

Valuation of Large Variable Annuity Portfolios: Monte Carlo Simulation and Benchmark Datasets

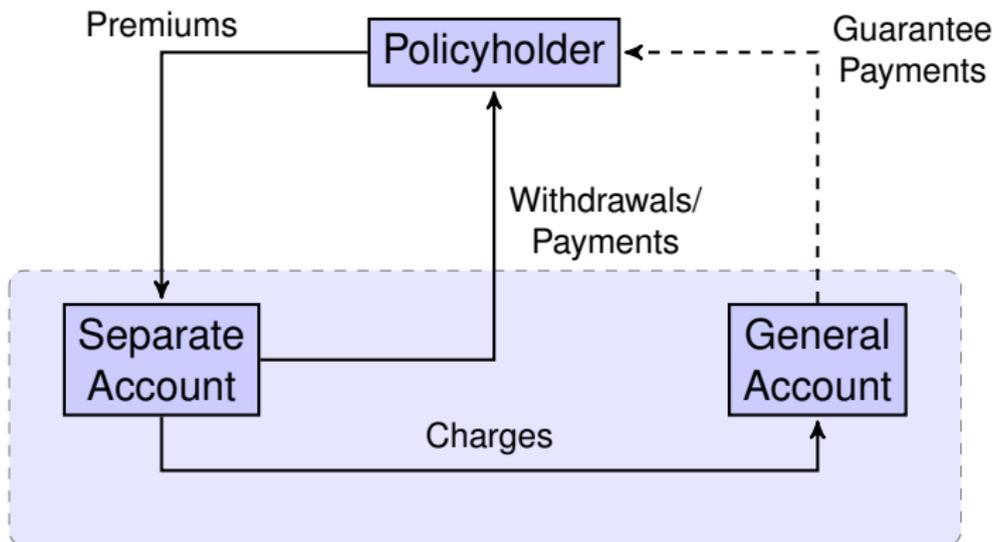
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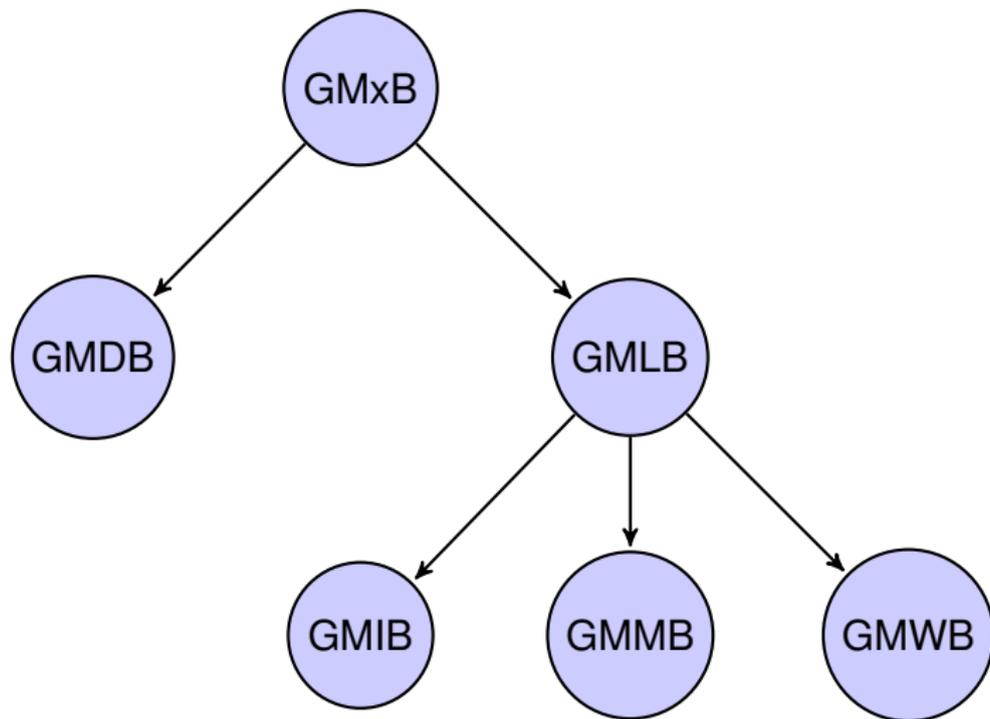
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- ▶ Computational problems from variable annuities (VA)
- ▶ A simulation engine
- ▶ Some datasets

A variable annuity is a contract between you and an insurance company



Variable annuities come with guarantees



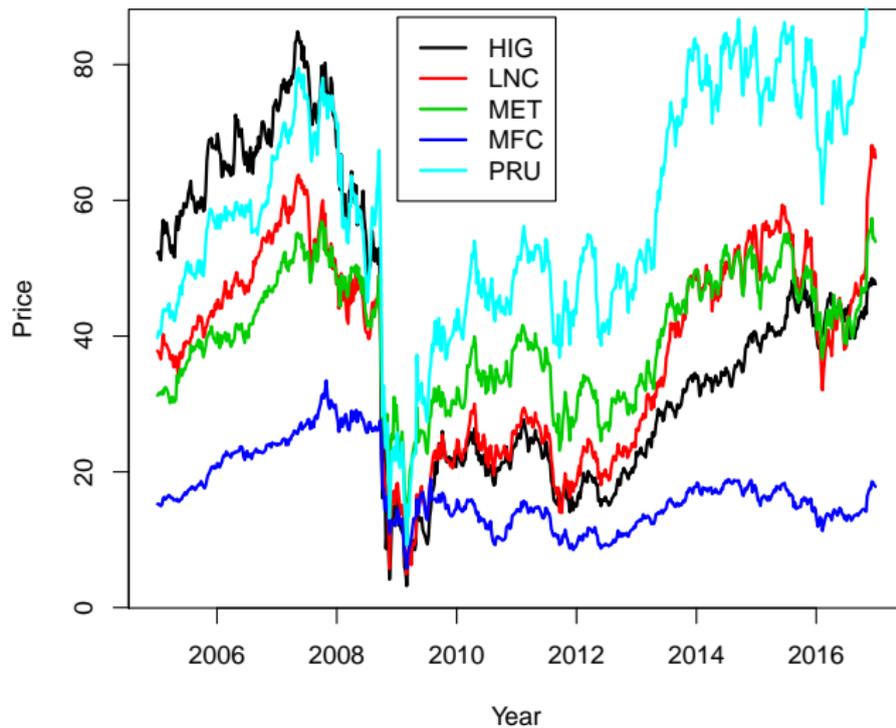
Insurance companies have to make guarantee payments under bad market conditions

Example (An immediate variable annuity with GMWB)

- ▶ Total investment and initial benefits base: \$100,000
- ▶ Maximum annual withdrawal: \$8,000

Policy Year	INV Return	Fund Before WD	Annual WD	Fund After WD	Remaining Benefit	Guarantee CF
1	-10%	90,000	8,000	82,000	92,000	0
2	10%	90,200	8,000	82,200	84,000	0
3	-30%	57,540	8,000	49,540	76,000	0
4	-30%	34,678	8,000	26,678	68,000	0
5	-10%	24,010	8,000	16,010	60,000	0
6	-10%	14,409	8,000	6,409	52,000	0
7	10%	7,050	8,000	0	44,000	950
8	r	0	8,000	0	36,000	8,000
⋮	⋮	⋮	⋮	⋮	⋮	⋮
12	r	0	8,000	0	4,000	8,000
13	r	0	4,000	0	0	4,000

The 2008 financial crisis



Dynamic hedging is a popular approach to mitigate the financial risk, but

- ▶ Dynamic hedging requires calculating the dollar Deltas of a portfolio of variable annuity policies within a short time interval
- ▶ The value of the guarantees cannot be determined by closed-form formula
- ▶ The Monte Carlo simulation model is time-consuming

Using the Monte Carlo method to value large variable annuity portfolios is time-consuming

Example (Valuing a portfolio of 100,000 policies)

- ▶ 1,000 risk neutral scenarios
- ▶ 360 monthly time steps

$$100,000 \times 1,000 \times 360 = 3.6 \times 10^{10}!$$

$$\frac{3.6 \times 10^{10} \text{ projections}}{200,000 \text{ projections/second}} = 50 \text{ hours!}$$

Potential solutions

- ▶ Hardware-based approaches (HPC, GPU)
- ▶ Software-based approaches (scenario reduction, replicating portfolio, metamodeling)

Metamodeling is promising

- ▶ select a small number of representative contracts
- ▶ use Monte Carlo simulation to calculate the fair market values (or other quantities of interest) of the representative contracts
- ▶ build a regression model (i.e., the metamodel) based on the representative contracts and their fair market values
- ▶ use the regression model to value the whole portfolio of variable annuity contracts

A problem

It is difficult for researchers to obtain real datasets from insurance companies to assess the performance of those metamodeling techniques.

Simulated datasets

- ▶ A synthetic portfolio of variable annuity contracts
- ▶ A Monte Carlo valuation engine used to produce FMV and Greeks

Synthetic portfolios

In particular, we create a synthetic portfolio of variable annuity contracts based on the following major properties typically observed on real portfolios of variable annuity contracts:

- ▶ Different contracts may contain different types of guarantees.
- ▶ The contract holder has the option to allocate the money among multiple investment funds.
- ▶ Real variable annuity contracts are issued at different dates and have different times to maturity.

Riders

Product	Description	Rider Fee
DBRP	GMDB with return of premium	0.25%
DBRU	GMDB with annual roll-up	0.35%
DBSU	GMDB with annual ratchet	0.35%
ABRP	GMAB with return of premium	0.50%
ABRU	GMAB with annual roll-up	0.60%
ABSU	GMAB with annual ratchet	0.60%
IBRP	GMIB with return of premium	0.60%
IBRU	GMIB with annual roll-up	0.70%
IBSU	GMIB with annual ratchet	0.70%
MBRP	GMMB with return of premium	0.50%
MBRU	GMMB with annual roll-up	0.60%
MBSU	GMMB with annual ratchet	0.60%
WBRP	GMWB with return of premium	0.65%
WBRU	GMWB with annual roll-up	0.75%
WBSU	GMWB with annual ratchet	0.75%
DBAB	GMDB + GMAB with annual ratchet	0.75%
DBIB	GMDB + GMIB with annual ratchet	0.85%
DBMB	GMDB + GMMB with annual ratchet	0.75%
DBWB	GMDB + GMWB with annual ratchet	0.90%

Investment funds

Fund	US Large	US Small	Intl Equity	Fixed Income	Money Market
1	1	0	0	0	0
2	0	1	0	0	0
3	0	0	1	0	0
4	0	0	0	1	0
5	0	0	0	0	1
6	0.6	0.4	0	0	0
7	0.5	0	0.5	0	0
8	0.5	0	0	0.5	0
9	0	0.3	0.7	0	0
10	0.2	0.2	0.2	0.2	0.2

Other settings

Feature	Value
Policyholder birth date	[1/1/1950, 1/1/1980]
Issue date	[1/1/2000, 1/1/2014]
Valuation date	1/6/2014
Maturity	[15, 30] years
Account value	[50000, 500000]
Female percent	40% (20% of each type)
Fund fee	30, 50, 60, 80, 10, 38, 45, 55, 57, 46bps for Funds 1 to 10, respectively
M&E fee	200 bps

Monte Carlo valuation engine

- ▶ Risk-neutral scenarios
- ▶ Cash flow projection

Risk-neutral scenario generator I

Suppose that there are k indices $S^{(1)}, S^{(2)}, \dots, S^{(k)}$ in the financial market and their risk-neutral dynamics are given by (Carmona and Durrelman, 2006):

$$\frac{dS_t^{(h)}}{S_t^{(h)}} = r_t dt + \sum_{l=1}^k \sigma_{hl} dB_t^{(l)}, \quad S_0^{(h)} = 1, \quad h = 1, 2, \dots, k \quad (1)$$

where $B_t^{(1)}, B_t^{(2)}, \dots, B_t^{(k)}$ are independent standard Brownian motions, r_t is the short rate of interest, and the matrix (σ_{hl}) is used to capture the correlation among the indices. The stochastic differential equations given in Equation (1) have the following solutions (Carmona and Durrelman, 2006):

$$S_t^{(h)} = \exp \left[\left(\int_0^t r_s ds - \frac{t}{2} \sum_{l=1}^k \sigma_{hl}^2 \right) + \sum_{l=1}^k \sigma_{hl} B_t^{(l)} \right], \quad h = 1, 2, \dots, k. \quad (2)$$

Risk-neutral scenario generator II

Let $t_0 = 0, t_1 = \Delta, \dots, t_m = m\Delta$ be time steps with equal space Δ . For $j = 1, 2, \dots, m$, let $A_j^{(h)}$ be the accumulation factor of the h th index for the period (t_{j-1}, t_j) , that is,

$$A_j^{(h)} = \frac{S_{j\Delta}^{(h)}}{S_{(j-1)\Delta}^{(h)}}. \quad (3)$$

Suppose that the continuous forward rate is constant within each period. Then we have

$$\exp(\Delta(f_1 + f_2 + \dots + f_j)) = \exp\left(\int_0^{t_j} r_s ds\right), \quad j = 1, 2, \dots, m,$$

where f_j is the annualized continuous forward rate for period (t_{j-1}, t_j) . The above equation leads to

$$f_j = \frac{1}{\Delta} \int_{t_{j-1}}^{t_j} r_s ds, \quad j = 1, 2, \dots, m.$$

Risk-neutral scenario generator III

Combining Equations (2) and (3), we get

$$A_j^{(h)} = \exp \left[\left(f_j - \frac{1}{2} \sum_{l=1}^k \sigma_{hl}^2 \right) \Delta + \sum_{l=1}^k \sigma_{hl} \sqrt{\Delta} Z_j^{(l)} \right], \quad (4)$$

where

$$Z_j^{(l)} = \frac{B_{j\Delta}^{(l)} - B_{(j-1)\Delta}^{(l)}}{\sqrt{\Delta}}.$$

By the property of Brownian motion, we know that $Z_1^{(l)}, Z_2^{(l)}, \dots, Z_m^{(l)}$ are independent random variables with a standard normal distribution.

Let n be the number of risk-neutral paths. For $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$, and $h = 1, 2, \dots, k$, let $A_{ij}^{(h)}$ be the accumulation factor of the h th index at time t_j along the i th path. Suppose

Risk-neutral scenario generator IV

that there are g investment funds in the pool and the fund mappings are given by

$$W = \begin{pmatrix} w_{11} & w_{12} & \cdots & w_{1k} \\ w_{21} & w_{22} & \cdots & w_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ w_{g1} & w_{g2} & \cdots & w_{gk} \end{pmatrix}.$$

Then the simple returns of the h th investment fund can be blended as

$$F_{ij}^{(h)} - 1 = \sum_{l=1}^k w_{hl} \left[A_{ij}^{(l)} - 1 \right], \quad h = 1, 2, \dots, g,$$

Risk-neutral scenario generator V

where $F_{ij}^{(h)}$ is the accumulation factor of the h th fund for the period (t_{j-1}, t_j) along the i th path. Since the sum of weights is equal to 1, we have

$$F_{ij}^{(h)} = \sum_{l=1}^k w_{hl} A_{ij}^{(l)}, \quad h = 1, 2, \dots, g.$$

Cash flow projection I

Without loss of generality, we assume that there are three types of cash flows: death benefit, guaranteed benefits, and risk charges for providing such guaranteed benefits. For a general variable annuity contract, we use the following notation to denote these cash flows that occur within the period $(t_{j-1}, t_j]$ along the i th risk-neutral path:

GB_{ij} denotes the guaranteed death or living benefit.

DA_{ij} denotes payoff of the guaranteed death benefit.

LA_{ij} denotes payoff of the guaranteed living benefit.

RC_{ij} denotes the risk charge for providing the guarantees;

$PA_{ij}^{(h)}$ denotes the partial account value of the h th investment fund, for $h = 1, 2, \dots, g$.

Cash flow projection II

TA_{ij} denotes the total account value. In general, we have

$$TA_{ij} = \sum_{l=1}^g PA_{ij}^{(l)}.$$

We use the following notation to denote various fees:

ϕ_{ME} denotes the annualized M&E fee of the contract;

ϕ_G denotes the annualized guarantee fee for the riders selected by the policyholder;

$\phi_F^{(h)}$ denotes the annualized fund management fee of the h th investment fund. Usually this fee goes to the fund managers rather than the insurance company.

Cash flow projection III

Then we can project the cash flows in a way that is similar to the way used by Bauer et al. (2008). For the sake of simplicity, we assume that events occur in the following order during the term of the contract:

- ▶ fund management fees are first deducted;
- ▶ then M&E and rider fees are deducted;
- ▶ then death benefit is paid if the policyholder dies;
- ▶ then living benefit is paid if the policyholder is alive.

We also assume that the fees are charged from the account values at the end of every month and the the policyholder takes withdrawal at anniversaries of the contracts.

Cash flow projection IV

Once we have all the cash flows, we can calculate the fair market values of the riders as follows:

$$\begin{aligned} V_0 = & \frac{1}{n} \sum_{i=1}^n \sum_{j=1}^m (j-1)\Delta p_{x_0} \cdot \Delta q_{x_0+(j-1)\Delta} DA_{i,j} d_j \\ & + \frac{1}{n} \sum_{i=1}^n \sum_{j=1}^m j\Delta p_{x_0} LA_{i,j} d_j, \end{aligned} \quad (5)$$

where x_0 is the age of the policyholder, p is the survival probability, q is the probability of death, and d_j is the discount factor defined as

$$d_j = \exp \left(-\Delta \sum_{l=1}^j f_l \right).$$

The risk charge value can be calculated as

$$RC_0 = \frac{1}{n} \sum_{i=1}^n \sum_{j=1}^m j \Delta p_{x_0} RC_{i,j} d_j. \quad (6)$$

In the following subsections, we describe how the cash flows of various guarantees are projected.

GMDB Projection I

For $j = 0, 1, \dots, m - 1$, the cash flows of the GMDB from t_j to t_{j+1} are projected as follows:

- ▶ The partial account values evolve as follows:

$$PA_{i,j+1}^{(h)} = PA_{ij}^{(h)} F_{i,j+1}^{(h)} \left(1 - \Delta\phi_F^{(h)}\right) (1 - \Delta[\phi_{ME} + \phi_G]) \quad (7)$$

for $h = 1, 2, \dots, g$, where Δ is the time step. Here we assume that the fees are deducted at the end of each period and the fund management fees are deducted before the insurance fees and withdrawal.

- ▶ The risk charges are projected as

$$RC_{i,j+1} = \sum_{h=1}^k PA_{ij}^{(h)} F_{i,j+1}^{(h)} \left(1 - \Delta\phi_F^{(h)}\right) \Delta\phi_G. \quad (8)$$

Note that the risk charge does not include the basic insurance fees.

GMDB Projection II

- ▶ If the guaranteed death benefit evolves as follows:

$$GB_{i,j+1} = \begin{cases} GB_{i,j}, & \text{if } t_{j+1} \text{ is not an anniversary,} \\ GB_{i,j}, & \text{if } t_{j+1} \text{ is an anniversary and} \\ & \text{the benefit is return of premium} \\ GB_{i,j}(1+r), & \text{if } t_{j+1} \text{ is an anniversary and} \\ & \text{the benefit is annul roll-up,} \\ \max\{TA_{i,j+1}, GB_{i,j}\}, & \text{if } t_{j+1} \text{ is an anniversary and} \\ & \text{the benefit is annul ratchet,} \end{cases} \quad (9)$$

with $GB_{i,0} = TA_{i,0}$.

- ▶ If the policyholder dies within the period $(t_j, t_{j+1}]$, then the payoff of the death benefit is projected as

$$DA_{i,j+1} = \max\{0, GB_{i,j+1} - TA_{i,j+1}\}. \quad (10)$$

- ▶ The payoff of the living benefit is zero, i.e., $LA_{i,j+1} = 0$.

- ▶ After the maturity of the contract, all the state variables are set to zero.

GMAB and DBAB Projection I

Here we follow the specification given in Hardy (2003) and consider GMAB riders that give policyholders to renew the policy at the maturity date. As a result, a policy with the GMAB rider may have multiple maturity dates.

At the maturity dates, if the guaranteed benefit is higher than the fund value, then the insurance company has to pay out the difference and the policy is renewed by resetting the fund value to the guaranteed benefit. If the guaranteed benefit is lower than the fund values, then the policy is renewed by resetting the guaranteed benefit to the fund value. Let $T_1 = T$ be the first renewal date. Let T_2, T_3, \dots, T_J be the subsequent renewal dates. Under such a GMAB rider, the guaranteed benefit evolves as follows:

$$GB_{i,j+1} = \begin{cases} \max\{GB_{i,j}, TA_{i,j+1}\} & \text{if } t_{j+1} \in \mathcal{T}, \\ GB_{i,j+1}^* & \text{if otherwise,} \end{cases} \quad (11)$$

GMAB and DBAB Projection II

where $GB_{i,j+1}^*$ is the benefit base adjusted for withdrawals and $\mathcal{T} = \{T_1, T_2, \dots, T_J\}$ is the set of renewal dates. We assume that the policyholder renews the policy only when the account value at a maturity date is higher than the guaranteed benefit. The payoff of the living benefit is calculated as follows:

$$LA_{i,j+1} = \begin{cases} 0, & \text{if } t_{j+1} \notin \mathcal{T}, \\ \max\{0, GB_{i,j+1} - TA_{i,j+1}\}, & \text{if } t_{j+1} \in \mathcal{T}. \end{cases} \quad (12)$$

The payoff of the death benefit is zero if the policy contains only the GMAB rider. For the DBAB policy, the death benefit is calculated according to Equation (10).

If the payoff is larger than zero, then the fund value is reset to the guaranteed benefit. In other words, the payoff is deposited to the investment funds. We assume that the payoff is

GMAB and DBAB Projection III

deposited to the investment funds proportionally. Specifically, the partial account values are reseted as follows:

$$PA_{i,j+1}^{(h)} = PA_{ij}^{(h)} F_{i,j+1}^{(h)} \left(1 - \Delta\phi_F^{(h)}\right) (1 - \Delta[\phi_{ME} + \phi_G]) + LA_{i,j+1}^{(h)} \quad (13)$$

for $h = 1, 2, \dots, g$, where $LA_{i,j}^{(h)}$ is the amount calculated as,

$$LA_{i,j+1}^{(h)} = LA_{i,j+1} \frac{PA_{ij}^{(h)} F_{i,j+1}^{(h)} \left(1 - \Delta\phi_F^{(h)}\right)}{\sum_{l=1}^p PA_{ij}^{(l)} F_{i,j+1}^{(l)} \left(1 - \Delta\phi_F^{(l)}\right)}.$$

GMIB and DBIB Projection I

A variable annuity policy with a GMIB rider gives the policyholder three options at the maturity date (Bauer et al., 2008; Marshall et al., 2010):

- ▶ get back the accumulated account values,
- ▶ annuitize the accumulated account values at the market annuitization rate, or
- ▶ annuitize the guaranteed benefit at a payment rate r_g per annum.

As a result, the payoff of the GMIB rider is given by

$$LA_{i,j+1} = \begin{cases} 0, & \text{if } t_{j+1} < T, \\ \max \left\{ 0, GB_{i,j+1} \frac{\ddot{a}_T}{\ddot{a}_g} - TA_{i,j+1} \right\}, & \text{if } t_{j+1} = T, \end{cases} \quad (14)$$

where \ddot{a}_T and \ddot{a}_g are the market price and the guaranteed price of an annuity with payments of \$1 per annum beginning at time

GMI_B and DBIB Projection II

T , respectively. In this paper, we determine \ddot{a}_T by using the current yield curve. We specify \ddot{a}_g by using a particular interest rate, i.e.,

$$\ddot{a}_g = \sum_{n=0}^{\infty} n p_x e^{-nr},$$

where r is an interest rate set to 5%.

GMMB and DBMB Projection I

For the GMMB and DBMB guarantees, account values, risk charges, and guaranteed benefits are projected according to the GMMB case specified in Equation (7), Equation (8), and Equation (9), respectively. The payoff of the living benefit is projected as

$$LA_{i,j+1} = \begin{cases} 0, & \text{if } t_{j+1} < T, \\ \max\{0, GB_{i,j+1} - TA_{i,j+1}\}, & \text{if } t_{j+1} = T. \end{cases} \quad (15)$$

For the GMMB guarantee, the payoff of the guaranteed death benefit is zero. For the DBMB guarantee, the payoff of the guaranteed death benefit is projected according to Equation (10).

GMWB and DBWB Projection I

To describe the cash flow project for the GMWB, we need the following additional notation:

WA_{ij}^G denotes the guaranteed withdrawal amount per year. In general, WA_{ij}^G is a specified percentage of the guaranteed withdrawal base.

WB_{ij}^G denotes the guaranteed withdrawal balance, which is the remaining amount that the policyholder can withdraw.

WA_{ij} denotes the actual withdrawal amount per year.

For $j = 0, 1, \dots, m - 1$, the cash flows of the GMWB from t_j to t_{j+1} are projected as follows:

GMWB and DBWB Projection II

- Suppose that the policyholder takes maximum withdrawals allowed by a GMWB rider at anniversaries. Then we have

$$WA_{i,j+1} = \begin{cases} \min\{WA_{i,j}^G, WB_{i,j}^G\}, & \text{if } t_{j+1} \text{ is an anniversary,} \\ 0, & \text{if otherwise.} \end{cases} \quad (16)$$

- The partial account values evolve as follows:

$$PA_{i,j+1}^{(h)} = PA_{ij}^{(h)} F_{i,j+1}^{(h)} \left(1 - \Delta\phi_F^{(h)}\right) \left(1 - \Delta[\phi_{ME} + \phi_G]\right) - WA_{i,j+1}^{(h)} \quad (17)$$

for $h = 1, 2, \dots, g$, where Δ is the time step and $WA_{i,j}^{(h)}$ is the amount withdrawn from the h th investment fund, i.e.,

$$WA_{i,j+1}^{(h)} = WA_{i,j+1} \frac{PA_{ij}^{(h)} F_{i,j+1}^{(h)} \left(1 - \Delta\phi_F^{(h)}\right)}{\sum_{l=1}^p PA_{ij}^{(l)} F_{i,j+1}^{(l)} \left(1 - \Delta\phi_F^{(l)}\right)}.$$

GMWB and DBWB Projection III

If the account values from the investment funds cannot cover the withdrawal, the account values are set to zero.

- ▶ The risk charges are projected according to Equation (8).
- ▶ If the guaranteed benefit is evolves as follows:

$$GB_{i,j+1} = GB_{i,j+1}^* - WA_{i,j+1}, \quad (18)$$

where

$$GB_{i,j+1}^* = \begin{cases} GB_{i,j}, & \text{if } t_{j+1} \text{ is not an anniversary,} \\ GB_{i,j}, & \text{if } t_{j+1} \text{ is an anniversary and} \\ & \text{the benefit is return of premium} \\ GB_{i,j}(1+r), & \text{if } t_{j+1} \text{ is an anniversary and} \\ & \text{the benefit is annul roll-up,} \\ \max\{TA_{i,j+1}, GB_{i,j}\}, & \text{if } t_{j+1} \text{ is an anniversary and} \\ & \text{the benefit is annul ratchet,} \end{cases} \quad (19)$$

GMWB and DBWB Projection IV

with $GB_{i,0} = TA_{i,0}$. The guaranteed benefit is reduced by the amount withdrawn.

- ▶ The guaranteed withdrawal balance and the guaranteed withdrawal amount evolve as follows:

$$WB_{i,j+1}^G = WB_{i,j}^G - WA_{i,j+1}, \quad WA_{i,j+1}^G = WA_{i,j}^G \quad (20)$$

with $WB_{i,0}^G = TA_{i,0}$ and $WA_{i,0}^G = x_W TA_{i,0}$. Here x_W is the withdrawal rate. The guaranteed base is adjusted for the withdrawals.

- ▶ The payoff of the guaranteed withdrawal benefit is projected as

$$LA_{i,j+1} = \begin{cases} \max\{0, WA_{i,j+1} - TA_{i,j+1}\}, & \text{if } t_{j+1} < T, \\ \max\{0, WB_{i,j+1}^G - TA_{i,j+1}\}, & \text{if } t_{j+1} = T. \end{cases} \quad (21)$$

GMWB and DBWB Projection V

It is the amount that the insurance company has to pay by its own money to cover the withdrawal guarantee. At maturity, the remaining withdrawal balance is returned to the policyholder.

- ▶ The payoff of the guaranteed death benefit for the GMWB is zero, i.e., $DA_{i,j+1} = 0$. For the DBWB, the payoff is projected according to Equation (10).
- ▶ After the maturity of the contract, all the state variables are set to zero.

Fair Market Value and Greek Calculation I

We use the bump approach (Cathcart et al., 2015) to calculate the Greeks. Specifically, we calculate the partial dollar deltas of the guarantees as follows:

$$\begin{aligned} & \text{Delta}^{(l)} \\ = & \frac{V_0 \left(PA_0^{(1)}, \dots, PA_0^{(l-1)}, (1+s)PA_0^{(l)}, PA_0^{(l+1)}, \dots, PA_0^{(k)} \right)}{2s} - \\ & \frac{V_0 \left(PA_0^{(1)}, \dots, PA_0^{(l-1)}, (1-s)PA_0^{(l)}, PA_0^{(l+1)}, \dots, PA_0^{(k)} \right)}{2s} \end{aligned} \quad (22)$$

for $l = 1, 2, \dots, k$, where s is the shock amount applied to the partial account value and $V_0(\dots)$ is the fair market value written as a function of partial account values. Usually, we use $s = 0.01$ to calculate the dollar deltas. The partial dollar delta measures the sensitivity of the guarantee value to an index and

Fair Market Value and Greek Calculation II

can be used to determine the hedge position with respect to the index.

We calculate the partial dollar rhos in a similar way. In particular, we calculate the l th partial dollar rho as follows:

$$Rho^{(l)} = \frac{V_0(r_l + s) - V_0(r_l - s)}{2s}, \quad (23)$$

where $V_0(r_l + s)$ is the fair market value calculated based on the yield curve bootstrapped with the l th input rate r_l being shocked up s bps (basis points) and $V_0(r_l - s)$ is defined similarly. A common choice for s is 10 bps.

Simulated Dataset

Quantity Name	Value	Quantity Name	Value
FMV	18,572,095,089	Rho2y	167,704
Delta1	(4,230,781,199)	Rho3y	85,967
Delta2	(2,602,768,996)	Rho4y	2,856
Delta3	(2,854,233,170)	Rho5y	(96,438)
Delta4	(2,203,726,514)	Rho7y	(546,045)
Delta5	(2,341,793,581)	Rho10y	(1,407,669)
Rho1y	40,479	Rho30y	(62,136,376)

Since the Monte Carlo simulation method is time-consuming, we used the HPC (High Performance Computing) cluster at the University of Connecticut with 80 CPUs together to calculate the fair market values and the greeks of the synthetic portfolio. It took these 80 CPUs about 2 hours to finish the calculations. If we add the runtime of all these CPUs, the total runtime was 389925.98 seconds or 108.31 hours.

Summary

- ▶ It is difficult to obtain real datasets to evaluate metamodeling techniques for valuing large portfolio of variable annuities.
- ▶ In this paper, we created a large synthetic portfolio of variable annuity contracts and developed a Monte Carlo simulation engine to calculate the Greeks.
- ▶ The simulated datasets can be used to measure the speed and accuracy of metamodeling techniques.

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