A Confirmation of Kocken's Proposition about the Intergenerational Risk Transfer within pension plans by Monte Carlo Simulations

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Using Monte Carlo simulations, this paper confirms two examples of intergenerational risk transfer asserted by Professor Theo Kocken based on financial economics in Kocken (2012). Investigated two examples are the defined-benefit (DB) corporate pension plans of state and local governments of the U.S. as well as the collective defined contribution (CDC) occupational pension plans in the Netherlands. Although Kocken's models are easy to understand since it does not use utility functions such as Gollier(2007), they might not give sufficient sense of reality to practitioners because they do not deal with annual contributions which our models explicitly incorporate.

With regards to the DB plans in the U.S., our simulations of matured pensions indicated that investing assets aimed at an investment-return higher than the risk-free rate with a 5% added risk premium has a 50% or higher probability of depleting pension assets. The reason for this is that the skewness of the probability distribution of future pension assets becomes large. In addition, it was found that the kurtosis increases with time, while the median continues to decrease. When pension assets are depleted, a reduction of benefits or additional contributions from the state and local governments becomes necessary, resulting in the occurrence of intergenerational risk transfer. However, results from other simulations confirmed that appropriate raises in premiums could prevent such depletion of assets.

The simulations for models of CDC of the Netherlands showed that under the agreed pension design, there is a possibility that pension assets may become depleted. Due to this depletion of pension assets, a risk transfer from the working generation to the post-retirement pensioners will occur. In the case of market-consistent CDC benefits proposed by Kocken, asset depletion will not occur.

Considering the above discussion, we also conclude that the traditional pension mathematics does not provide sufficient information to the plan sponsors or employers without enough money to raise premiums. Traditional pension mathematics states that low discount rate means high premium, low discount rate means high premium. I think it would be kind to advice additional future contribution calculated with Monte Carlo Simulation if the liability is measured with high discount rates.

Keywords: pension, collective DC, investment risk, target return, DC, Monte Carlo simulation

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1. Introduction

Using Monte Carlo simulations, this paper confirms two examples of intergenerational risk transfer asserted by Professor Theo Kocken based on financial economics in Kocken (2012),who thinks risk premiums should be given in accordance with risk taken. Investigated two examples are the defined-benefit (DB) corporate pension plans of state and local governments of the U.S. as well as the collective defined contribution (CDC) occupational pension plans of the Netherlands. Although Kocken's models are easy to understand since it does not use utility functions, they might not give sufficient sense of reality to practitioners because they do not deal with annual contributions which our models explicitly incorporate.

The remainder of the paper is organized as follows. In section 2, we briefly summarize Kocken's discussion about two examples of interegenerational risk transfer. Section 3 discusses the risk of high discount rate using Monte Carlo simulation. In section 4, we demonstrate the risk of current generous benefits of Dutch CDC. Section 5 concludes the validity of Kocken's assertion as well as several findings.

2. Summary of Kocken's reasoning

We refer to the following text in the abstract of Kocken (2012) as "Kocken's proposition": "Some techniques in use today underestimate liabilities and benefit current retirees at the expense of other plan stakeholders, undermining the sustainability of risk-sharing pension plans by shifting concealed deficits to future generations." We also refer to Kocken's proposition applied to U.S. State and local pension plans as "Kocken's proposition 1", and Kocken's proposition applied to Dutch CDC as "Kocken's proposition 2".

From the above definition, Kocken's proposition 1 is "U.S. State and local pension plans underestimate liabilities and benefit current retirees at the expense of other plan stakeholders, undermining the sustainability of risk-sharing pension plans by shifting concealed deficits to future generations." This relates the possibility of interegenerational risk transfer in State and local pension plans in the U.S., which are public pensions for state and local government employees. These plans cover wide range of occupations including teachers, fire fighters, police, members of judiciary, and many other state and local employees. They are pure DB systems that guarantee a benefit to their beneficiaries. Kocken asserts that from the beneficiaries' viewpoints, they are risk-free and the total present value of pension payments discounted against the term structure of risk-free rate, equals the market-consistent value of liabilities. However, in reality, these payments are discounted based on generally aggressive asset return assumptions such as 8%. As a result, many plans now face rapidly running out of assets, which will turn them into almost depleted plans for the generations to come. Funding ratios have fallen below 100% with risk-free discount rates, but retirees are still paid 100% of their promised pensions.

Kocken's proposition 2 is "Dutch CDCs underestimate liabilities and benefit current retirees at the expense of other plan stakeholders, undermining the sustainability of risk-sharing pension plans by shifting concealed deficits to future generations." From the viewpoint of financial economics, Kocken criticizes the Dutch Pension Accord of June 19,2011, which is consistent with FTK2 , revised version of old regulation FTK and replaced before implementation by nFTK. The Pension Accord proposed to add the expected risk premium on top of the risk-free rate as a discount factor, reasoning that the pensions have become uncertain and therefore the expected return – risk-free rate plus expected risk premium – can be applied. The accord has produced a collective risk-sharing system, where any shock in financial market returns or unanticipated changes in longevity are allocated to the members by means of 10-year smoothing period. Assume, for example, that inflation rate is 2%, the risk premium is 2%, and the realized return at the end of year 1 equals the risk-free rate. Owing to 10-year smoothing, the riskiness of retirement income is equivalent to retirees' having 90% invested in risk-free bonds and 10% invested in risk assts. If the realized return is -4%, pension payment for retirees should reflect $-4\% \times 10\% = -0.4\%$ return, but reality is the endowment of 1.4%=2-0.6%=2-(2%+4%) $\times 10\%$. The excess payment of 1.8% in the example above means that retirees are consuming the risk premium of risks they did not take. It generates a material income redistribution from younger to older people.

3. The risk of high discount rate

3.1 Assumptions

We verify Kocken's Proposition 1 by Monte Carlo simulation. We construct simple models by extracting the essence of U.S. State and local pensions, and show that the model pensions will deplete even if they are fully funded with discount rates including risk premiums. We assume that the contributions are 10 and the benefits are 15 every year, and both are occurred at the middle of each year. This means we simulate about the matured plan of which contributions are less than benefits. Considering current global low interest rate situation, we assume risk free rate to be 0% In the Kocken's U.S. example, the risk free rate is 3%, and risk premium is 5% which we also adopt. The recurrence formula of pension fund F_τ is given by

$$F_\tau = F_{\tau-1}(1 + r_\tau) + (P - B)(1 + r_\tau)^{\frac{1}{2}}, \qquad (3.1)$$

where r_τ is return of pension fund for year τ , P is contributions, B is benefits. If r_τ is equal to its expected value, and F_τ is stationary:

$$F_{\tau+1} = F_\tau = \cdots = F_0 \ , \qquad (3.2)$$

then the initial amount of pension asset F_0 is derived by solving the following recurrence equation of F_0:

$$F_0 = F_0(1 + \mu) + (P - B)(1 + \mu)^{\frac{1}{2}} \ . \qquad (3.3)$$

The solution is given by

$$F_0 = \frac{B-P}{\mu}(1+\mu)^{\frac{1}{2}} \quad . \tag{3.4}$$

For μ=1%, 2%and 5%, the value of F_0 is presented in Table 3-1 below, where P=10 and B=15. Our main case is μ= 5%. Case with 2% is provided for comparison. F_0 with 1% is provided for determining contribution suspensions in case of larger assets compared with F_0.

Table 3-1. Discount rates and amount of assets in equilibrium

Expected return	Amount of asset F_0 in equilibrium
1%	502.49378
2%	252.48762
5%	102.46951

Taking 5% as an example, form the static point of view, as shown in the following (3.5) formula, the equilibrium amount 102.46951 of assets is always maintained because investment returns from assets is equal to the amount of benefits excess of contributions, as shown in the following (3.5) formula.

$102.46951 \times 1.05 + (10\text{-}15) \times \sqrt{1.05} = 102.46951$ (3.5)

However, the result is quite different when you assume the risks associated with the return achieved as shown in subsection 3.2 to 3.5.

We assume 3.2% to be the portfolio risk (standard deviation) to achieve the 2%, 10% to be the portfolio of risk (standard deviation) to achieve a 5%. These risks are the standard deviations of risk-minimizing portfolio calculated based on the expectation of asset returns and risks for Japanese market as shown in the table 3-2a, but possible values in the U.S.

Table 3-2a. Expectation of returns, risks, and correlations for asset classes

Asset class	Expected return	Expected risk	Expected correlation				
Cash	0.20%	0.12%	1.000	0.265	-0.161	-0.014	-0.039
Domestic bonds	0.90%	2.71%	0.265	1.000	-0.229	0.073	-0.094
Domestic stocks	6.80%	17.97%	-0.161	-0.229	1.000	0.260	0.600
Foreign bonds	3.30%	10.96%	-0.014	0.073	0.260	1.000	0.579
Foreign stockd	8.30%	19.12%	-0.039	-0.094	0.600	0.579	1.000

The asset allocations of portfolios to attain 2% return and 5% return are shown in the table 3-3.

Table 3-2b.The asset allocation of portfolios targeting 2% and 5% returns

Asset Class	Target Return: 2%	Target Return: 5%
Cash	9%	0%
Domestic Bonds	73%	40%
Domestic Stocks	9%	22%
International Bonds	1%	0%
International Stockes	8%	38%

One of example of asset returns, risks, and correlation matrix in the U.S. market can be found in page 236 of Reilly & Brown (2011), as follows:

Table 3-2c. Example of asset returns, risks, and correlation matrix in the U.S. market

Asset Class	Return	Standard Deviation	Correlation Matrix			
			U.S.Stocks	U.S.Bonds	U.S.Real Estate	U.S. Treasury Bills
U.S.Stocks	12.0%	21.0%	1.00			
U.S.Bonds	8.0	10.5	0.14	1.00		
U.S.Real Estate	12.0	9.0	-0.04	-0.03	1.00	
U.S. Treasury Bills	7.0	0.0	-0.05	-0.03	0.25	1.00

We adjust the return vector considering the 4-week T-bill rate on April 4, 2016 is 0.2%, as shown in table 3-2d, after the subtraction of 2.8% from the return vector of table 3-2c,

Table 3-2d.Assumed asset returns, risks, and correlation matrix in the U.S. market as of April 4, 2016

Asset Class	Return	Standard Deviation	Correlation Matrix			
			U.S.Stocks	U.S.Bonds	U.S.Real Estate	U.S. Treasury Bills
U.S.Stocks	8.2%	21.0%	1.00			
U.S.Bonds	4.2	10.5	0.14	1.00		
U.S.Real Estate	8.2	9.0	-0.04	-0.03	1.00	
U.S. Treasury Bills	0.2	0.0	-0.05	-0.03	0.25	1.00

The standard deviation of returns of risk minimizing portfolios targeting 2% return and 5%

return are 1.7% and 4.6% respectively fairly smaller than our assumption 3.2% and 10%, but 3.6% and 9.5% respectively if we do not invest in real estate, therefore standard deviation 3.2% and 10% is the possible value even in the U.S. market.

As we assume that portfolio return r_τ follows a normal distribution with mean μ and standard deviation, r_τ can be written as

$$r_\tau = \mu + \sigma \times standardized\ normal\ random\ number\ . \qquad (3.5)$$

3.2 Basic Cases

Assuming normal distribution for the return of a portfolio, we perform Monte Carlo simulation10 million times. The normal random numbers for simulations are made after the application of antithetic variables method to numbers generated by the invers function method from uniform random numbers produced from a Mersenne twister in R language. Table 3-3a provides the result of a simulation run, consisting of 1,000,000 replicates with portfolios targeting 5% return. For reference, Figure 1 shows 10 sample paths of a simulation with the vertical axis as amounts of the fund and the horizontal axis as years.

Figure 1 Sample Paths of a Simulation

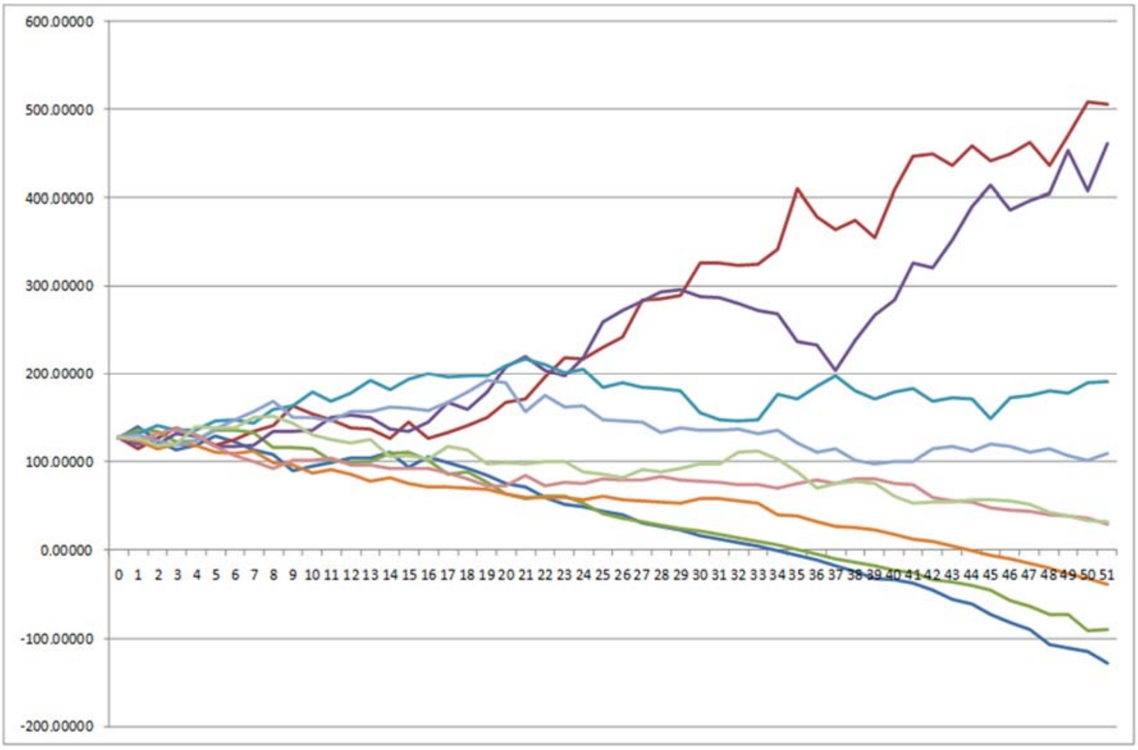


Table 3-3a Basic case (Target return 5%)

Statistics	beginning of 1st year	end of 50th year	end of 100th year
Mean	102	104	120
Percentage of depletion	0.0%	49.9%	64.0%
Standard deviation	0	428	6,181
Standard error	0.000	0.428	6.181
Skewness	-	3	6
Kurtosis	-	21	154
Minimum amount of asset	102	-1,903	-91,623
Maximum amount of asset	102	12,415	473,367
Median	102	1	-763

It is surprising that percentage of depletion is 64% after 100 years in the stochastic simulation, although equilibrium are maintained in the static model. In spite of the increasing tendency of means, the probability of depletion, skewness and kurtosis increase, median decreases. The tendency is the same in the case of 2% target return as shown in table 3-3b. Kocken attribute these deficits to the constant pension payment regardless of the investment return. By solving equation (3.1), the necessary condition for $F_\tau = F_{\tau-1}$ is

$$F_{\tau-1} r_\tau = (B - P)(1 + r_\tau)^{\frac{1}{2}} \ , \qquad (3.6)$$

which tells investment returns from pension assets at the beginning of the year offset the sum of the difference in which benefits exceeds contribution and its investment return for half a year. This condition can be easily satisfied in the deterministic model where $\mathrm{r}_\tau = \mu$ and F_0 as (3.4). But in our stochastic model, r_τ may be smaller than μ, therefore (3.6) is not satisfied any more. Namely,

$$F_{\tau-1} r_\tau < (B - P)(1 + r_\tau)^{\frac{1}{2}} \ . \qquad (3.7).$$

Thus F_τ is smaller than $F_{\tau-1}$,therefore the probability for pension assets to turn back to F_0 is lower than 50%, because starting amount of assets is smaller than F_0. This causes the wide range of distribution of pension assets. Table 3-3b provides the average of statistics of ten separate simulation runs, each consisting of 1,000,000 replicates with portfolios targeting 2% return.

We can conclude that mean values are not sufficient to evaluate the results of the simulation, illustrating the words in Waring(2012) “Long term investors can’t expect to “get” the expected return; they receive a highly random and uncertain draw from an increasingly wide distribution of possible realized returns.”

Table 3-3b Basic case (Target return 2%)

Statistics	Beginning of 1st year	End of 50th year	End of 100th year
Mean	252	253	253
Percentage of depletion	0.0%	0.0%	18.7%
Standard deviation	0	101	297
Standard error	0.000	0.101	0.297
Skewness	-	1	1
Kurtosis	-	1	2
Minimum amount of asset	252	-39	-647
Maximum amount of asset	252	1,155	3,814
Median	252	241	206

The negative value of pension asset means borrowing from the sponsoring company, and that indicates the reduction of pension benefits if the sponsoring company is not willing to pay additional contributions. The reduction of benefits means the future pension amount for young workers is smaller than that of retired pensioners, this is the risk transfer from young employee to old pensioners, which supports Kocken's assertion.

3.3 Nonnegative constraint for pension assets

The above-mentioned basic case permitted negative pension assets, which is usually unrealistic, because if the fund depletes, plan sponsor usually adds necessary contribution or abolish pension plan, instead of lending money to pension fund. Therefore we provide a simulation run consisting of 1,000,000 replicates in which the amount of assets is equal to 0 after the amount of assets reaches to zero or negative. Table 3-4a provides the result of the simulation, where the portfolio aims to attain 5% return. Table 3-4b provides average of statistics for portfolio with target rate 2%.

Table 3-4a Case with nonnegative constraint (Target rate 5%)

Statistics	Beginning of 1st year	End of 50th year	End of 100th year
Mean	102	187	1,680
Percentage of depletion	0.0%	49.9%	64.0%
Standard deviation	0	371	5,223
Standard error	0.000	0.371	5.223
Skewness	-	4	10
Kurtosis	-	35	281

Minimum amount of asset	102	-6	-5
Maximum amount of asset	102	12,356	455,991
Median	102	1	0

Table 3-4b Case with nonnegative constraint (Target rate 2%)

Statistics	Beginning of 1st year	End of 50th year	End of 100th year
Mean	252	253	271
Percentage of depletion	0.0%	0.0%	18.7%
Standard deviation	0	101	274
Standard error	0.000	0.101	0.274
Skewness	-	1	1
Kurtosis	-	1	3
Minimum amount of asset	252	-5	-5
Maximum amount of asset	252	1,105	3,848
Median	252	241	206

3.4 Cases with amortization of deficit

In the above-mentioned case 3.2 and 3.3, the depletion of assets occurred because of no additional contribution in spite of deficits, the difference between planned assets and actual assets. However in the practice, additional contribution to amortize deficit is usually paid. To investigate the effect of additional contribution, we provide 10 simulation runs, each consisting 1,000,000 replicates with additional contribution the amount of which is 10% of deficits. Table 3-5a provides the statistics for target rate 5%. Owing to the additional contribution, the depletion disappeared. The average of additional contribution for 100 years is 72; 7.2 times annual contribution 10. The standard deviation of additional contribution is 0.04.

Table 3-5a Case allowing the amortization of deficits (target rate 5%)

Statistics	Beginning of 1st year	End of 50th year	End of 100th year
Mean	102	275	2,220
Percentage of depletion	0.0%	0.0%	0.0%
Standard deviation	0	356	5,388
Standard error	0.000	0.356	5.388
Skewness	-	5	10

Kurtosis	-	40	254
Minimum amount of asset	102	28	29
Maximum amount of asset	102	13,542	452,465
Median	102	141	455

As for the case with 2% target return presented in Table 3-5b, owing to the additional contribution, the depletion disappeared. The average of additional contribution for 100 years is 46; 4.6 times of annual contribution 10. The standard deviation of additional contribution is 0.03.

Table 3-5b Case allowing the amortization of deficits (target rate 2%)

Statistics	Beginning of 1st year	End of 50th year	End of 100th year
Mean	252	300	413
Percentage of depletion	0.0%	0.0%	0.0%
Standard deviation	0	72	211
Standard error	0.000	0.072	0.211
Skewness	-	2	2
Kurtosis	-	4	8
Minimum amount of asset	252	175	177
Maximum amount of asset	252	1,266	4,099
Median	252	279	340

3.5 10% amortization of deficit, with contributions suspended if the assets exceed a prescribed amount

We provide simulation with contribution suspended if the assets exceed 502.4937, equilibrium asset F_0 at the discount rate 1% presented in Table 3-1, because case3.4 above shows large amount of pension assets which might be unnecessary. Table 3-6a presents the average of 10 run of simulation with 1,000,000 replicates, having target rate 5%. The average additional contribution for100 years in 10 separate simulation, each consisting of 1,000,000 replicates is 72, 7.2 times of annual normal contribution 10. The standard deviation of additional contribution for 10 cases is 0.06. The average amount of suspended contribution for 100 years is 164 with standard deviation 0.2.

Table 3-6a Case allowing the amortization of deficits and contribution holiday (target rate 5%)

Statistics	Beginning of 1st year	End of 50th year	End of 100th year

Mean	102	253	1,457
Percentage of depletion	0.0%	0.0%	0.0%
Standard deviation	0	283	3,631
Standard error	0.000	0.283	3.631
Skewness	-	4	11
Kurtosis	-	35	344
Minimum amount of asset	102	29	30
Maximum amount of asset	102	10,643	334,590
Median	102	142	433

Table 3-6b presents the case with target rate 2%. The average additional contribution for100 years in 10 separate simulation, each consisting of 1,000,000 replicates is 46, 4.6 times of annual normal contribution 10. The standard deviation of additional contribution for 10 cases is 0.03. The average amount of suspended contribution for 100 years is 33 with standard deviation 0.07.

Table 3-6b Case allowing the amortization of deficits and contribution holiday (target rate 2%)

Statistics	Beginning of 1st year	End of 50th year	End of 100th year
Mean	252	299	363
Percentage of depletion	0.0%	0.0%	0.0%
Standard deviation	0	68	105
Standard error	0.000	0.068	0.105
Skewness	-	1	0
Kurtosis	-	1	-1
Minimum amount of asset	252	175	177
Maximum amount of asset	252	779	1,414
Median	252	279	340

3.6 Conclusion for Kocken's Proposition 1

From the above simulations, we can conclude that high discount rates may cause depletion of pension assets especially when it is difficult for the plan sponsors to raise the premiums, even if the initial liability is fully funded. To avoid depletion, additional contributions, benefit reductions are necessary, which means the risk transfer from old pensioner to young workers.

4. Probability of depletion in CDC

4.1 Assumptions

There are a number of academic research with respect to CDC (Gollier (2008), Jiajia et al. (2011), de Jong et al. (2011), Bams et al. (2013), Sender, S (2012)) ,but they are difficult to understand because it uses utility functions. There is no guarantee for members in pension funds adopt the utility function. For example, Gollier(2008) adopts a utility function of which variable is each person's wealth only, disregarding the possibility of the member's tendency for the comparison with the amount of pensions of other generations. Kocken (2012)'s paper is easy to understand because he does not use the utility function than that. We also do not use utility functions.

As for FTK2 there are simulations of the Dutch Central Planning Agency (CPB (2012)). These simulations adopts APG, Ortec and KNW scenario sets. They implemented the stochastic simulation for 80 years, and they compared the profit among generations without using utility function. The report is affirmative for 10-year smoothing because of the decreased the fluctuation of the benefits, but it does not focus on the probability of financial difficulties due to smoothing which our simple model demonstrates.

In this section, we simulate to confirm Kocken's Proposition 2 - second assertion about CDCs. For simplicity, one participant is supposed to enter the pension plan at age 20 working until just before age 60, and they do not die or withdraw. Pensions are supposed to paid from age 60 to age 79, namely they are annuity 20 years certain. In short, money are accumulated for 40 years with interest, and they are after 20 years from age 60. Pensioners are supposed not to die during those 20 years. The amount of pension for each year varies according to the return of the pension fund for previous years. Contribution for each active member is 1 every year, thus total amount of all contributions are 40. Contributions and Payments are supposed to perform at the middle of each year.

We simulate three cases, the first case reflecting investment rate directly, the second case reflecting smoothing for 10 years according to Dutch pension accord, the third case being Kocken's market consistent consideration which account for 1/10 of returns according to risk. We will call the first case "no smoothing", second case" smoothed" ,and third case "market consistent". The rate of investment return for year τ is denoted by r_τ.

As with the previous section, we simulate on portfolios with target return 2% and 5%, corresponding to risk 3.2% and 10% respectively. Examples of returns and risks for asset classes for Netherlands can be found Alphen et al. (1997). For example, the returns and risks from Frank Russell presentation were shown in table 4-1a.

Table 4-1a. Returns and Risk of Frank Russell in Alphen et al. (1997)

Asset Class	Expected return (%)	Expected Standard Deviation (%)
Inflation(Wages)	4.5	5.0

Inflation(Prices)	2.4	4.0
Dutch Bonds	6.5	7.0
Dutch Stocks	9.5	21.0
International Bonds	6.5	10.0
International Stocks	9.0	7.0

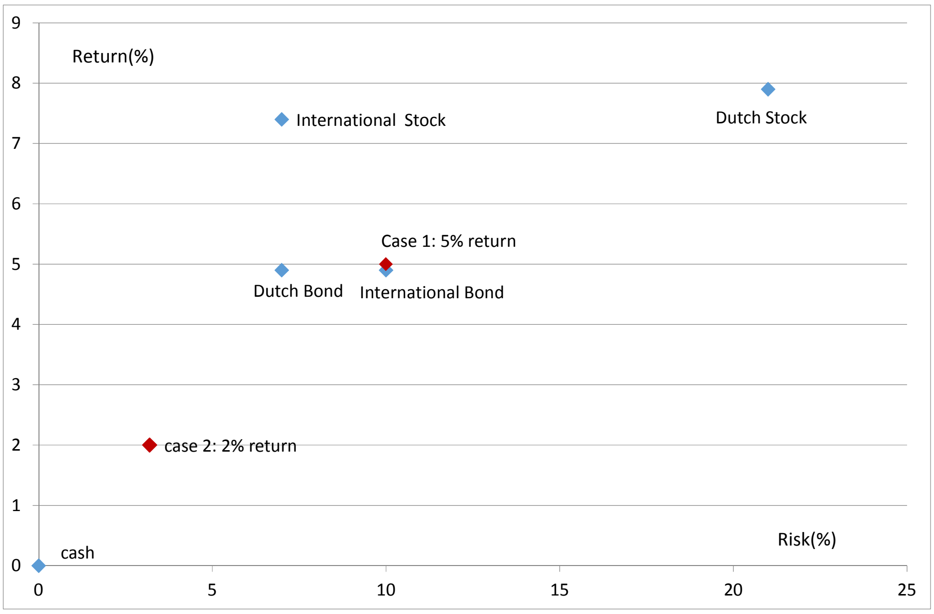
The inflation rate in 1996, when the presentation of Frank Russell reported in Alphen et al. was performed in 1996, was 1.96%. We suppose current expected returns by subtracting 1.64 %(1.96%-0.32%) from the above return vector, as shown in Table 4-1b.

Table 4-1b. Expected Returns and Risk of Dutch market as of April 4, 2016

Asset Class	Expected return (%)	Expected Standard Deviation (%)
Dutch Bonds	4.9	7.0
Dutch Stocks	7.9	21.0
International Bonds	4.9	10.0
International Stocks	7.4	7.0

Though we do not acquire Dutch correlation matrix, we can conclude return risk combination, (2%, 3.2%) and (5%, 10%) are possible ones from the above table. For reference Figure 2 shows the returns and risks of Dutch asset classes. We can estimate that our return-risk combination (5%,10%) and (2%,3.2%) are both within the efficient frontier from the Figure.

Figure 2 Returns and Risks of Dutch asset classes



4.2 Formula of benefits

4.2.1 Case of no smoothing

4.2.1.1 Hypothetical Account of active members

Although the pension fund is invested jointly, we can define hypothetical account as the sum of contributions and their investment return for each member. The amount of hypothetical account for a participant age x at the beginning of year τ is denoted by $A(x+1)_\tau$ considering his or her age being $x+1$ at the end of the year. The hypothetical amount for a member age 20 at the beginning of year τ is the sum of contribution 1 and the investment return for half a year τ, namely, for $x >= 21$,

$$A(21)_\tau = (1+r_\tau)^{\frac{1}{2}} \quad . \tag{4.1}$$

$A(x+1)_\tau$is the sum of $\mathrm{A(x)}_{\tau-1}with$ return for full year, and contribution 1 with return for half a year. Namely,

$$A(x+1)_\tau = A(x)_{\tau-1}(1+r_\tau) + (1+r_\tau)^{\frac{1}{2}} \quad . \tag{4.2}$$

4.2.1.2 Pension

As is well known, the annuity value for annuity certain for 20 years paying 1 at the middle of each year with interest rate $r_{\tau-1}$ is given by

$${}_{\tau-1}\bar{a}_{20]} = \frac{1}{(1+r_{\tau-1})^{\frac{1}{2}}} + \frac{1}{(1+r_{\tau-1})^{1+\frac{1}{2}}} + \cdots + \frac{1}{(1+r_{\tau-1})^{19+\frac{1}{2}}} \quad . \tag{4.3}$$

Using this value, we can calculate the amount of annuity for year τ denoted by ${}_\tau\alpha\left(60+\frac{1}{2}\right)$ for a pensioner age 60 at the beginning of the year as the amount of hypothetical account $A(60)_{\tau-1}$ devided by ${}_{\tau-1}\bar{a}_{20]}$. Namely,

$${}_\tau\alpha\left(60+\frac{1}{2}\right) = \frac{A(60)_{\tau-1}}{{}_{\tau-1}\bar{a}_{20]}} \quad . \tag{4.4}$$

For $x \geq 61$,

$${}_\tau\alpha\left(x+\frac{1}{2}\right) = \frac{A(x)_{\tau-1}}{{}_{\tau-1}\bar{a}_{20-(x-60)]}} \quad . \tag{4.5}$$

After the beginning of pension payment, namely at age older than or equal to 60, the transition formula of hypothetical account can be written as,

$$A(x+1)_{\tau-1} = A(x)_{\tau-1}(1+r_\tau) - {}_\tau\alpha\left(x+\frac{1}{2}\right)(1+\mathrm{r}_\tau)^{\frac{1}{2}} \quad . \tag{4.6}$$

As the amount of benefits for year τ is the sum of pension benefits from age 60 to 79, the total amount of benefits for year r can be given by,

$$\mathrm{B}_\tau = \sum_{x=60}^{79} {}_\tau\alpha\left(x+\frac{1}{2}\right) \quad . \tag{4.7}$$

4.2.2 Smoothing

The smoothed rate of return s_τ for Dutch pension accord described by Kocken can be given by

$$\mathrm{s}_\tau = \mu + \frac{1}{10}\sum_{i=0}^{9}(r_{\tau-i}-\mu) = \frac{1}{10}\sum_{i=0}^{9} r_{\tau-i} \quad . \tag{4.8}$$

We simulate transition of pension fund with smoothed return by replacing s_τ for r_τ in the above formula from (4.1) through (4.6).

4.2.3 Market consistent valuation

The market consistent valuation of benefits can be realized replacing r_τ in the above formula (4.1) through (4.6) to m_τ below which is the return after market consistent smoothing by Kocken:

$\mathrm{m}_\tau = r_\tau/10.$

4.3 Formula of Pension Assets

Considering the contribution 40 and benefit payments being occurred at the middle of each year, the recurrence formula of the amount of pension assets F_τ for 3 cases can be written as

$$F_\tau = F_{\tau-1}(1+r_\tau) + (40 - B_\tau)(1+r_\tau)^{\frac{1}{2}} \ . \tag{4.9}$$

The difference among the three cases lies in B_τ.The initial amount of pension assets F_0 is calculated by solving the following formula:

$$F_0 = F_0(1+\mu) + (40 - B_\tau)(1+\mu) \ , \tag{4.10}$$

which shows the stationary situation with expected rate or return μ .

4.4 Result of simulation

The result of simulation for target rate 2% is summarized in table 4-2. In the smoothed case, percentage of depletion is positive, but small.

Table 4-2 Simulation of CDC with target rate 2%

Policy	Statistics	Beginning of 1st year	End of 50th year	End of 100th year
No smoothing	Mean	1,711	1,677	1,552
	Percentage of depletion	0.00%	0.00%	0.90%
	Standard deviation	0	1,066	2,209
	Standard error	0.00	1.07	2.21

	Skewness	-	0.61	0.54
	Kurtosis	-	0.78	1.52
	Minimum amount of asset	1,711	478	-3,281
	Maximum amount of asset	1,711	4,536	8,411
	Median	1,711	1,645	1,499
Smoothed	Mean	1,711	1,677	1,552
	Percentage of depletion	0.00%	0.00%	0.90%
	Standard deviation	0	1,066	2,209
	Standard error	0.00	1.07	2.21
	Skewness	-	0.61	0.54
	Kurtosis	-	0.78	1.52
	Minimum amount of asset	1,711	478	-3,281
	Maximum amount of asset	1,711	4,536	8,411
	Median	1,711	1,645	1,499
Market consistent	Mean	1,241	3,107	8,113
	Percentage of depletion	0.00%	0.00%	0.00%
	Standard deviation	0	1,614	6,201
	Skewness	0.00	1.61	6.20
	Standard error	-	0.59	0.87
	Kurtosis	-	0.65	1.39
	Minimum amount of asset	1,241	1,521	2,782

	Maximum amount of asset	1,241	6,899	26,804
	Median	1,241	3,059	7,855

The result of simulation for target rate 5% is summarized in table 4-3

Table 4-3 Simulation of CDC with target rate 5%

Policy	Statistics	Beginning of 1st year	End of 50th year	End of 100th year
No smoothing	Mean	3,312	3,237	3,111
	Percentage of depletion	0.00%	0.00%	0.00%
	standard deviation	0	4,103	3,874
	Standard error	0.00	4.10	3.87
	Skewness	-	1.42	1.39
	Kurtosis	-	3.79	3.69
	Minimum amount of asset	3,312	607	-922
	Maximum amount of asset	3,312	18,581	21,097
	Median	3,312	2,990	2,879
Smoothed	Mean	3,153	1,640	-18,846
	Percentage of depletion	0.00%	39.78%	67.73%
	Standard deviation	0	21,597	303,228
	Standard error	0.00	21.60	303.23
	Skewness	-	1.80	0.94
	Kurtosis	-	16.89	72.65
	Minimum	3,153	-89,969	-3,446,024

	amount of asset			
	Maximum amount of asset	3,153	166,820	5,084,282
	Median	3,153	919	-12,922
Market consistent	Mean	1,306	13,130	148,118
	Percentage of depletion	0.00%	0.00%	0.00%
	Standard deviation	0	27,364	517,557
	Standard error	0.00	27.36	517.56
	Skewness	-	2.44	4.94
	Kurtosis	-	11.91	63.48
	Minimum amount of asset	1,306	1,041	2,382
	Maximum amount of asset	1,306	174,153	8,805,533
	Median	1,306	10,908	99,552

As presented above, smoothed cases for CDC show 0.91% probability of depletion for target rate 2%, and 68.03% for target rate 5%. Like the case of 3.2, negative value of pension assets means loans, additional contributions, the reduction of benefits, or winding up of the plan. If the benefits decrease, risk transfer from old pensioner to young workers could be present, which support Kocken's Proposition 2. However the probability of depletion for target rate 2% is less than 1%, and can be evaded by the raise of premiums according to financial standards (FTK for the Netherlands). Kocken's market consistent policy exclude the worry about asset depletion, but the amount of surplus should be distributed fairly, which is another problem to solve.

5. Conclusion

We confirmed Kocken's proposition by Monte Carlo simulation with additional findings. High target rate causes depletion of pension asset especially when it is difficult for the plan sponsor to raise the premiums. Market consistent policy for CDC proposed by Kocken prevent pension funds from depletion successfully with a huge amount of surplus, which should be fairly distributed to active and retired members of the pension fund. We demonstrated that Monte Carlo simulation is useful not only Asset Liability Management to determine strategic asset allocation, but also the risk management of pension fund as the bridge between financial economics and practical consultations because Monte Carlo simulations will present how a stochastic world is different from a deterministic world.

We also conclude that the traditional pension mathematics does not provide sufficient information to the plan sponsors or employers without enough money to raise premiums. Traditional pension mathematics states that low discount rate means high premium, low discount rate means high premium. I think it would be kind to advice additional future contribution calculated with Monte Carlo Simulation if the liability is measured with high discount rates.

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