SYNOPSIS

TITLE The distribution function of the sum of dependent risks: a geometric-combinatorial approach.

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Key words:
dependent risks, VaR and ES, stop-loss reinsurance, copulas, measure of simplexes, computation algorithm.

Purpose of your paper:
The problem of evaluating the distribution of the sum of dependent risks has been treated recently through a new numerical approach called AEP algorithm. We consider a theoretical result extending the proof of the convergence of this algorithm to any dimension and analyze its use for applications in finance and insurance.

Synopsis:
The evaluation of the sum of dependent risks is a main issue for many applications in finance and insurance. One of the reasons for such an interest is the approach of the regulatory standard on bank capital adequacy introduced by the so called Basel II [3], which is based on a quantile (Value-at-Risk) of the distribution function of the potential losses of a bank, collecting several risky positions. There would be no need to recall that in insurance applications the evaluation of aggregate losses of many risks is a very common task, as is the case, for example, with stop-loss reinsurance treaties.
Such a subject is addressed by a wide range of literature contributions exploiting the concept of copula, which permits to separate the analysis of the dependence structure from that of the marginal distributions. Whereas the theory based on copula functions allows to measure the effects of dependence among different risks, especially when stress-testing is required, another relevant problem is to assess bounds for the probability of the sum of dependent risks exceeding some threshold. To this purpose [1], [2], [6] introduced a new numerical procedure. In fact the authors proposed an algorithm whose implementation required a bounded density of the multivariate distribution and whose convergence was proven for dimension less or equal than 5.
However, in a quite recent breakthrough, [5] has been proven the convergence of the algorithm for any dimension. Building on that, exploiting a combinatorial approach, we propose an improvement of the algorithm which allows to consider a summation of a smaller number of terms. This way we aim at a considerably faster numerical procedure, which, avoiding some memory constraints, can make the algorithm manageable and competitive with Monte Carlo methods also for higher (i.e. larger than 5) dimensions.
The layout of the paper is the following. In the first paragraph we summarize the AEP algorithm and its main results. Then we give a sketch of Galeotti's theorem, which extends the proof of the convergence of the AEP algorithm to any dimension. In the third paragraph some combinatorial details are fully developed in order to find some efficient ways for the numerical implementation of the AEP algorithm. Then we propose the application of the AEP to financial risk evaluation, as Value At Risk and Expected Shortfall evaluation, and to insurance risk evaluation, in particular for the Stop-Loss reinsurance treaty, which analytical description coincides with Expected Shortfall one. We end by a short list of comments and open problems.

Main references

[1] Arbenz P., Embrechts P., Puccetti G., (2011) The AEP algorithm for the fast computation of the distribution of the sum of dependent random variables. Bernoulli 17(2), 562-591.

[2] Arbenz, P., Embrechts, P., and G. Puccetti (2012), The GAEP algorithm for the fast computation of the distribution of a function of dependent random variables. Stochastics 84(5-6), 569-597.

[3] Basel Committee on Banking Supervision (2006), International Convergence of Capital Measurement and Capital Standards.

[4] Durante F., Sarkoci P., Sempi C. (2009), Shuffles of copulas. Journal of Mathematical Analysis and Applications 352(2), 914-921.

[5] Galeotti M. (2015), Computing the distribution of the sum of dependent random variables via overlapping hypercubes. Decisions in Economics and Finance 38(2), 231-255.

[6] Puccetti, G. and L. Rüschendorf (2015), Computation of sharp bounds on the expected value of a supermodular function of risks with given marginals. Commun. Stat. Simulat. 44(3), 705-718.

Note: If you are not presenting a paper for this Colloquium, please include as much detail as possible in your Synopsis (maximum three pages) to enable delegates to prepare for your session.