}^{(s)} - \hat{B}_{n-1}^{(s)}\right)\exp\left(r_{M,n-1}^{(s)} - m\right) & (n \ge 2) \end{cases}$$

Evaluation of utility

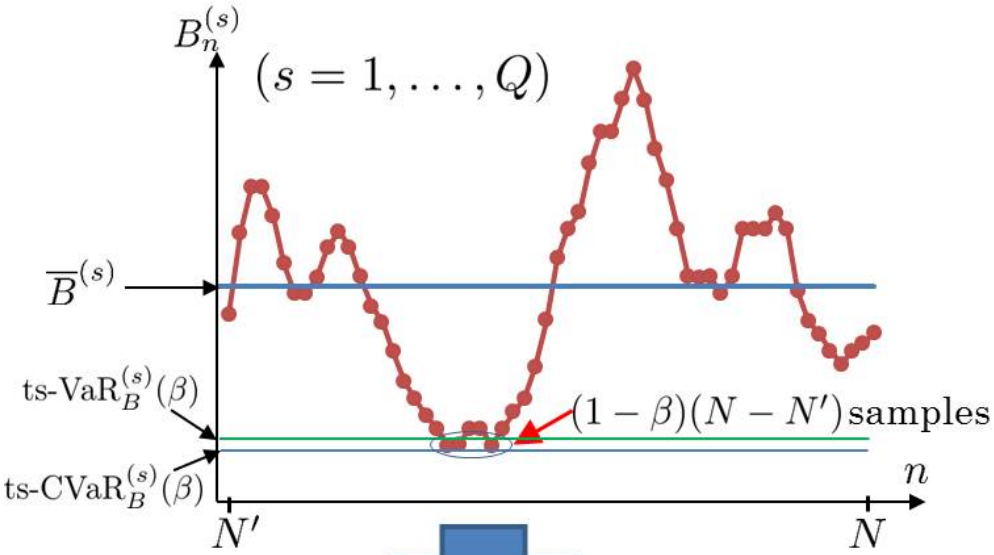
- We use the distributions in the latter part of simulation period to evaluate the utility under the steady-state condition, because the distributions in the former period are dependent on the initial value.
  — N: simulation period
  — N': exempted former period
  (Numerical analysis: $N = 100$ and $N' = 40$).

- Stakeholders share investment risk
  — Participants and retirees: mean and CVaR of benefit
  — Sponsoring company: mean and CVaR of contribution

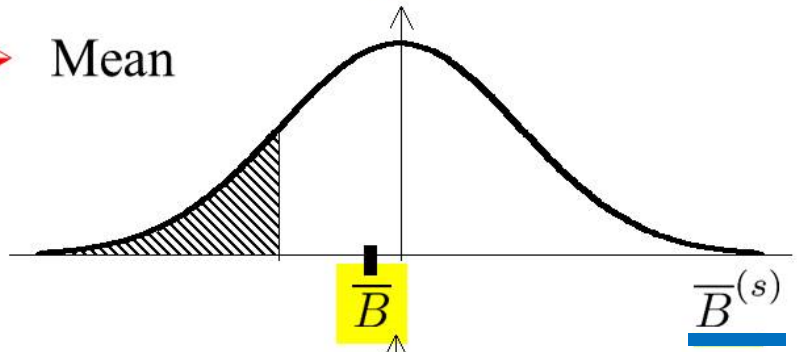
- We consider that it is more important to evaluate time-series state variability on each scenario than variability of states at specific time.

Evaluation measures

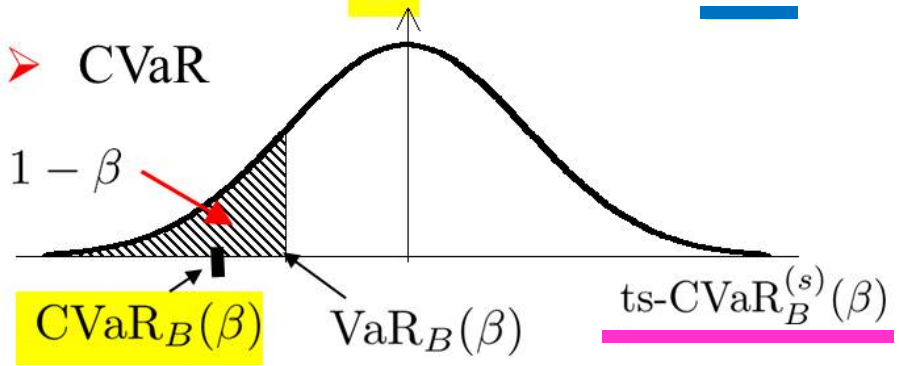
✓ Benefit



➢ Mean



➢ CVaR



Time-series mean (average of $N - N'$ samples)

$$\overline{B}^{(s)} = \frac{\sum_{n=N'+1}^{N} B_n^{(s)}}{N - N'}$$

Time-series CVaR(average of $(1-\beta)(N-N')$ samples)

$$\text{ts-CVaR}_B^{(s)}(\beta) = \frac{\sum_{n=N'+1}^{N} \mathbf{1}_{\{B_n^{(s)} < \text{ts-VaR}_B^{(s)}\}} B_n^{(s)}}{(1-\beta)(N-N')}$$

$$\text{ts-VaR}_B^{(s)}(\beta) = \max\{X^{(s)} | \Pr(X^{(s)} \le B_n^{(s)}) \le 1-\beta\}$$

Mean (average of time-series mean)

$$\overline{B} = \frac{\sum_{s=1}^{Q} \overline{B}^{(s)}}{Q}$$

CVaR (CVaR of times-series CVaR)

$$\text{CVaR}_B(\beta) = \frac{\sum_{s=1}^{Q} \mathbf{1}_{\{\text{ts-CVaR}_B^{(s)} < \text{VaR}_B\}} \text{ts-CVaR}_B^{(s)}(\beta)}{(1-\beta)Q}$$

$$\text{VaR}_B(\beta) = \max\{X | \Pr(X \le \text{ts-CVaR}_B^{(s)}(\beta)) \le 1-\beta\}$$

Numerical Analysis (1)

◆ Numerical Analysis using the Monte Carlo simulation approach

- Base analysis: Comparison of risk-sharing plans with DC, DB and CB plans
- Sensitivity analysis of five parameters of risk-sharing plan

Parameters	values
Number of years of working period	$T_L = 45$
Number of years of payment period for pension benefit	$T_R = 15$
Number of years of simulation period	$N = 100$
Number of years of exemption period	$N' = 40$
Management fee of DC plan Management fees of DB, CB, and RS plans	m =150bp(1.5%) m =50bp(0.5%)
Number of scenarios over a hundred years	$Q = 10,000$
Trigger parameters of risk-sharing plan	$T^{(1)} = 1.05$, $T^{(2)} = 1.3$
Sharing parameters of risk-sharing plan	$K^{(0)} = 0.5$, $K^{(1)} = 0.2$ $K^{(2)} = 0.5$
Additional parameter of DB and CB plans	$K^{(1)} = 0.2$
Number of assets	$U = 5$

Numerical Analysis (2)

- Generating random samples of nominal rate of return, which are normally distributed

Expected return, standard deviation and correlation

	DS	DB	FS	FB	SA	IR	10Y-GB
Expected return	6.0%	3.4%	6.4%	3.7%	1.1%	2.8%	3.4%
Standard deviation	25.1%	4.7%	27.3%	12.6%	0.5%	1.9%	1.5%
Correlation	DS	DB	FS	FB	SA	IR	10Y-GB
Domestic stock (DS)	1.00	−0.16	0.64	0.04	−0.10	0.12	−0.10
Domestic bond (DB)	−0.16	1.00	0.09	0.25	0.12	0.18	0.12
Foreign stock (FS)	0.64	0.09	1.00	0.57	−0.14	0.10	−0.14
Foreign bond (FB)	0.04	0.25	0.57	1.00	−0.15	0.07	−0.15
Short-term asset (SA)	−0.10	0.12	−0.14	−0.15	1.00	0.35	1.00
Inflation rate (IR)	0.12	0.18	0.10	0.07	0.35	1.00	0.35
Government bond (10Y-GB)	−0.10	0.12	−0.14	−0.15	1.00	0.35	1.00

- These parameters are based on the parameters used in order to derive the new policy asset mix for the third medium-term plan published by Government Pension Investment Fund in Japan

Numerical Analysis (3)
– Setting for sensitivity analysis –

Expected rates of return

	DS	DB	FS	FB	SA	IR	10Y-GB
Return (A)	6.0%	3.4%	6.4%	3.7%	1.1%	2.8%	3.4%
Return (B)	3.0%	1.7%	3.2%	1.85%	0.55%	1.4%	1.7%
Return (C)	0.0%	3.4%	0.0%	3.7%	1.1%	2.8%	3.4%

※ Return (A): Base parameter, Return (B): 0.5 × Return (A),
Return (C): 0% expected rate of return of stocks in Return (A)

Portfolios

	DS	DB	FS	FB	Stock:Bond
Portfolio [a]	25%	35%	25%	15%	5:5
Portfolio [b]	5%	55%	5%	35%	1:9

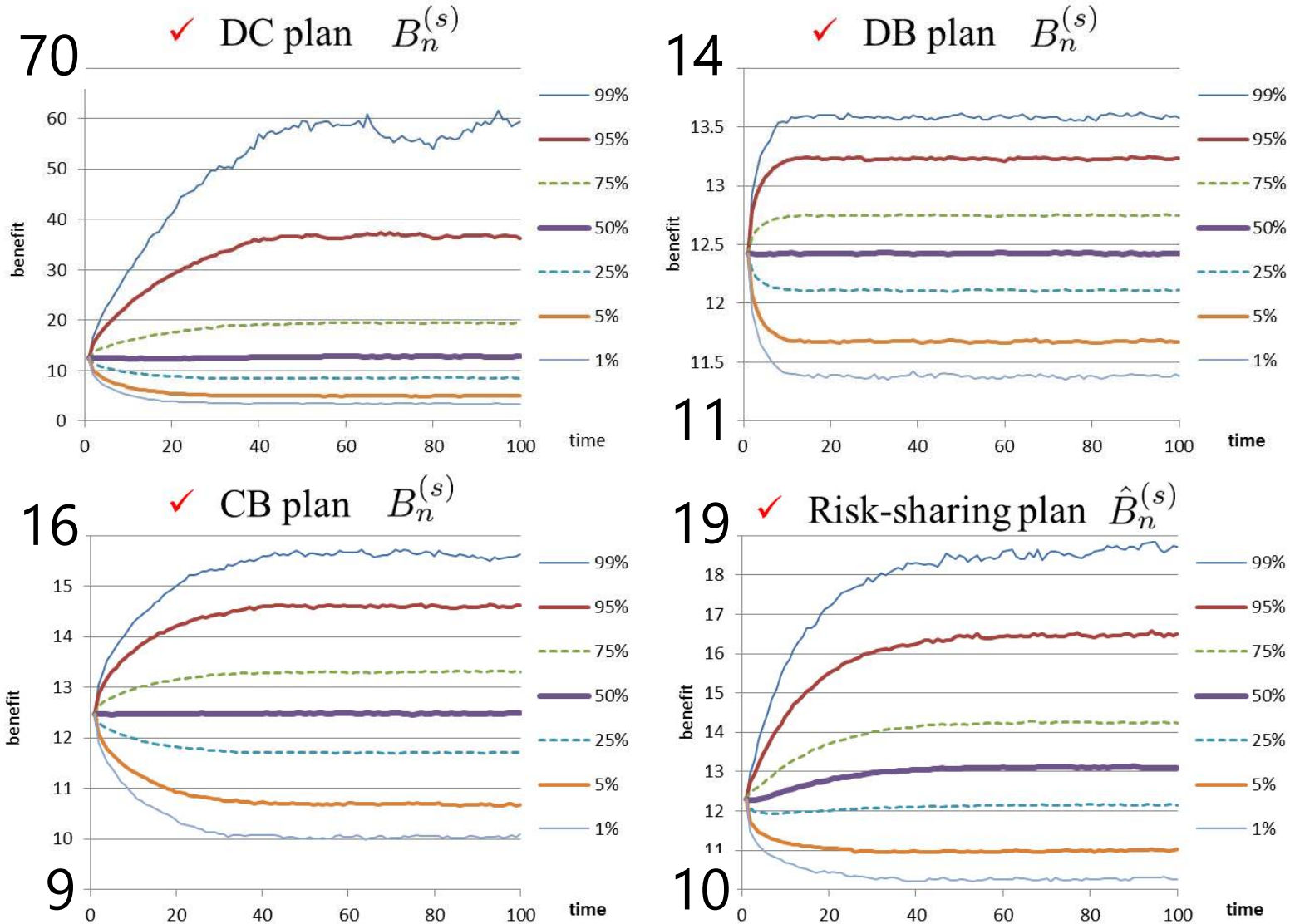
Expected real rate of return and standard deviation

	Expected real rate of return			Standard deviation
	Return (A)	Return (B)	Return (C)	
Portfolio [a]	[Aa] 2.045%	[Ba] 1.0225%	[Ca] −1.055%	12.766%
Portfolio [b]	[Ab] 0.985%	[Bb] 0.4925%	[Cb] 0.365%	6.661%

Base analysis (1)

– Comparison of DC, DB, CB and risk-sharing (RS) plans –

Percentiles of the distributions of benefit in Case Aa



- Variabilities among four plans

  $\mathrm{DB} < \mathrm{CB} < \mathrm{RS} < \mathrm{DC}$

- ✓ DC plan: Benefit is based on actuarial liabilities calculated using portfolio return.
- ✓ DB plan: Benefit is fixed to 1 at retirement.
- ✓ CB < RS: Benefit of RS plan is adjusted by the funding ratio.

Base analysis (2)

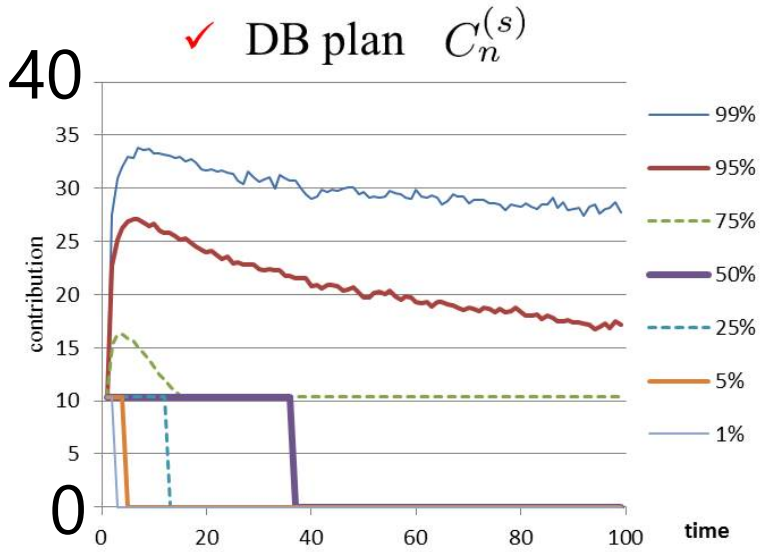
– Comparison of DC, DB, CB and risk-sharing (RS) plans –

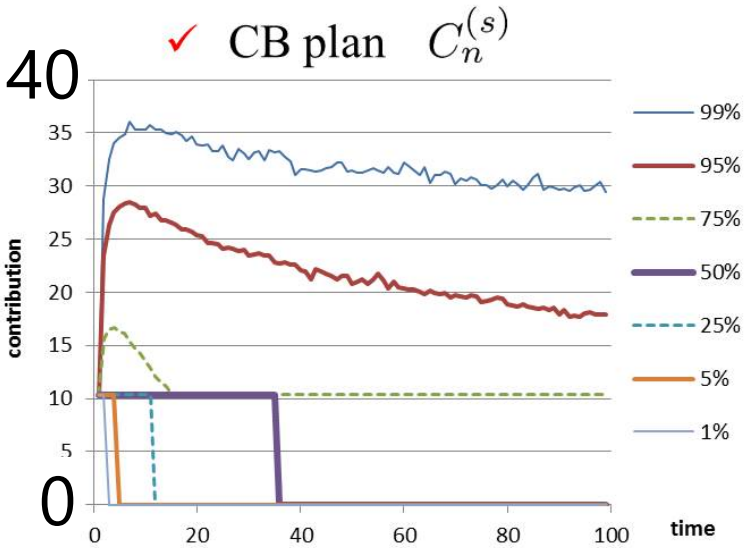
Percentiles of the distributions of contribution in Case Aa

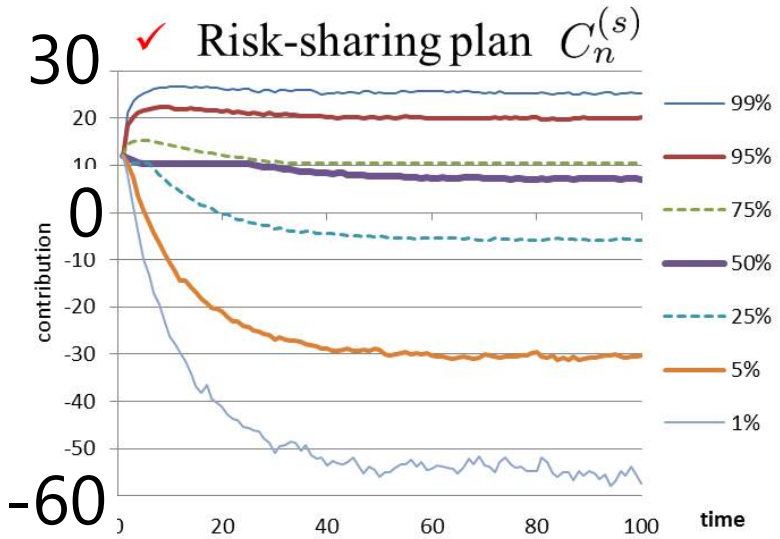
✓ DC plan : constant

$$C_n^{(s)} = 10.379$$

$$(C_n^{1(s)} = 10.379,\ C_n^{2(s)} = 0)$$







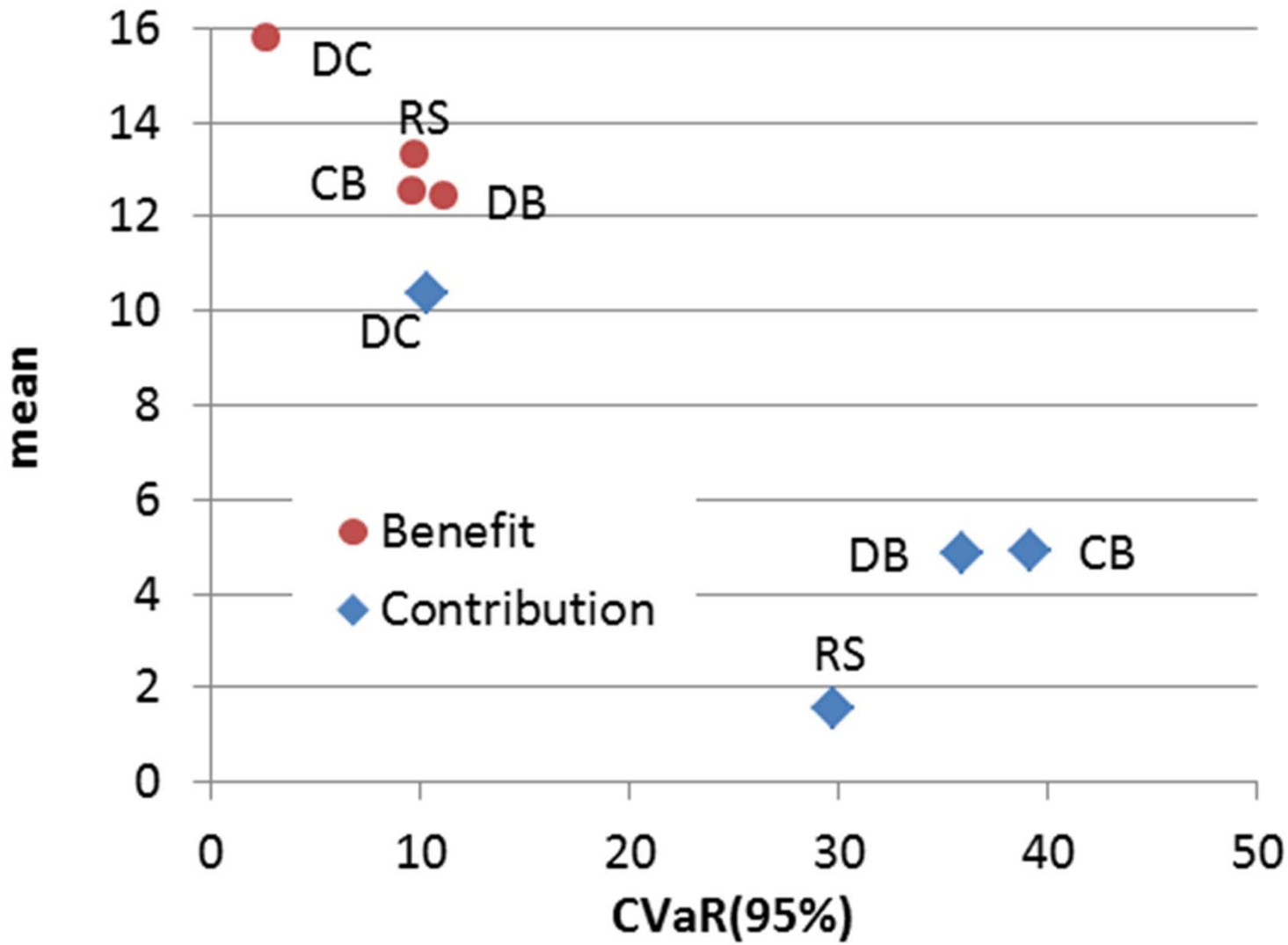
✓ DB and CB plans: Amortizations are similar, and contributions do not become negative.

✓ Risk-sharing plan: Contributions can become negative, and it is possible to decrease cost of a sponsoring company.

Base analysis (3)

– Comparison of DC, DB, CB and risk-sharing (RS) plans –

Mean-CVaR diagram in Case Aa



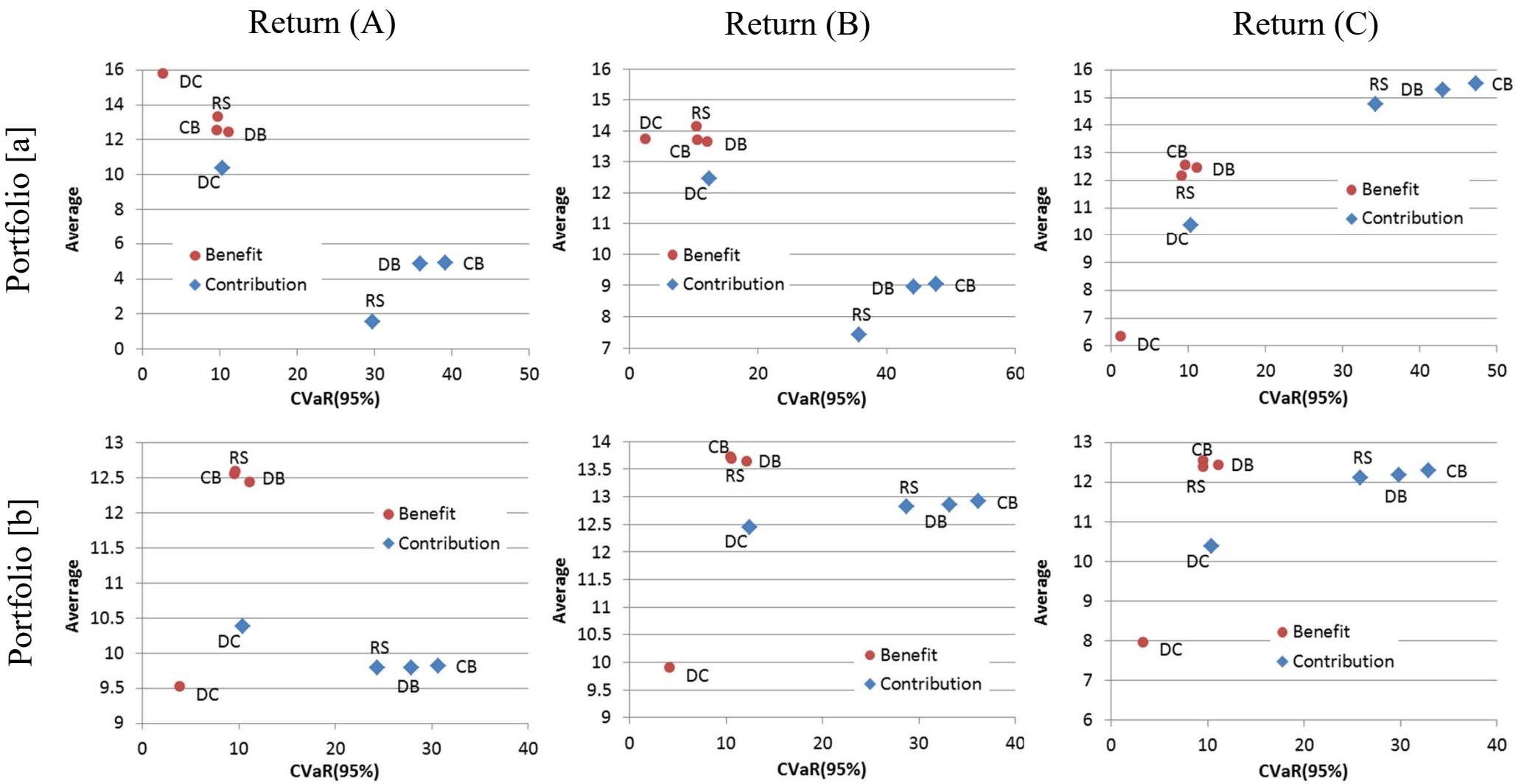
- ✓ Distributions become stable when about forty years pass
- ✓ We evaluate means and CVaRs in the latter sixty years of the simulation period
- ✓ Relationship of CVaRs among four plans
  - ✓ Benefit

    $\mathrm{DC} < (\ \mathrm{RS} \approx \mathrm{CB}\ ) < \mathrm{DB}$
  - ✓ Contribution

    $\mathrm{DC} < \mathrm{RS} < \mathrm{DB} < \mathrm{CB}$

Base analysis (4)

– Comparison of four plans: Mean-CVaR diagram –



Base analysis (5)

– Comparison of DC, DB, CB and risk-sharing (RS) plans –

case	expected return	mean benefit	mean contribution
Aa	2.0450%	DB < CB < RS < DC	RS < DB < CB < DC
Ba	1.0225%	DB < CB < DC < RS	
Ab	0.9850%	DC < DB < CB < RS	
Bb	0.4925%	DC < DB < RS < CB	DC < RS < DB < CB
Cb	0.3650%	DC < RS < DB < CB	
Ca	−1.0550%		

- ✓ CVaR of Benefit

  DC < (RS ≈ CB) < DB

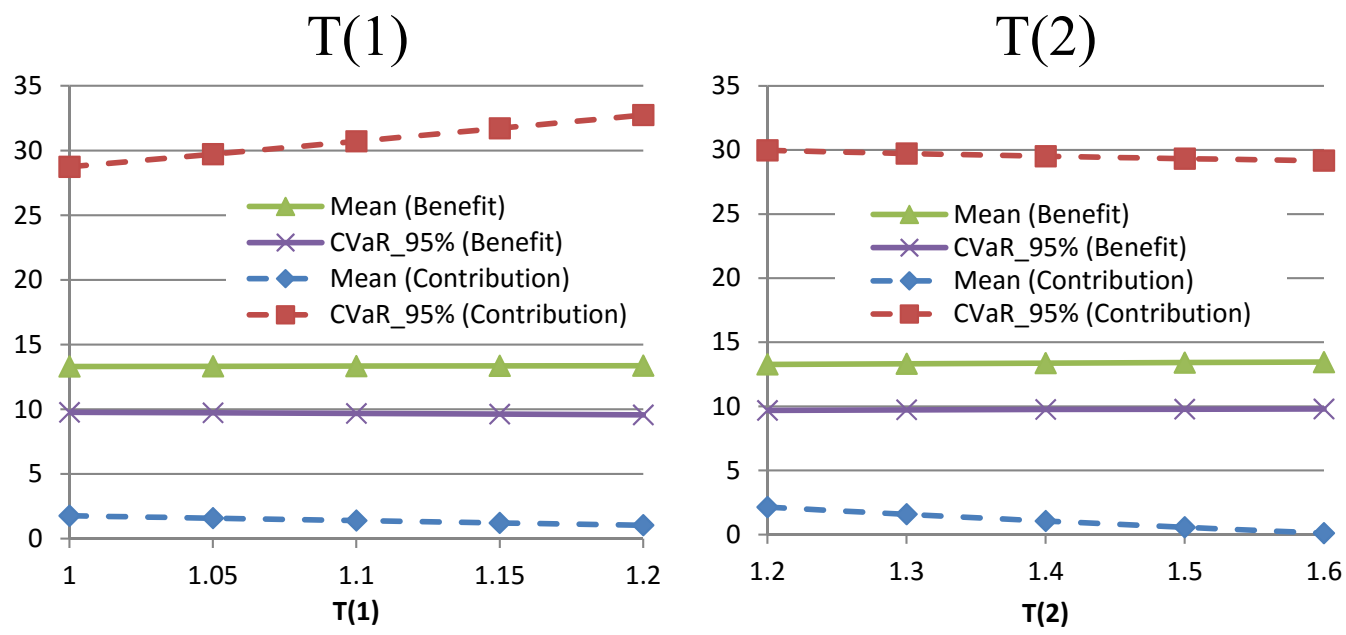
- ✓ CVaR of Contribution

  DC < RS < DB < CB

The relationship of CVaRs of four plans is not dependent on the portfolio returns

Sensitivity analysis (1)

– Trigger parameters in Case Aa –



- ✓ The parameters T(1) and T(2) are related with the funding ratio which triggers the deficiency/surplus-sharing.
- ✓ The CVaR of the contribution is sensitive to T(1) due to deficiency-sharing, but the other measures are a little bit sensitive to T(1) and T(2).

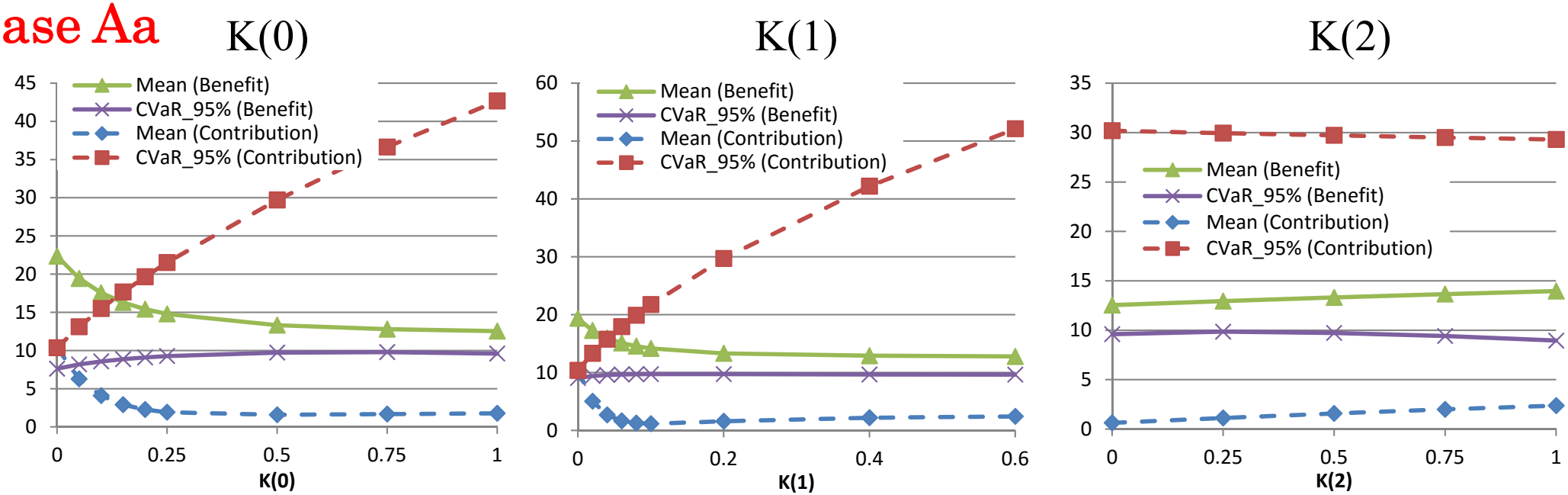
Sensitivity analysis (2)

– Sharing parameters –

	benefit		contribution		Who share the fraction?	
	mean	CVaR	mean	CVaR	K(i)	1 – K(i)
K(0)	−/+	+	−/+	+	sponsor	participants/retirees
K(1)	−/+	+	−/+	+	sponsor	
K(2)	+/−	−	+/−	−	retiree	participants

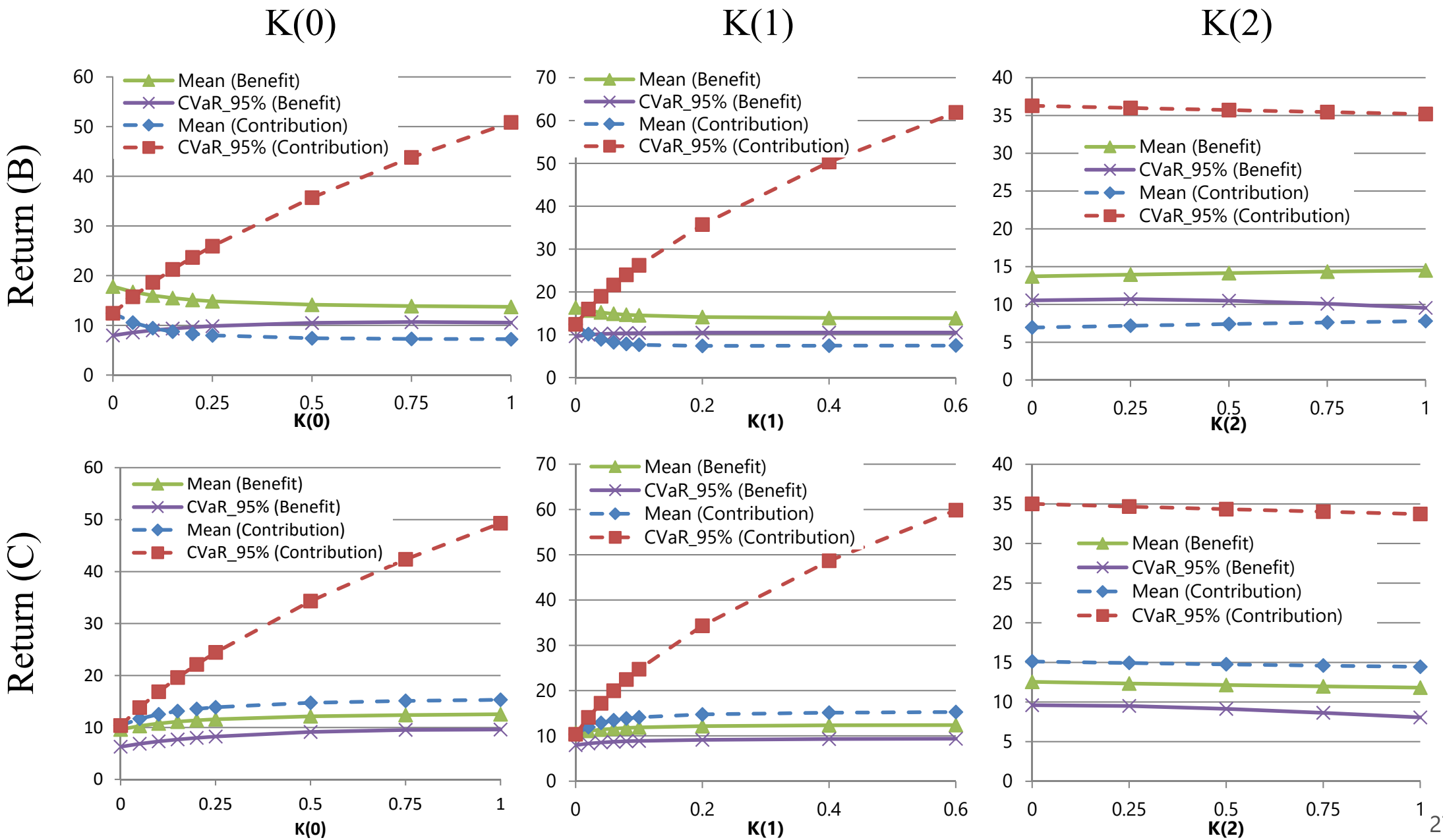
※ 'mean' is dependent on portfolio return: high return/low return

Case Aa



Sensitivity analysis (3)

– Sharing parameters in Cases Ba and Ca –



Backtesting – Setting –

- ✓ Backtest period: From March 1995 to March 2015 (twenty years)
- ✓ Historical data
  - Domestic stock (DS): TOPIX (Tokyo Stock Price Index)
  - Domestic bond (DB): JPGBI (Citigroup Japan Government Bond Index)
  - Foreign stock (FS): S&P500 (Standard & Poor's 500 Stock Index)
  - Foreign bond (FB): USGBI (Citigroup USA Government Bond Index)
  - Wage growth rates, calculated using monthly labor survey
  - 10-year government bond yield
  - Dollar-yen exchange rate (center value of interbank spot rate)
  - Japanese yen interest rate: one-year Euroyen TIBOR
  - Dollar interest rate: one-year Eurodollar interest rate

	DS	DB	FS	FB	IR	10Y-GB
index	TOPIX	JPGBI	S&P500	USGBI	wage growth rate	bond yield
mean	3.87%	2.73%	12.07%	8.01%	−0.36%	1.59%
st. dev.	25.71%	2.37%	25.73%	12.86%	1.72%	0.80%

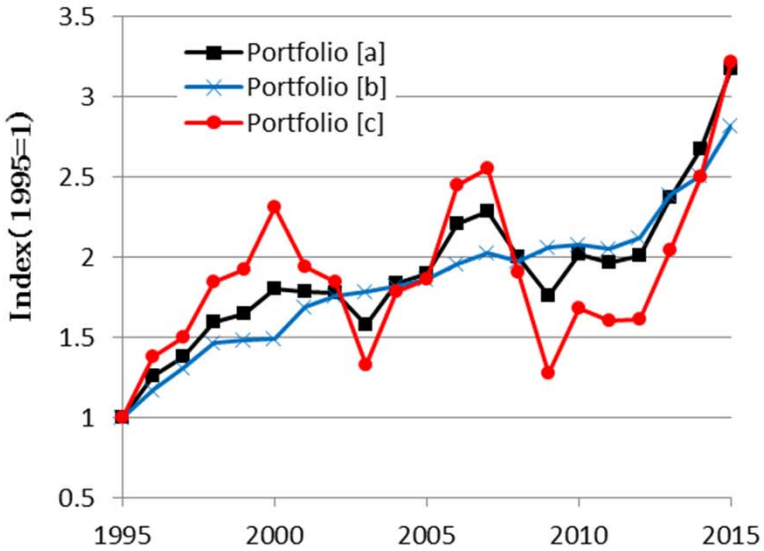
Backtesting – Result(1) –

✓ Portfolio weights (Constant rebalance strategy)

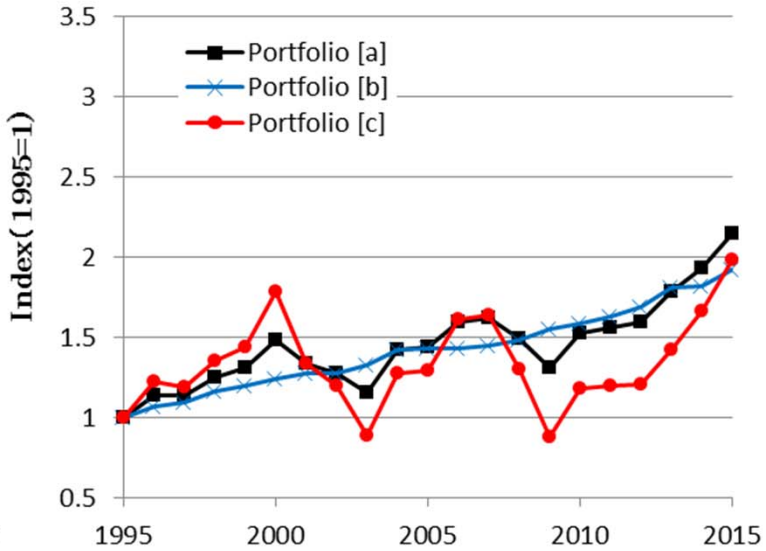
	DS	DB	FS	FB	Stock:Bond
Portfolio [a]	25%	35%	25%	15%	5:5
Portfolio [b]	5%	55%	5%	35%	1:9
Portfolio [c]	50%	0%	50%	0%	10:0

✓ Cumulative returns (on a yen basis)

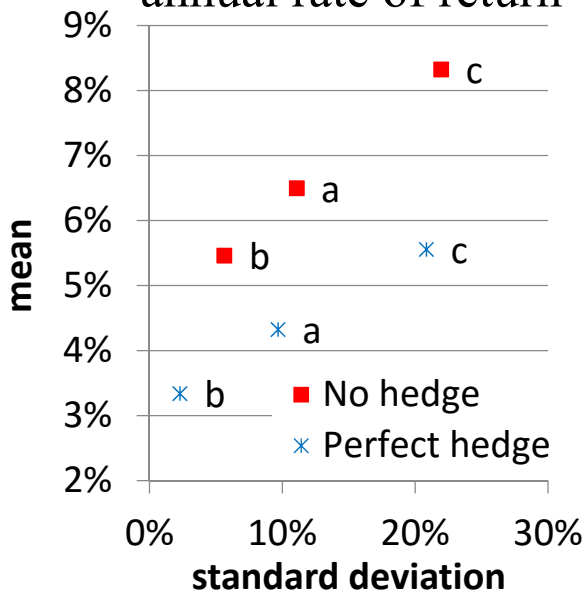
No hedge



Perfect hedge



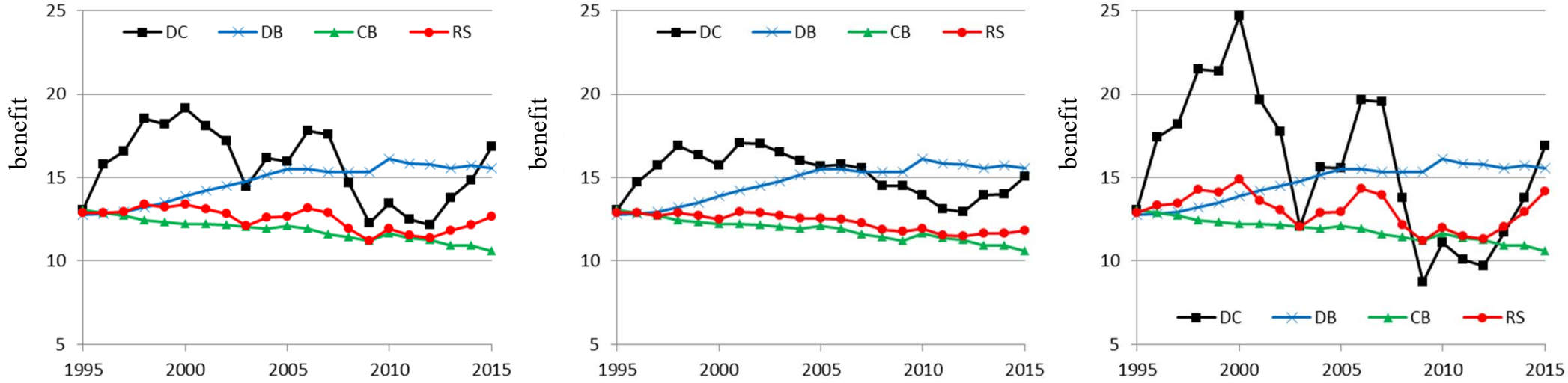
Mean and standard deviation of annual rate of return



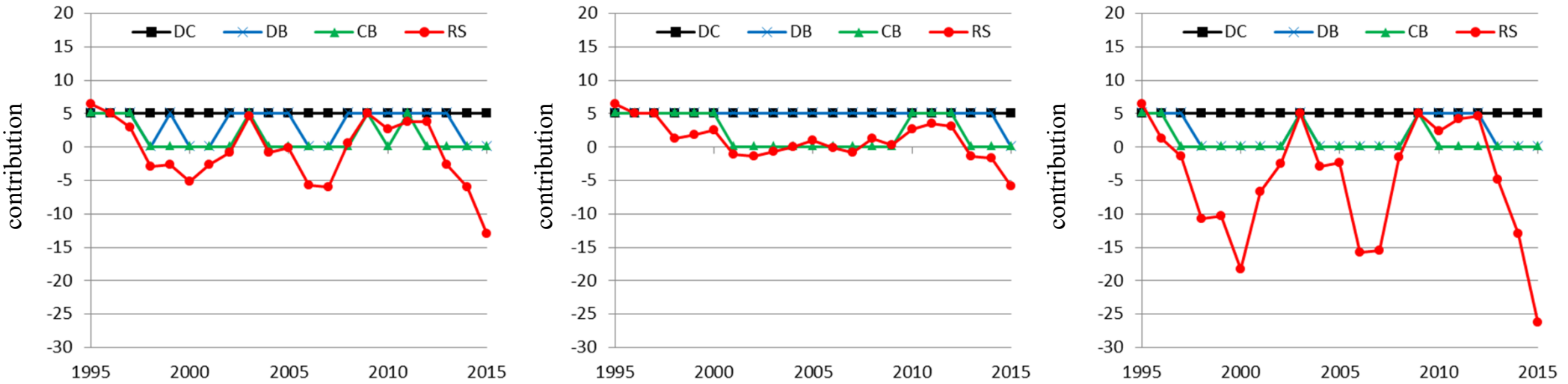
Backtesting – Result(2): No hedging strategy –

Portfolio [a] Portfolio [b] Portfolio [c]

Benefit

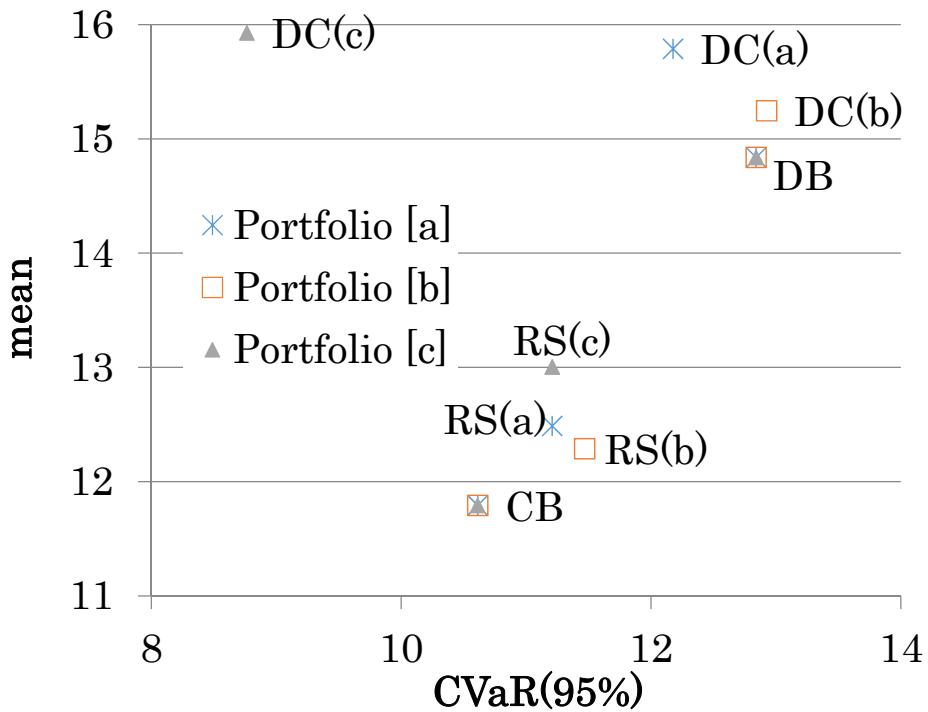


Contribution

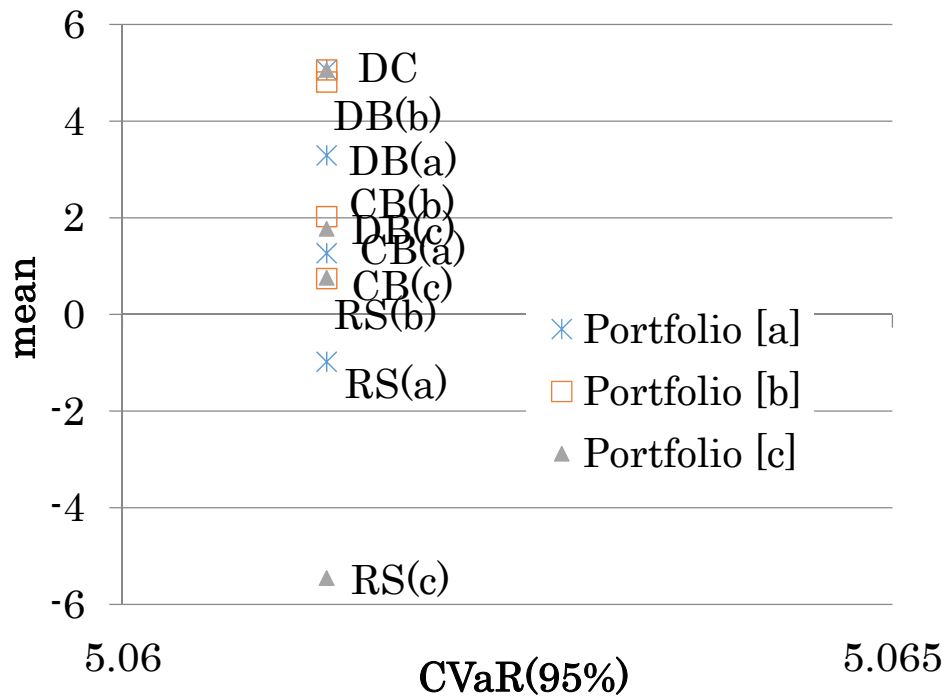


Backtesting – Result(3): Mean-CVaR diagram –

Benefit



Contribution



- ✓ Mean benefit among the four plans

  $$\mathrm{DC} > \mathrm{DB} > \mathrm{RS} > \mathrm{CB}$$

- ✓ CVaRs of benefits

  $$\mathrm{DC(b)} > \mathrm{DB} > \mathrm{DC(a)} > \mathrm{RS} > \mathrm{CB} > \mathrm{DC(c)}$$

- ✓ Mean contribution among the four plans

  $$\mathrm{RS} < \mathrm{CB} < \mathrm{DB} < \mathrm{DC}$$

- ✓ CVaRs of contributions

  $$\mathrm{RS} = \mathrm{CB} = \mathrm{DB} = \mathrm{DC}$$

Conclusion

- In this paper, we design the risk-sharing pension plan using the five parameters which control the level of risk sharing.
- We implement the Monte Carlo simulation for a long-term period, and evaluate the uncertainty of benefits and contributions.
- Moreover, we compare the RS plan with the existing DC, DB, and CB plans, and we find the benefit and contribution of the RS plan are not only between the DB and DC plans, but also superior to them in some cases.
- In the future research, we compare the risk-sharing plan proposed in this paper with the intermediate plan which consists of the weighted plan of the DB and DC plans. In addition, we need to formulate the optimization model, which solve the problem to find the optimal parameter values of controlling the risk-sharing.

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