ASTIN / AFIR-ERM COLLOQUIUM
PANAMA CITY, PANAMA
20 - 24 AUGUST, 2017

SYNOPSIS

TITLE Multivariate Stochastic Prioritization of Dependent Actuarial Risks in Agricultural Insurance

AUTHOR(S) Ezgi Nevruz, Kasirga Yildirak, Ashis SenGupta

Key words: Aggregate claims, Partial order theory, Schur-convexity, Stochastic majorization, Crop-hail insurance.

Purpose of your paper: To investigate the aggregate claims of different risk classes in agricultural insurance in terms of their comparability and orderability under the dependency assumption

Synopsis:

Risk ranking is an important component for optimization of risk managament efforts. Prioritization of environmental risks is a specific area which ranks risks geographically with fair and accurate standards. Analytic tools are not always helpful for the prioritization. In some cases, especially when we are interested in environmental risk prioritization, we may consider the geographic information of our data. Therefore, we can use geographic information system (GIS) as a tool for prioritizing risks. In this study, we aim to investigate the aggregate claims of different risk classes in agricultural insurance in terms of their comparability and orderability under the dependency assumption. For this aim, we firstly classify actuarial risks of an agricultural insurance portfolio according to spatial and temporal characteristics of hazard regions. After that, we use various stochastic ordering relations such as stochastic majorization, stochastic dominance and stop-loss dominance that are proposed in the frame of partial order theory. We take into account the dependency of the individual claims exposed to similar environmental risks.

Majorization, which is an ordering relation of real-valued vectors, turns out to be a useful tool. The vectors do not need to be totally ordered, which is very advantageous for our study. “Order-preserving” functions are very beneficial in this context, since we use risk measures defined as functions to evaluate risks. A real-valued function which preserves the ordering of majorization is said to be “Schur-convex” function [1]. For the risk assessment, it is significant to use a measure reflecting the risk of a portfolio sufficiently and accurately. Therefore, we choose a risk measure that fulfils the properties of Schur-convexity and we use it to order the aggregate claims with the stochastic majorization relation.

In order to introduce our model setting, let us consider a crop-hail insurance portfolio. We suppose that there are m risk classes and p_i crop classes for i-th risk class with $i = 1, 2, \dots, m$. The aggregate claims for the i-th risk class can be considered as a p_i-variate random vector (r.vector) as follows:

$$S^{(i)} = \left(S_1^{(i)}, S_2^{(i)}, \dots, S_{p_i}^{(i)}\right)'.$$

ASOCIACIÓN CENTROAMERICANA de ACTUARIOS

AFIR/ERM

ASTIN

Here, the aggregate loss of the i-th risk class and j-th crop class can be represented by the random variable (rv) $S_j^{(i)}$ and it is obtained as

$$S_j^{(i)} = \sum_{k=1}^{N_j} X_{jk}^{(i)},$$

where N_j is the claim number of the j-th crop class and $X_{jk}^{(i)}$ is the claim amount of the k-th individual loss in the j-th crop class with $i = 1, 2, \dots, m$, $j = 1, 2, \dots, p_i$, and $k = 1, 2, \dots, N_j$.

After introducing our model setting, the following definition clearly demonstrates the convenience of the majorization relation for prioritizing the aggregate claim vectors. For n-dimensional vectors $x, y \in \mathbb{R}^n$, the ordering $x \lesssim_{maj} y$ denotes that y majorizes x (or x is majorized by y), and it is defined by [2] as follows:

$$x \lesssim_{maj} y \text{ iff } \begin{cases} \sum_{i=1}^{k} x_{[i]} \leq \sum_{i=1}^{k} y_{[i]}\,; k = 1, 2, \dots, n-1 \\ \sum_{i=1}^{n} x_{[i]} = \sum_{i=1}^{n} y_{[i]} \end{cases}. \tag{1}$$

Here, $x_{[i]}$ denotes i-th component of $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ in the decreasing order, i.e. the i-th element of the vector $x \downarrow = \left(x_{[1]}, x_{[2]}, \dots, x_{[n]}\right)$ where $x_{[1]} \geq x_{[2]} \geq \cdots \geq x_{[n]}$.

This condition (1) is equivalent to the condition below:

$$x \lesssim_{maj} y \text{ iff } \begin{cases} \sum_{i=1}^{k} x_{(i)} \geq \sum_{i=1}^{k} y_{(i)}\,; k = 1, 2, \dots, n-1 \\ \sum_{i=1}^{n} x_{(i)} = \sum_{i=1}^{n} y_{(i)} \end{cases}. \tag{2}$$

Here, $x_{(i)}$ denotes i-th component of $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ in the increasing order, i.e. the i-th element of the vector $x \uparrow = \left(x_{(1)}, x_{(2)}, \dots, x_{(n)}\right)$ where $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$[1].

In 1923, Schur introduced the Schur-convex function that is also referred as the term "Schur-increasing function". Marshall et al. (2009) define these functions that preserve the majorization ordering as follows.

A real function $\varphi: \mathcal{A} \rightarrow \mathbb{R}$ for some set $\mathcal{A} \subset \mathbb{R}^n$ is said to be Schur-convex on $\mathcal{A}$ if

$$x \lesssim_{maj} y \text{ on } \mathcal{A} \Leftrightarrow \varphi(x) \leq \varphi(y). \tag{3}$$

φ is strictly Schur-convex on $\mathcal{A}$ if $x \lesssim_{maj} y$ on $\mathcal{A} \Leftrightarrow \varphi(x) < \varphi(y)$ when x is not a permutation of y.

Likewise, φ is said to be Schur-concave on $\mathcal{A}$ if

$$x \lesssim_{maj} y \text{ on } \mathcal{A} \Leftrightarrow \varphi(x) \geq \varphi(y). \tag{3}$$

φ is strictly Schur-concave on $\mathcal{A}$ if $x \precsim_{maj} y$ on $\mathcal{A} \Leftrightarrow \varphi(x) > \varphi(y)$ when x is not a permutation of y. In addition, $\varphi(x)$ is Schur-convex on $\mathcal{A}$ if and only if (iff) $-\varphi(x)$is Schur-concave on $\mathcal{A}$.

Risk measures are classified into two as safety risk measures evaluating wealth under risk and dispersion measures assessing the uncertainty level [3]. We use the "sample variance" as a dispersion measure in order to rank the aggregate claims of m risk classes through majorization relation. In order to do that, we set the prioritization of the aggregate claims as follows:

$$S^{(k)} \precsim_{maj} S^{(l)} \Leftrightarrow \varphi\left(\mathbb{V}\left(S^{(k)}\right)\right) \leq \varphi\left(\mathbb{V}\left(S^{(l)}\right)\right),$$

where φ is any Schur-convex function. The sample variance defined as

$$\varphi_1(x) = \varphi_1(x_1, x_2, \dots, x_n) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

is strictly Schur-convex wrt $x = (x_1, x_2, \dots, x_n)$. For the application part, we use the claim data of the crop-hail insurance in Turkey, and prioritize the determined risk classes using the sample variance dispersion measure which fulfils the conditions of the Schur-convexity.

References

[1] Marshall, A.W., Olkin, I., Arnold, B.C., *Inequalities: Theory of Majorization and Its Applications*, Springer Series in Statistics, 2nd, New York, Dordrecht, Heidelberg, London, 2009.

[2] Hardy, G.H., Littlewood, J.E., Pólya, G., *Inequalities*, Cambridge University Press, 1st, 2nd edition, 1934, 1952. London, New York.

[3] Ortobelli, S., Rachev, S.T., Stoyanov, S., Fabozzi, F.J., Biglova, A., *The proper use of risk measures in portfolio theory*, International Journal of Theoretical and Applied Finance, 8(8):1107-1133, 2005.