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Pricing and Hedging Guaranteed Minimum Withdrawal Benefits under a General Levy Framework using the COS Method

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SYNOPSIS

TITLE Pricing and Hedging Guaranteed Minimum Withdrawal Benefits under a General Lévy Framework using the COS Method

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Key words: Pricing, Hedging, Variable Annuities, GMWB, Levy, COS Method

Purpose of your paper: The global market for variable annuities (VAs) represents a huge pool of assets. For instance, the market share of VAs in the U.S. as of the second quarter of 2015 was estimated to be US$1.98 trillion (IRI, 2015). These VAs are a popular retirement product for several reasons, including equity exposure, longevity protection, and the various guaranteed minimum benefits (GMBs) that insurers offer to protect their customers from downside market risks (Hanif et al., 2007; Condron, 2008).

Guaranteed minimum withdrawal benefits (GMWBs) are the most popular form of GMBs, which come in various forms including the guaranteed lifelong withdrawal benefit (GLWB), an alternative that guarantees a fixed periodic withdrawal amount until death of the policyholder (Bauer et al., 2008; Ledlie et al., 2008; Fung et al., 2014). These ensure a minimum withdrawal amount at each withdrawal date over the term of the contract, regardless of the status of the VA investment account. The insurer funds this guarantee with proportional, periodic charges to the investment account.

In this paper, we provide an efficient algorithm for pricing VAs embedded with GMWB riders using the COS method. The algorithm we develop demonstrates superior computational efficiency as it can be adapted to the general class of Lévy processes. These processes are general enough to include a wealth of patterns and thus they account for the smile and skew effects observed in option prices (Papapantoleon, 2008). We also extend the use of the COS method to develop hedging strategies that seek to minimise a moment or quantile-based risk measure, such as the variance of the hedging outcomes or the 95% Value at Risk (VaR) of the hedged portfolio loss distribution. We show that the COS method is computationally more efficient in comparison with valuation methodologies in existing literature for the same level of accuracy. The framework developed is general enough to incorporate complex policyholder behavior decisions and sophisticated contract features such as the reset provision. The local risk minimization strategies developed can incorporate short-selling and budgeting constraints while remaining robust. The framework developed proves to be compatible to both pricing, delta-gamma hedging, risk minimization and VaR calculations, making it a strong candidate for quick and accurate valuations for the industry.

Synopsis: This paper extends the Fourier-cosine (COS) method (Fang and Oosterlee, 2008) to the pricing and hedging of variable annuities embedded with guaranteed minimum withdrawal benefit (GMWB) riders. The COS method facilitates efficient computation of prices and hedge ratios of the GMWB riders when the underlying fund dynamics evolve under the influence of the general class of Lévy processes (Papapantoleon, 2008). Formulae are derived to value the contract at each withdrawal date using a backward recursive dynamic programming algorithm. Numerical

comparisons are performed with results presented in Bacinello et al. (2014) and Luo and Shevchenko (2014) to confirm the accuracy of the method. The efficiency of the proposed method is assessed by making comparisons with the approach presented in Bacinello et al. (2014). We find that the COS method presents highly accurate results with notably fast computational times. The valuation framework forms the basis for GMWB hedging. A local risk minimisation approach to hedging inter-withdrawal date risks is developed (Kolkiewicz and Liu, 2012). A variety of risk measures are considered for minimisation in the general Lévy framework. While the second moment and variance have been considered in existing literature, we show that the value-at-risk may also be of interest as a risk measure to minimise risk in variable annuities portfolios.

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