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SYNOPSIS

TITLE Comparative evaluation of multi attribute decision making methods on determination of optimal reinsurance

AUTHOR(S) Başak BULUT KARAGEYIK & Şule ŞAHIN

Key words: Optimal reinsurance, De Vylder's method, ultimate ruin probability, multi attribute decision making methods.

Purpose of your paper:

The purpose of this study is to obtain optimal reinsurance level and measure the changes of optimal levels according to benefit and cost criteria under the minimum ruin probability constraint by using Multi Attribute Decision Making methods.

Synopsis:

In this study, we investigate the comparative evaluation of multi attribute decision making methods on determination of the optimal reinsurance level.

The aggregate claim amounts are assumed to be distributed as compound Poisson and the individual claim amounts are distributed exponentially. The probability of ultimate ruin is approximated by using De Vylder's method. In De Vylder's method, it is assumed that the risk process for which is intended to calculate the ultimate ruin probability is denoted by $\{U(t)\}_{t\geq 0}$. This risk process is approximated by a classical risk process $\{\widetilde{U}(t)\}_{t\geq 0}$ which has the following characteristics: $\widetilde{U}(0) = u$, number of claims has a Poisson process with $\tilde{\lambda}$ Poisson parameter, the premium income per unit time is $\tilde{c}$, The individual claim amount has an exponential distribution with $\widetilde{F}(x) = 1 - exp\{-\tilde{\alpha}x\},\ x \geq 0$ [1],[2].

Since the individual claim has an exponential distribution, the ultimate ruin probability of the risk process $\{\widetilde{U}(t)\}_{t\geq 0}$ is obtained as

$$\psi(u) = \frac{\tilde{\lambda}}{\tilde{\alpha}\tilde{c}} \exp\{-(\tilde{\alpha} - \tilde{\lambda}/\tilde{c})\, u\}.$$

The parameters $\tilde{\lambda}, \tilde{c}$ and $\tilde{\alpha}$ are calculated by matching first three central moments of two surplus processes $\{U(t)\}_{t\geq 0}$ and $\{\widetilde{U}(t)\}_{t\geq 0}$ as given

$$\alpha = 3m_2/m_3\,, \qquad \tilde{\lambda} = \frac{9\lambda m_2^3}{2m_3^2} \qquad \text{and} \qquad \tilde{c} = c - \lambda m_1 + \tilde{\lambda}/\tilde{\alpha}.$$

The ultimate ruin probability under De Vylder's method can be described according to two functions f_1 and f_2 as follows: $\psi(u) = f_1 \exp\{f_2 u\}$, where $f_1 = \frac{\tilde{\lambda}}{\tilde{\alpha}\tilde{c}}$ and $f_2 = \tilde{\alpha} - \tilde{\lambda}/\tilde{c}$.

There are many important factors that are used to ensure the continuity of the insurance company. The expected profit, ruin probability, risk measures, variance of the insurer or reinsurer's risk and insurance/reinsurance premium can be given as examples of the essential factors for the insurer's income and profit after a reinsurance arrangement. In the literature, most of the studies focus on determination of the reinsurance level according to a single criterion. In this study, optimal reinsurance levels are determined according to several criteria which are related to each other.

We aim to determine the optimal reinsurance which maximises benefit criteria and minimizes cost criteria while the ultimate ruin probability is fixed at a minimum level. The optimal initial surplus and reinsurance level can be obtained with respect to these benefit or cost criteria. In determination of the optimal initial surplus and reinsurance level, we take the advantage of the decision theory.

Decision making investigates identifying and choosing the best alternatives based on the values and preferences of the decision maker. MADM is an approach which applies to solving problems by selection among a finite number of alternatives.

MADM method specifies how attribute information is to be processed in order to arrive at a choice [3]. For MADM problem, a decision matrix is used to express a multiple criteria decision problem as a matrix format. There are m alternative options which need to be assessed on n attributes (criteria).

The application of outranking is based on Multi-Attribute Decision Making (MADM) methods: The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), The analytic hierarchy process (AHP), Multi criteria Optimization and Compromise Solution (VIKOR), Elimination and Choice Expressing Reality (ELECTRE).

TOPSIS method is suggested by Hwang and Yoon [4] to determine the best alternative based on the concepts of a compromise solution. The compromise solution can be regarded as choosing the solution with the shortest Euclidean distance from the ideal solution and the farthest Euclidean distance from the negative ideal solution. The ideal solution is defined such as the one which maximizes the benefit criteria and minimizes the cost criteria.

AHP first decomposes the decision problem into a system of hierarchies of objectives, attributes (criteria), and alternatives. The decision-maker valuates the relative importance of the various elements by pairwise comparisons [5].

VIKOR method is developed for Multiple-Criteria optimization of complex systems. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It introduces a multiple criteria ranking index based on the particular measure of closeness to the ideal solution [6].

Hwang and Yoon [4] clarify ELECTRE method such as alternatives can be compared by using the pairwise comparison of alternatives based on the degree to which evaluations of the alternatives. The preference weights are clarified that confirm or contradict the pairwise dominance relationship between alternatives.

The optimal reinsurance levels are compared for each MADM method so that it is decided which MADM method is more appropriate for insurance and reinsurance agreements. In order to determine the sufficient and more accurate reinsurance level, MADM methods are compared with respect to both the various benefit and cost criteria as well as the possibility of minimum ultimate ruin probability.

We compare optimal reinsurance levels which are obtained according to different MADM methods. In this study, we have investigated which MADM method is more appropriate in determination of the optimal reinsurance level under the actuarial point of view. We measure the effect of benefit or cost criteria on optimal reinsurance level by using sensitivity analysis. It is also be investigated which of the benefit or cost criteria have more significant effect in determining the optimal reinsurance.

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