Solvency Capital Estimation, Reserving Cycle and Ultimate Risk

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SCOR

Agenda

Objective and Motivation

- Our objective is to estimate the Solvency Capital Requirement, and the Risk Margin, as prescribed in the Solvency II regulation for a non-life (re)insurance portfolio.
  - Similar quantities as in the SST

- How are the SCR and the RM defined?

- Companies holds reserves to guaranty that the losses generated by the contracts they signed will be paid.
  - Technical provisions are made of a best estimate of the reserves plus the risk margin.
  - The risk margin is the difference between the market and the “face” value of the reserves.

- What is the SCR? The SCR is the capital required to cover the risk of a large increase of the technical provision (TP) from one year to the other.
  - $\mathrm{SCR}_n = \mathbf{VaR}_{99.5\%}(\mathrm{TP}_{n+1} - \mathrm{TP}_n | \mathrm{F}_n)$, where F_n is the available information at year n.
  - Note that at time $t = 0$ the SCR_0 is a number whereas SCR_n at a future year $n \geq 1$ is a random variable.

Objective and Motivation

- What is the RM? The RM is the difference between the market and the “face” value of the reserves. It is defined as the present value of the sum of all the future capitals costs which need to be hold in order to be solvent during the run-off of the insurance portfolio:

$$\mathrm{RM}_n = \mathrm{CoC} \sum_{k=n}^{m-1} \frac{\mathbf{E}(\mathrm{SCR}_k \mid \mathrm{F}_n)}{(1 + r_{n,k-n+1})^{k-n+1}}$$

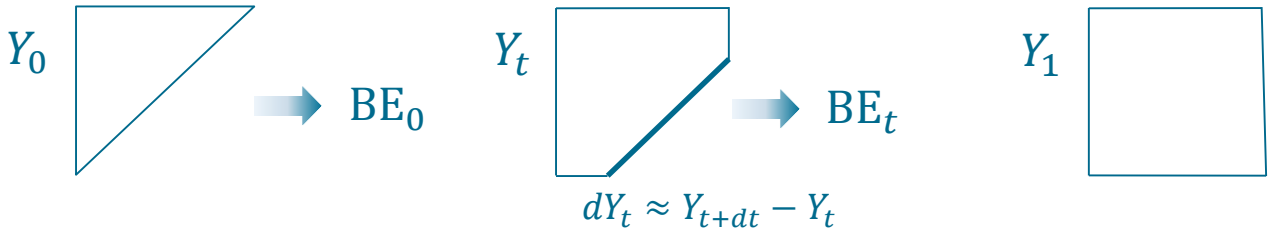
where the Cost of Capital is assumed to be constant $\mathrm{CoC} = 6\%$ and the run-off lasts m years.

- The most common method used in practice for the SCR estimation is the Merz-Wüthrich formula.
  - The hypothesis behind the MW formula are often violated.
  - MW formula does not actually provide estimations for the SCR and the Risk Margin because does not provide an estimation for the tail risk measures.
  - The MW formula is not reliable if used outside its applicability perimeter.

Agenda

The Ideas behind our Model

- Let Y_1 be the ultimate (attritional) loss of a run-off portfolio. We want to model the dynamics which brings the losses, and thus the corresponding best estimates of the ultimate loss, from $\mathrm{t} = 0$ to $\mathrm{t} = 1$.



- At time $t > 0$ the realized losses Y_s, $s \leq t$, determine the estimation of BE_t.
  - For example, we may project Y_t to BE_t with the chain-ladder method.

- However, in reality we trust our estimations when things behave normally but we know that exceptionally things may happen which make our estimations wrong.
  - If Y_s, $s \leq t$, oscillate up and down around what we expected, then we are confident with our estimations and may even make occasional prudent reserves release.
  - If Y_s, $s \leq t$, are systematically above expectation over a certain period of time $[T^s, T^e]$, then we may distrust our estimations and thus make a material correction.
  - This change of regime marks the reserving cycle.

Our Model – The Losses over Time

- We assume that Y_1 has Log-Normal distribution.
  - This is appropriate because Y_1 is the attritional losses component.

- In order to model the two regimes of the reserving cycle we assume that the relative loss developments dY_t/Y_t have uncertainties around what expected which are:
  - uncorrelated and have normal distribution on $[0,1] \setminus [T^s, T^e]$, like in a Brownian motion,
  - positively correlated and have normal distribution on $[T^s, T^e]$, like in a fractional Brownian motion with dependency exponent h between 0.5 and 1.

- In mathematical terms,

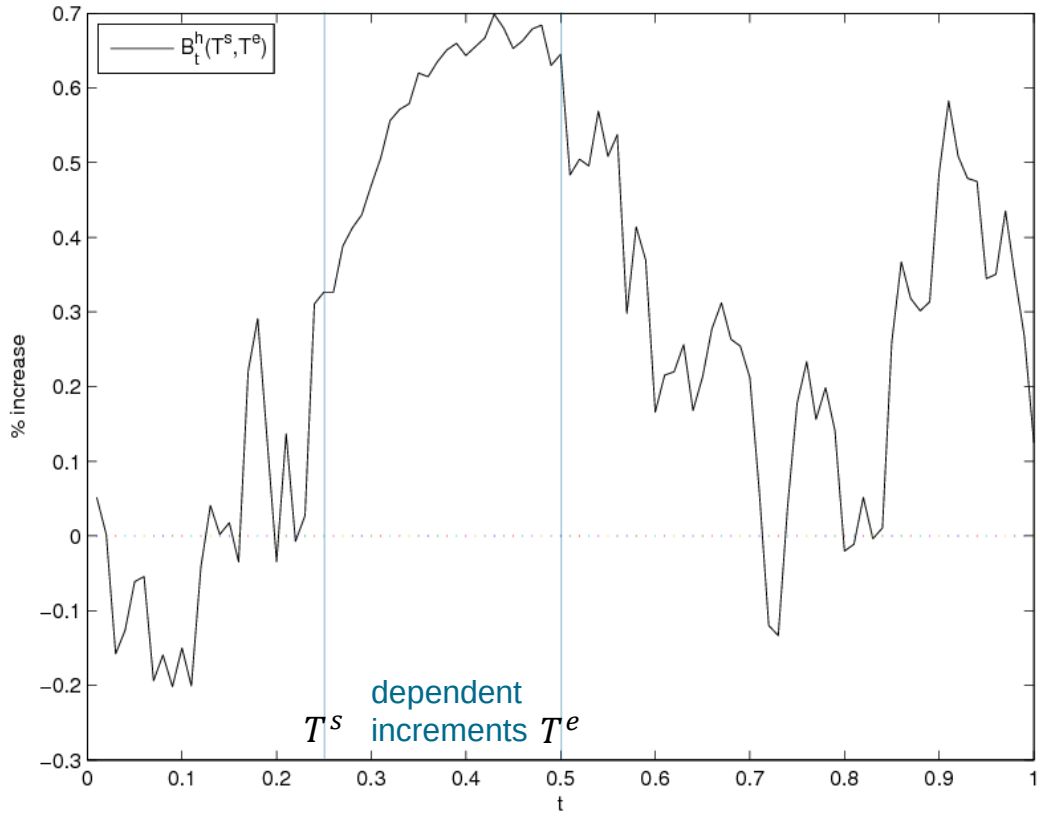
$$dY_t/Y_t = p_t dt + dB_t^h(T^s, T^e), \quad t \in [0,1],$$

  with initial loss $Y_0 > 0$, where $p_t dt$ is what expected and $dB_t\ (T^s, T^e)$ is the uncertainty.
  - The variance of the relative loss developments is assumed to be proportional to the expected incremental loss.

- The time T^e is when a sudden material reserves increase may occur as a result of a period $[T^s, T^e]$ of systematic under-estimation of the losses.

Our Model – The Losses over Time

$$dY_t/Y_t = p_t dt + dB_t^h(T^s, T^e)$$



Our Model – The Best Estimate of the Ultimate Loss over Time

- If γ is the relative size of a reserves jump, then we model the evolution of the best estimate of the ultimate loss over time by the stochastic differential equation

$$\boxed{d\mathrm{BE}_t = d\mathbb{E}(Y_1|\mathfrak{F}_t) + \gamma(\mathrm{BE}_t - Y_t)dJ_t(T^e), \qquad \text{for } t \in [0,1],}$$

  with initial value $\mathrm{BE}_0 = \mathbf{E}(Y_1)$, where:
  - F_t is the available information at time t, provided by Y_t and T^e,
  - $dJ_t(T^e)$ is approximately always null but in T^e where $dJ_{T^e}(T^e)$ may be 1, if a reserve strengthening occurs, otherwise is 0.

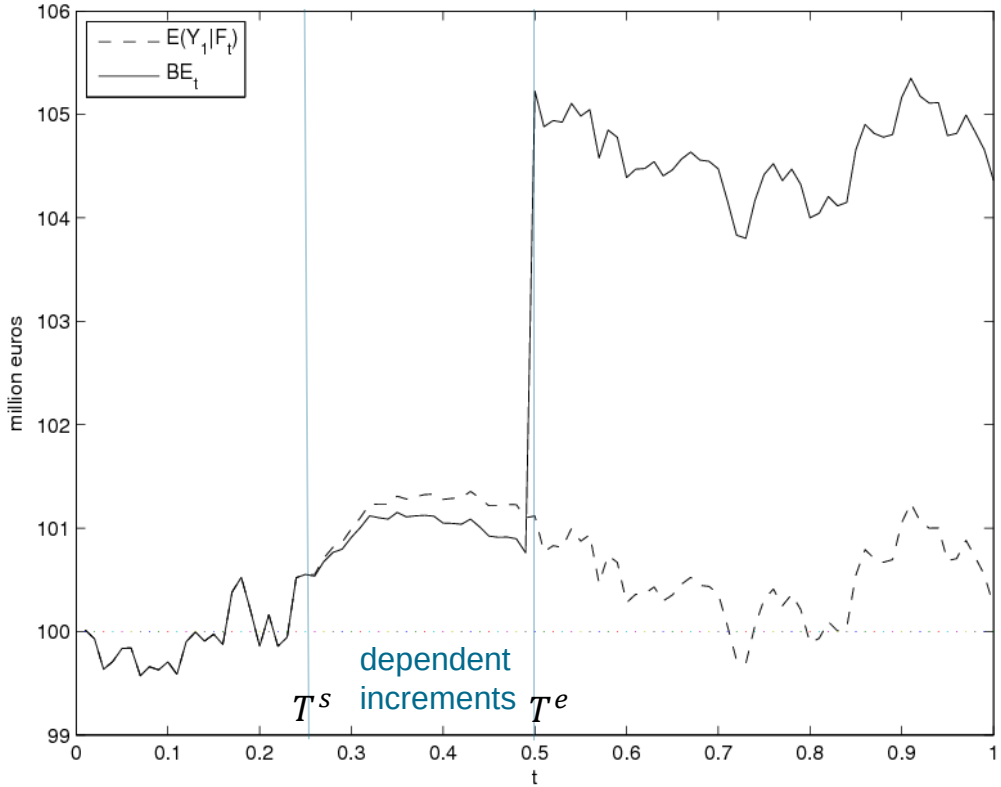
- A plausible reserving actuary criteria f_α triggering the reserve strengthening is: if the realized losses during $[T^s, T^e]$ exceed what expected by ξ_α-times the standard deviation, then the best estimate is increased by $\gamma(\mathrm{BE}_{T^e} - Y_{T^e})$.
  - Any such a criteria has an associated probability of occurrence α.

- $\{\mathrm{BE}_t\}$ is a martingale, as it should be, i.e. $\mathbf{E}(\mathrm{BE}_t|F_s) = \mathrm{BE}_s$, for $s \leq t$.

- The model is formulated in terms of stochastic differential equations. However it can be equivalently formulated in a simpler way which does not make use of stochastic differential equations.

Our Model – The Best Estimate of the Ultimate Loss over Time

$$d\mathrm{BE}_t = d\mathbb{E}(Y_1|\mathfrak{F}_t) + \gamma(\mathrm{BE}_t - Y_t)dJ_t(T^e), \qquad \text{for } t \in [0,1],$$



- In the figure, $Y_0 = 50,\ \mathrm{BE}_0 = 100,\ \sigma_0 := \mathrm{Std}(Y_1)/\mathrm{BE}_0 = 3\%$ and $\gamma = 18\%,\ \alpha = 0.05,\ m = 10.$

Our Model - Comments

- The quantity $\mathrm{BE}_t - Y_t$, which represents the reserves, tends to decrease over time, hence the reserves jump size $\gamma(\mathrm{BE}_t - Y_t)$ decreases too.

- As in reality we do not know a priori but only a posteriori if the loss developments have started to be dependent,T^s is not part of the available information F_t.

- The best estimate evolution is composed by two parts, a smooth part and a jump part.

$$d\mathrm{BE}_t = \underbrace{d\mathbb{E}(Y_1|\mathfrak{F}_t)}_{\text{smooth part}} + \underbrace{\gamma(\mathrm{BE}_t - Y_t)dJ_t(T^e)}_{\text{jump part}}, \qquad \text{for } t \in [0,1]$$

- Summarizing, our model describes a reserving cycle.
  - $B_t^h(T^s, T^e)$ captures the first phase of the cycle in which a systematic under-estimation of the losses may occur.
  - $J_t(T^e)$ captures the second phase of the cycle in which a sudden material deterioration of the reserves occurs as a result of the preceding systematic under-estimation.

Agenda

SCR and Risk Margin simplifications

- $\mathrm{TP}_{n+1} - \mathrm{TP}_n$ is replaced by $\mathrm{BE}_{n+1} - \mathrm{BE}_n$, which is justified by the fact that the Risk Margin is approximately constant from one year to the other.

- $\mathbf{VaR}_{99.5\%}$ is replaced by $\mathbf{tVaR}_{99\%}$ because $\mathbf{VaR}_{99.5\%}$ is not robust and not coherent.

- $\mathbf{E}[\mathbf{tVaR}_{99\%}(\mathrm{BE}_{n+1} - \mathrm{BE}_n|\mathrm{F}_n)]$ is replaced by $\mathbf{tVaR}_{99\%}(\mathrm{BE}_{n+1} - \mathrm{BE}_n|\mathrm{F}_0)$ in the Risk Margin at $t = 0$ because the first quantity is cumbersome.
  - The Risk Margin is a second order quantity with respect to the SCR, and however the proposed simplification is more prudent since $\mathbf{tVaR}_{99\%}(\mathrm{BE}_{n+1} - \mathrm{BE}_n|\mathrm{F}_0) \geq \mathbf{E}[\mathbf{tVaR}_{99\%}(\mathrm{BE}_{n+1} - \mathrm{BE}_n|\mathrm{F}_n)]$.

$$\mathrm{RM}_0 = \mathrm{CoC} \sum_{k=0}^{m-1} \frac{\mathbf{E}(\mathrm{SCR}_k \mid \mathrm{F}_0)}{(1 + r_{0,k+1})^{k+1}} \lesssim \mathrm{CoC} \sum_{k=0}^{m-1} \frac{\mathbf{tVaR}_{99\%}(\mathrm{BE}_{k+1} - \mathrm{BE}_k|\mathrm{F}_0)}{(1 + r_{0,k+1})^{k+1}}$$

SCR and Risk Margin approximation

- Our model has three parameters: the reserves jump size γ, the reserves jump probability α, the loss developments dependency exponent h.
  - The volatility parameter of the ultimate attritional loss is σ_0.

- If $\gamma\alpha, \sigma_0$ are small, then it can be shown that

$$\mathrm{BE}_t \simeq \begin{cases} \mathbb{E}(Y_1|\mathfrak{F}_{T^e}) + \gamma(\mathrm{BE}_{T^e} - Y_{T^e}) & \text{if } t \in (T^e, 1], f_\alpha(T^e) = 1, \\ \mathbb{E}(Y_1|\mathfrak{F}_t) & \text{otherwise.} \end{cases}$$

- Our model is formulated with continuous-time but it can be easily discretized.

- It can then be shown that, with $\lambda := \alpha/(m1\%)$, which represents the contribution of the "jump" part, and $c_n := (e^{p_n} - 1)/(e^{p_m} - 1)$, which represents the cumulative expected calendar loss until year n relative to the total,

$$\boxed{\underbrace{\mathrm{tVaR}_{99\%}(\mathrm{BE}_{n+1} - \mathrm{BE}_n)}_{\text{one-year risk}} \simeq \underbrace{[(c_{n+1} - c_n)^h(1-\lambda)}_{\text{smooth part}} + \underbrace{(1-c_n)\lambda]}_{\text{jump part}} \underbrace{\mathrm{tVaR}_{99\%}(\mathrm{BE}_m - \mathrm{BE}_0)}_{\text{ultimate risk}}.}$$

Our Model in practice

- To use our in model in practice we need the following inputs:
  - the ultimate loss model $\mathrm{BE_m}$, which could either include or not the large loss component,
  - the cumulative calendar year expected loss pattern $(c_1, \dots, c_m)$.

- The parameters of the model:
  - the reserves jump size γ; if the ultimate model includes the large losses component, then we can approximate γ by $\mathbf{tVaR}_{99\%}(\mathrm{BE}_m - \mathrm{BE}_0)/(\mathrm{BE}_0 - Y_0)$, otherwise γ can be quantified by experts,
  - the reserves jump probability α,
  - the loss developments dependency exponent h.

$$\mathrm{SCR}_0 = [(c_1)^h(1-\lambda)+\lambda]\gamma(\mathrm{BE}_0 - Y_0)$$

$$\mathrm{RM}_0 = 6\%\left[\frac{(c_1)^h(1-\lambda)+\lambda}{1+r_{0,1}} + \sum_{n=1}^{m-1}\frac{(c_{n+1}-c_n)^h(1-\lambda)+(1-c_n)\lambda}{(1+r_{0,n+1})^{n+1}}\right]\gamma(\mathrm{BE}_0 - Y_0)$$

Agenda

Dice Model

- We throw n dices in n steps in the following way
  - At each step, we choose randomly a number of dices according to a uniform distribution over all remaining possibilities:

$$\begin{aligned} N_1 &\sim U(\{0,\dots,n\}) \\ N_2 &\sim U(\{0,\dots,n-N_1\}) \\ &\vdots \\ N_{n-1} &\sim U(\{0,\dots,n-N_1-\dots-N_{n-2}\}) \\ N_n &= n-N_1-\dots-N_{n-1} \end{aligned}$$

  - Our loss is then 1 (hundred/thousand/million euros) for each “6” obtained
  - The loss at step i is therefore distributed as $B(N_i,p)$, with p=1/6
  - The ultimate distribution is $B(n,p)$

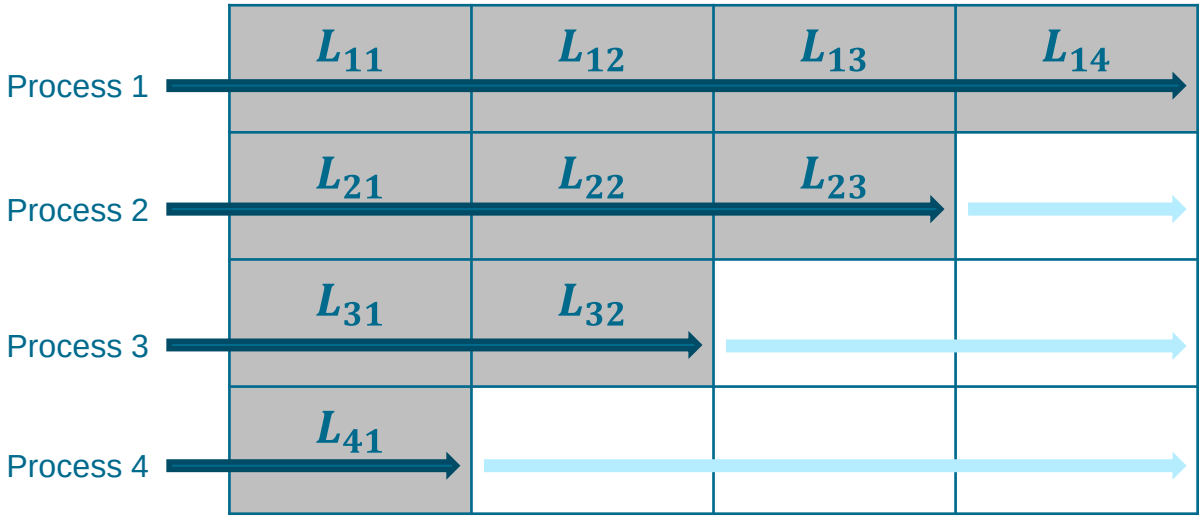
Fixed-sum insurance contracts with the Dice Model

- The so called fixed-sum insurance contracts, for which a fixed amounts is paid to the contract owner in case of accident, like the IDA Auto insurance in France, the MRH Multi-Risk Property insurance in France, the Personal Accident insurance in Japan, certain products linked to the Health insurance in Switzerland, etc..., can be modelled with the Dice Model

- The company is exposed to the risk n times. Different exposures come in different time steps, each exposure has a probability p to incur in a fixed loss, say 1 hundred/thousand/million euros, and at the end of the n steps the total number of exposures to the risk is equal to n, where n could be seen as the number of contract policies sold by the insurance company

Triangles of Losses

- We consider triangles for which the losses of each underwriting year are driven independently by the Dice Model loss process. Note that the different representations of the process are at different stages of development.



Comparison Statistics

- We simulate a “large” number of triangles, on which we calculate the capital requirement of the first year using the Merz-Wüthrich method, the SCOR method and a benchmark method that is an approximation of the theoretical value.

- We calculate each time the mean value of the capital requirement over all triangles and the standard deviation around that mean value.

- We also calculate the following measures of deviation from the benchmark value:
  - Mean absolute deviation: $E(|\text{"estimate"} - \text{"true"}|)$
  - Mean relative absolute deviation: $E(|\text{"estimate"} - \text{"true"}| / \text{"true"})$

- We simulate 500 triangles

Calculations

- The Merz-Wüthrich method provides a method to estimated the one-year standard deviation and not the TVaR. We then use the Normal approximation to convert standard deviation into TVaR. This choice is consequence of the fact that the ultimate distributions for the triangle of losses with the dice model could be reasonably approximated by normal distributions.

- The Merz-Wüthrich method, being multiplicative, it fails if there is a 0 in the triangle, which could happens with the dice model. We therefore use a trick that consists in taking a truncated version of a dice process generated triangle with n large.
  - The truncation consists in taking the top left triangle of size m = log(n) + 6.57, and n=100000
  - For the Dice Model, the expected termination time is approximately log(n) + 1.57
  - For the Dice model we use a more realistic p = 0.1%.

- For the SCOR methodology we compute two main different parameterizations, one with “jumps” and the other without.

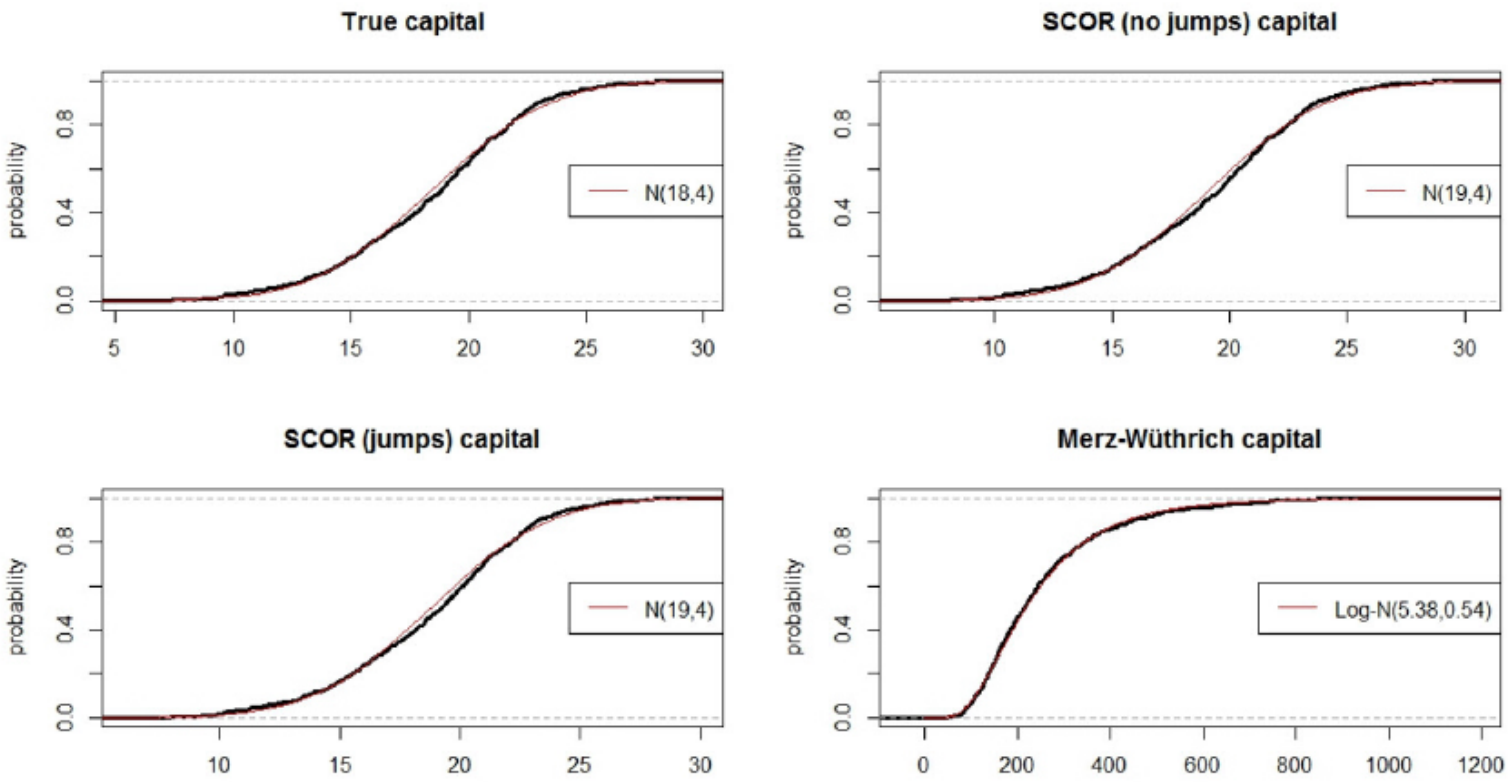
Dice Model Results

- The mean reserves are 100 and the capital intensity around 18%, which is a realistic value.
- The SCOR method get excellent results with both parameterizations. The Merz-Wüthrich however completely fails, even in estimating the order of magnitude of the capital.

Method	Mean	Std. dev.	MAD	MRAD
Theoretical value	18.37	3.92	0	0%
SCOR, without jumps	19.08	3.93	0.71	4.14%
SCOR, with jumps	18.81	3.86	0.43	2.47%
Merz-Wüthrich	252.89	149.6	234.5	1365.6%

Dice Model Results

- Empirical distributions of the one-year capital estimations for the 500 simulations
- SCOR method provides normally distributed estimates, as in the true case, whereas MW provides skewed estimates



Dice Model Results

- Correlations between the one-year capital estimation for the 500 simulations
- SCOR method's estimates move basically the same way as the true estimates, whereas MW estimates move even in the opposite directions.

Standard corr	True value	SCOR, no jumps	SCOR, jumps	Merz-Wüthrich
True value	100%	99.98%	99.97%	-37.64%
SCOR, no jumps		100%	99.99%	-37.65%
SCOR, jumps			100%	-37.64%
Merz-Wüthrich				100%

Dice Model Results

- Why MW is failing so badly?

- The Merz-Wüthrich method is multiplicative and assumes that any value, regardless his size, has the same probability to be multiplied by a large factor. The dice model however is an additive model, which makes small values more likely to be “multiplied” by a large factor.

- In addition, the MW hypotheses assume that $Var\left(L_{j,i+1} \middle| F_{j,i}\right) = \sigma_i^2 \, L_{j,i}$. Hence, the larger the previous loss, the more volatility there is in the remaining process. For the dice model however, due to the “fixed number of dices” property, it is the opposite. Indeed, a large previous loss means that a large number of dices has most likely already been thrown which implies less remaining risk. This explains the negative correlation.

Agenda

Conclusion

- Our model is more robust than MW, i.e. can be used in a reliable way in more cases then MW.

- Our method ensures consistency between ultimate and one-year risk.
  - The better understood ultimate risk can be maintained throughout the entire model.
  - The availability of consistent ultimate and one-year risk estimations enhances the potential use cases (solvency, pricing, capital allocation, planning and retro optimization, …).

- Our model addresses known limitations of the MW method:
  - captures the dependency between loss developments,
  - captures the reserving actuary behavior and the reserving cycle,
  - provides estimates for the SCR and the Risk Margin as opposed to the mean square error.

- Our model makes use of the knowledge of the ultimate risk.

Conclusion

- Our method has the practical advantage to be used for portfolios with limited credibility.
  - Given the ultimate attritional loss model and parameters γ, α and h, the SCR and Risk Margin can be estimated with our methodology.
  - However, while the calibration of the parameters γ, α can be elicited through expert judgment, the parameter h, i.e. the dependency between loss developments, would require credible data. It is noted though that such calibration can be benchmarked using data from similar portfolios.

- Behind our SCR and Risk Margin formulas we have the following assumptions:
  - $\gamma\alpha, \sigma_0$ small and γ around $6\sigma_0$,
  - losses and the best estimates behave as described in the model.

- The content discussed in this presentation are contained in:
  - Ferriero, “Solvency capital estimation, reserving cycle and ultimate risk”, IME 68 (2016), 162-168.
  - Dacorogna - Ferriero - Krief, “Taking the one-year change from another angle”, 2014, preprint

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Appendix

The Merz-Wüthrich method

- The Merz-Wüthrich method calculates the one-year risk in the form of the standard deviation of the first one year change

- It is based on the Mack method to calculate the ultimate uncertainty in the same framework.

- It makes the following hypotheses:
  - The rows of the triangle are independent
  - $E\left(L_{j,i+1} \middle| F_{j,i}\right) = f_i \, L_{j,i}$
  - $Var\left(L_{j,i+1} \middle| F_{j,i}\right) = \sigma_i^2 \, L_{j,i}$

Our Model – The Losses over Time

- We assume that Y_1 has Log-Normal distribution.
  - This is appropriate because Y_1 is the attritional losses component.

- We define the continuous-time dynamics which brings the losses from $\mathrm{t} = 0$ to $\mathrm{t} = 1$ with the stochastic process given by

$$Y_t = Y_0 e^{p_t + B_t^h(T^s, T^e)}, \quad \text{for } t \in [0, 1],$$

  with initial loss $Y_0 > 0$, where
  - p_t is the expected loss development, which is an increasing concave function with $p_0 = 0$,
  - $B_t^h(T^s, T^e)$ is the uncertainty around p_t, which is a Brownian motion on $[0,1] \setminus [T^s, T^e]$ and a fractional Brownian motion on $[T^s, T^e]$ with dependency exponent h between 0.5 and 1, with mean such that $\mathbf{E}(Y_t) = Y_0 e^{p_t}$ and variance proportional to the expected outstanding loss.

- The random time T^e is when a sudden material reserves increase may occur as a result of a period $[T^s, T^e]$ of systematic under-estimation of the losses.
  - T^e is uniformly distributed on $[0,1]$, and T^s is a r.v. on $[T^e - 1, T^e]$ with exponential distribution, i.e. $\mathbf{P}(T^s \leq t) = a^{t - T^e}, a > 1$, so that times close to T^e are more probable.

Our Model – The Best Estimate of the Ultimate Loss over Time

- If γ is the relative size of a reserves jump, then we model the evolution of the best estimate of the ultimate loss over time by the Itô stochastic differential equation

$$\boxed{d\mathrm{BE}_t = d\mathbb{E}(Y_1|\mathfrak{F}_t) + \gamma(\mathrm{BE}_t - Y_t)dJ_t(T^e), \qquad \text{for } t \in [0,1],}$$

with initial value $\mathrm{BE}_0 = \mathbf{E}(Y_1)$, where F_t is the σ-algebra generated by $\{Y_s|s \leq t\}$ and $\{T^e \leq s|s \leq t\}$,

$$J_t(T^e) := -k_t + \begin{cases} 1, & \text{if } t \in (T^e, 1], f_\alpha(T^e) = 1, \\ 0, & \text{otherwise,} \end{cases}$$

with reserving actuary criteria

$$\boxed{f_\alpha(T^e) := \begin{cases} 1, & \text{if } Y_{T^e}/(Y_{\bar{T}^s} e^{p_{T^e} - p_{\bar{T}^s}}) \geq 1 + \xi_\alpha \mathrm{Std}[Y_{T^e}/(Y_{\bar{T}^s} e^{p_{T^e} - p_{\bar{T}^s}})], \\ 0, & \text{otherwise,} \end{cases}}$$

$\bar{T}^s = [T^s]^+$, $\xi_\alpha \geq 0$ is such that $\mathbf{P}(f_\alpha(T^e) = 1|T^e = 1) = \alpha$, and $k_t := \int_0^{t \wedge T^e} \frac{\mathbb{P}(f_\alpha(T^e) = 1|T^e = s)}{1-s} ds.$

- The reserving actuary criteria means that, if the realized losses during $[T^s, T^e]$ exceed what expected by ξ_α-times the standard deviation, then the best estimate is increased by $\gamma(\mathrm{BE}_{T^e} - Y_{T^e})$.

Our Model - Comments

- The model is formulated in terms of Itô's stochastic differential equation. However it can be equivalently formulated by the following simpler relation

$$\mathrm{BE}_t = \mathbb{E}(Y_1|\mathfrak{F}_t) - A_t + \begin{cases} \gamma[\mathbb{E}(Y_1 - Y_{T^e}|\mathfrak{F}_{T^e}) - A_{T^e}], & \text{if } t \in (T^e, 1], f_\alpha(T^e) = 1, \\ 0 & \text{otherwise,} \end{cases}$$

where $A_t := e^{-\gamma k_{t\wedge T^e}} \int_0^{t\wedge T^e} [\mathbb{E}(Y_1|\mathfrak{F}_s) - Y_s] de^{\gamma k_s}$.

- Note that BE_1 differs from Y_1. The reason being that Y_t represents the attritional losses only, whereas BE_t contains also the large losses behind the reserves jump.

- Summarizing, our model describes a reserving cycle.
  - $B_t^h(T^s, T^e)$ captures the first phase of the cycle in which a systematic under-estimation of the losses may occur.
  - $J_t(T^e)$ captures the second phase of the cycle in which a sudden material deterioration of the reserves occurs as a result of the preceding systematic under-estimation.

Model Parameters

- Our model has three parameters for the ultimate-to-one-year relation:
  - the reserves jump size γ,
  - the reserves jump probability α,
  - the loss developments dependency exponent h.

- The volatility parameter of the ultimate attritional loss is σ_0.

- If $\gamma\alpha, \sigma_0$ are small, then the model is approximately equal to

$$\mathrm{BE}_t \simeq \begin{cases} \mathbb{E}(Y_1|\mathfrak{F}_{T^e}) + \gamma\mathbb{E}(Y_1 - Y_{T^e}|\mathfrak{F}_{T^e}), & \text{if } t \in (T^e, 1], f_\alpha(T^e) = 1, \\ \mathbb{E}(Y_1|\mathfrak{F}_t) & \text{otherwise.} \end{cases}$$

Indeed, A_t is small if $\gamma\alpha$ is small, and $\mathbf{E}(Y_1|\mathrm{F}_t) - \mathbf{E}(Y_1|\mathrm{F}_{T^e})$ is small if σ_0 is small.

SCR and Risk Margin approximation

- Our model is formulated with continuous-time but it can be easily discretized. We only need to restrict T^s, T^e to assume values on a discrete subset of equidistant points in $[0,1]$.

- If $\gamma\alpha, \sigma_0$ are small, then

$$\boxed{\mathrm{BE}_{n+1} - \mathrm{BE}_n \simeq \begin{cases} \gamma\mathbb{E}(Y_m - Y_n|\mathfrak{F}_n), & \text{if } t_{n+1} = T_*^e, f_\alpha(T_*^e) = 1, \\ \mathbb{E}(Y_m|\mathfrak{F}_{n+1}) - \mathbb{E}(Y_m|\mathfrak{F}_n), & \text{otherwise,} \end{cases}}$$

- Suppose that α and γ are such that $\alpha/m < 1\%$ and $\gamma(\mathrm{BE}_0 - Y_0) \geq \mathbf{tVaR}_{99.5\%-|\alpha/m-0.5\%|}[Y_m - \mathbf{E}(Y_m)]$ and $\gamma(\mathrm{BE}_0 - Y_0) \leq \mathbf{tVaR}_{99.5\%+|\alpha/m-0.5\%|}[Y_m - \mathbf{E}(Y_m)]$. Then, with $\lambda := \alpha/(m1\%)$,

$$\begin{aligned} \mathrm{tVaR}_{99\%}(X) &\simeq \mathrm{tVaR}_{99\%+\alpha/m}[Y_m - \mathbb{E}(Y_m)](1-\lambda) + \gamma Y_0(e^{p_m} - 1)\lambda, \\ &= \mathrm{tVaR}_{99\%+\alpha/m}[Y_m - \mathbb{E}(Y_m)](1-\lambda) + \gamma(\mathrm{BE}_0 - Y_0)\lambda \\ &\simeq \gamma(\mathrm{BE}_0 - Y_0), \end{aligned}$$

and

$$\mathrm{tVaR}_{99\%}(\mathrm{BE}_{n+1} - \mathrm{BE}_n) \simeq \mathrm{tVaR}_{99\%+\alpha/m}[\mathbb{E}(Y_m|\mathfrak{F}_{n+1}) - \mathbb{E}(Y_m|\mathfrak{F}_n)](1-\lambda) + \gamma Y_0(e^{p_m} - e^{p_n})\lambda.$$

SCR and Risk Margin approximation

- In addition, with $c_n := (e^{p_n} - 1)/(e^{p_m} - 1)$,

$$\mathrm{tVaR}_{99\%+\alpha/m}[\mathbb{E}(Y_m|\mathfrak{F}_{n+1}) - \mathbb{E}(Y_m|\mathfrak{F}_n)] \simeq (c_{n+1} - c_n)^h \, \mathrm{tVaR}_{99\%+\alpha/m}[Y_m - \mathbb{E}(Y_m)].$$

Indeed a fBM B_t^h with dependency exponent h is such that $B_{ct}^h \sim c^h B_t^h$, for any $c > 0$.

- We therefore obtain

$$\boxed{\underbrace{\mathrm{tVaR}_{99\%}(\mathrm{BE}_{n+1} - \mathrm{BE}_n)}_{\text{one-year risk}} \simeq \underbrace{[(c_{n+1} - c_n)^h(1-\lambda)}_{\text{smooth part contribution}} + \underbrace{(1-c_n)\lambda]}_{\text{jump part contribution}} \underbrace{\mathrm{tVaR}_{99\%}(\mathrm{BE}_m - \mathrm{BE}_0)}_{\text{ultimate risk}}}$$