A practical model for pricing optimization in car insurance

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A PRACTICAL MODEL FOR OPTIMIZATION OF CAR INSURANCE RATE

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Abstract

A methodology is proposed to calculate the optimal premium for car insurance of a hypothetical portfolio of clients. The methods use statistical tools to estimate the potential value of car insurance customers, their price elasticity of demand and simple optimization algorithms to simulate results. The impact of customer sensitivity to variations in the price of the insurance policy and the outcome of applying optimal premium rates in several customer segments are shown.

I. Introduction

The sensitivity of car insurance customers to price and how this affects their retention has been a subject of intense analysis in the insurance market research literature. Different methodologies have been developed to determine the most valuable customer groups in an insurance company with the application of models for estimating the potential value of customers, and which, given their price elasticity, it would be profitable to offer differential rates for keeping them as clients of the insurance portfolio. Applications have been particularly interesting in the case of car insurance.

A number of reviews have assessed these applications of statistical and actuarial methodologies to calculate the potential value of the clients of a car insurance portfolio and their reliability as estimates of retention. Gupta *et al.* (2006) reviewed the models used in the literature to estimate the potential value of the customer as the present value of the net benefits (income minus costs) to the company in its relationship with the customer. Harrison and Ansell (2002) and Reinhart and Kumar (2003) used survival functions to determine the likelihood that a car insurance customer will renew over time their policy and also to calculate the likelihood of cross-selling for high probability client retention.

Donkers *et al.* (2003) propose the use of multivariate probit models to estimate the customer's decision-making when purchasing several products, Venkatesan and Kumar (2004) use GLM models to estimate the likelihood of purchasing several insurance products from (cross-selling) to determine the potential value of the customer.

The use of data mining tools for client segmentation in insurance portfolios can be found in Smith *et al.* (2003), Godfrey *et al.* (2006), Garcia and Hannabi (2002), Yeo *et al.* (2001), Kamakura *et al.* (2003).

The relationship between price elasticity and customer retention probability has been analysed in Pfeifer and Farris (2003), who also employ duration models. Along similar lines, Barone and Bella (2004) apply the segmentation of customers and the calculation of price elasticity in the car insurance market.

Once the probability of retention has been determined, the customer has purchased other insurance products, at that point the potential value of the client is determined, the portfolio of clients has been clustered among those clients of high potential value and of low potential value, then the natural step is to seek retention strategies and maximize the profitability of high potential value clients.

Given the sensitivity of car insurance customers to the price of the policy, premium rate optimization is a tool that has been used in the car insurance markets. Price optimization uses the economic concept of "price elasticity of demand" which is a measure of the value of the quantity of a product or a service that changes in response to changes in the price.

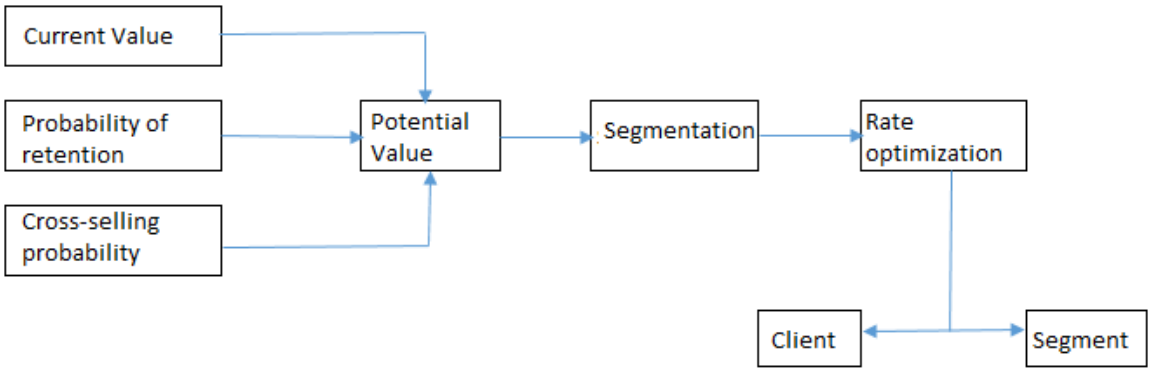
Through data mining and a program of optimization of the insurance premium, Yeo *et al.* (2002) proposes a model of segmentation of clients according to the probability of retention of the customer segments or clusters and their sensitivity to changes in the price. Krikler *et al.* (2004) detailed the methodology of the commercial software EARNIX. It starts from the calculation of demand functions using GLM models to estimate the sensitivity to the price of the customers and then applies premium optimization algorithms. With this estimated demand function, the margin of profitability of the customer portfolio is maximised while maintaining the retention of the customer portfolio.

Although there are different computer programs available that allow the segmentation of car insurance customer portfolios and the optimization of its yearly renewal rate, this article proposes the application of different statistical and actuarial tools available in a unified framework. Allowing for the segmenting of a client portfolio according to the potential value of these and to optimize the premium, accounting for the elasticity of the price in the different segments, so as to maximize profitability by keeping the retention indicators fixed[1].

The application of the proposed methodology is carried out on a hypothetical car insurance client portfolio of a Colombian company.

[1] The used algorithms were design in the statistical software R

Figure 1. Client Value and Optimization Methodology



II. Estimation of the Potential Value of the Client

The first step in the car insurance premium optimization methodology is to segment the customer portfolio according to the potential value of the customer. According to the specialized literature previously reviewed, the customer's potential value measure was defined as the sum of the current value and the future value of the customer.

The current value of a customer corresponds to the difference between: the total sum of the premiums paid during the entire relationship with the insurance company for all the insurance products **less** the sum of the claims incurred during the same period (adjusting values by the effect of inflation) and the acquisition and issue expenses of these same insurance policies.

Some assumptions for the calculation of the client value are: The possibility of a customer with several insurance products, some of the customers who hold individual car coverage will have other additional products with the same company (such as home, health, or life policies). The present value corresponds to the sum of the value of the customer from its first acquisition of company's products up to the analysis period.

The future value of the customer is proposed as the expected value of the margin that the client will leave in his future relationship with the company (premiums less expected losses, expenses and commissions). The calculation of this future value should be based on the following elements:

- The likelihood that the client will continue in the future with the Company

- The likelihood that the customer will purchase additional products that he/she currently has with the Company

- Expected loss ratio of purchased insurance products
- A discount interest rate

The future value will be calculated as the expected value of the average premiums of the customers of a specific segment **multiplied** by the probability that the customers will acquire it, in addition to the car insurance coverage, any of the others considered in the analysis, each adjusted to the likelihood that the client will maintain his relationship with the insurance company.

Following the methods in the literature, the probability of customer retention was estimated. For the exercise presented in this document, the historical information of the current and cancelled individual car insurance client data base of the last 7 years was used with a Kaplan-Meier function, the cross-sell probability is proposed to be estimated using a MULTINOMIAL LOGIT model.

The formal equation for calculating the Customer's Potential Value for *k* insurance products included in the possibility of cross-selling will be as follows:

$$potential\ value = current\ value + \sum_{j=1}^{k}(p_j margin_j)\sum_{t=1}^{T}\left(\frac{1}{1+r}\right)^t\left(\frac{l_t}{l_0}\right)$$

III. Segmentation of Client Portfolio

Once the potential value variable for each client is calculated, to estimate several statistical and econometric models in order to analyse which segments or relevant variables may be used to cluster the clients with the highest potential value. Some of the models that could be used are linear regression analysis, percentile regression, ordered probit models or clustering models such as Correspondence Analysis, Discriminant Analysis, and Tree Analysis and Neural Networks.

The final objective of the estimation from several of these models, with the descriptive analysis of the Customer Value, is to find those characteristics of the clients or particular clusters of clients that allow to determine groups of high potential value. Appropriately through an analysis of basic statistics, to analyse the differences between the average potential value of different interest categories. Some examples are:

- To find the differences between the average potential value of clients by sex, insured, vehicle brand, broker or duration in customer years.
- To present graphically the proportion of customers with more than one product that has been acquired in the Company.

- To find the proportion of customers with a high potential value per branch.

This analysis can be as extensive as the user considers appropriate and will be done in order to present analysis tools that can approximate the ultimate goal of customer value analysis, that is, to find those segments or characteristics that define the most valuable customers.

IV. Tariff Optimization Models

Once the potential value of each customer has been determined, it is possible to cluster and optimize the premium for each customer segment.

Based on bibliographic references, the premium optimization model was defined in order to estimate the optimal renewal increment that should be applied to each customer or predefined cluster of customers. This increase will be applied to the renewal premium associated with each client; i.e., the net of discounts or surcharges for claims factors or commercial considerations prior to analysis.

To calculate the optimum increase it is necessary to account for two relevant factors:

- Margin, defined as the expected net premium of expected claims, commissions and associated expenses.
- Retention rate of the client portfolio

The objective of the mathematical optimization program is to maximize the margin, subject to two constraints:

- The portfolio retention rate should be maintained above a predefined value.
- The optimal increment for each customer (or group of customers) must be kept within a predefined range. The minimum limit and the maximum limit of the premium increase rate range for each customer segment is a parameter determined by the Management.

The optimization program that executes the proposed algorithms maximizes the margin of the portfolio renewal rate, denoted as r and equivalent to the probability that the client does not renew the policy, let r be greater than a defined value and the optimal percentage increase of the premium (d) in a pre-established range:

$$Max\ \pi_i = f(r_i, d_i)$$
$$\text{s.a. } r_i \geq \bar{r}$$
$$d_{min} \leq d_i \leq d_{max}$$
$$\pi_i = premium_{t,i}(1 + d_i) - E(losses) - E(expenses\ and\ comissions\ )$$

The designed algorithms seek to maximize the margin, subject to retention of the portfolio and to the minimum and maximum limits of premium increase. The final result of the process will be the optimal increase to be charged on the next renewal to each car insurance customer.

The optimal premium results from adjusting the premium offered by the optimal margin. This optimal increase will be the one that the commercial or subscription area will offer in the renewal of the *i*th client to offer to each client.

The optimization algorithm proposed in this exercise is a static optimization application by simulation. For technical details, see Robert and Casella (2010) Chapter 5.

The first step of the premium optimization algorithm is to estimate the demand function, which will estimate the price elasticity of the customer portfolio analysed. The demand function corresponds to a GAM-LOGIT[2] model that estimates the probability that the customer renews with the company, according to relevant explanatory variables. It is mandatory that one of these explanations be the premium issued to be paid by the client.

The GAM models correspond to an extension of the GLM models. A GLM model is defined by specifying two components: the response function that must be a member of the exponential distribution and the link function that describes how the mean response is related and a linear combination of predictors.

In a GLM the distribution of the variable *y* belongs to the exponential family of distributions that takes the following general form:

$$f(y|\theta,\phi) = exp\left[\frac{y\theta - b(\theta)}{a(\phi)} + c(y,\phi)\right]$$

In this equation, θ is the canonical parameter and represents the location, while ϕ is the scatter parameter and represents the scale. Exponential family distributions have mean and variance as follows:

$$E(y) = \mu = b'(\theta)$$
$$var(y) = b''(\theta)a(\phi)$$

[2] See details in Jong and Zeller (2008), and Wood (2012).

The mean is a function that depends only on θ while the variance depends on the product of two functions, localization and scale ones.

The link function g, describes how the average response $E(y) = \mu$ to the covariables is linked through the linear predictor:

$$\eta = g(\mu)$$

We can express the effect of predictors on the response through a linear predictor of the form:

$$\eta = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p = x^T \beta$$

Some link functions and its family function are as follows:

Family Function	Link	Variance function
Normal	$\eta = \mu$	1
Poisson	$\eta = \log\mu$	μ
Binomial	$\eta = \log \mu/1 - \mu$	$\mu(1-\mu)$
Gama	μ^{-1}	μ^2
Inverse Gauss	μ^{-2}	μ^3

The parameters β of a GLM can be estimated using maximum likelihood, the logarithm of the maximum likelihood function for a single observation, where $a_i(\phi) = \phi/w_i$ is:

$$logL(\theta_i, \phi; y_i) = w_i \left[\frac{y_i\theta_i - b(\theta_i)}{\phi}\right] + c(y_i, \phi)$$

In this way, for independent observations, the logarithm of the likehood function will be: $\sum_i logL(\theta_i, \phi; y_i)$.

In the case of the GAM model, a non-parametric approach is applied, estimated as:

$$y = f(x_1, \dots, x_p) + \varepsilon$$

Once the demand function (or demand functions by client cluster) is estimated, the proposed static optimization algorithm is applied, so that each customer or segment of customers will have an increase in their renewal premium rate considered optimal, given the above considerations.

The optimization algorithm by simulation can be constructed such that it is correlated with the company's probability of renewal; the optimal premium increase is greater when the client's turnover probability is higher (otherwise, the average probability of the client cluster, according on model variation). In this way, customers with lower price sensitivity will have a greater increase in their premium.

However, it is possible that the optimization algorithm allocates the optimum rate increase differentially for each segment, keeping the renewal rate constant between segments. The utility of this second alternative is that the user may consider it relevant to analyse different classifications of the client portfolio. For example, to create client cluster according to their Potential Value, the first segment incorporates those customers with Lower Potential Value and the last segment those of Higher Potential Value. By applying this algorithm, the impact of applying differentiated premium increases is quantifiable with a portfolio characteristic, or in general, by a desired user characteristic.

From a theoretical viewpoint, the optimization model seeks to affect the retention rate of a client portfolio.

In general terms, the optimization algorithm starts from generating random numbers (for each individual or each cluster) with the distribution chosen by the user (by default, uniform) between the lower limit and upper limit of the defined premium increase ranges.

With the *i*th iteration of premium growth rates, the proposed renewal premium is estimated, when this is used the estimated demand function is evaluated. The result of evaluating the demand function will be the probability of renewal for each individual in the given segment.

If the probability of the portfolio renewal or that of the determined estimated clusters by the demand function is lower than that required by the model, the algorithm passes to the next iteration. In case that the renewal rate exceeds the required threshold, this renewal premium evaluates the objective function, which is the sum of the margin of all policyholders in all segments of the insurance portfolio.

This procedure is repeated *n* times. The optimal renewal premium results from multiplying the current premium by the optimal increased premium, such that it generates the highest margin value of all iterations of the algorithm. (For computational details of the algorithm see Robert and Casella (2010), Chapter 5).

Finally, the user could assign a retention rate, which will be used as a constraint of the optimization algorithm. In particular, the proposed algorithm evaluates in each replicate the increment assigned to clients. If, when the optimal increment is applied, the retention rate is less than the predefined value, this increment vector is rejected and a new increment vector is evaluated.

If the user does not choose a target retention rate, the algorithm will take as an evaluation threshold the estimated retention rate as the average of the probability of renewing the policy for the set of clients in the analysed database.

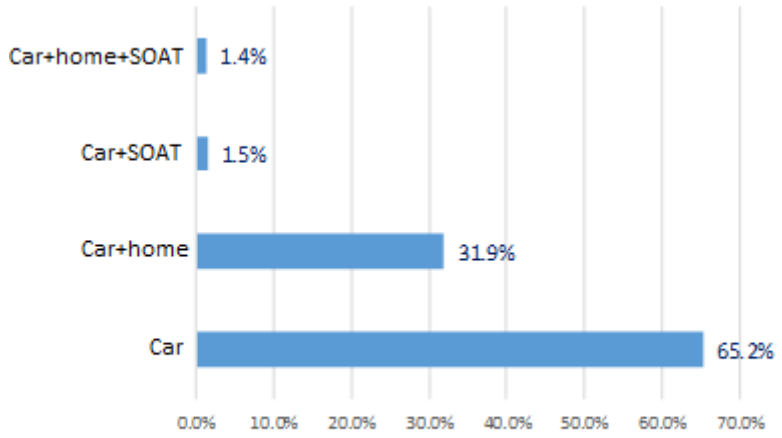
When using the algorithm of optimization for determined segments, the same optimum increase will be assigned for the clients that belong to the segment, while different optimum increments will be assigned for different customer segments.

V. Application of the Methodology

Theoretically, the potential customer value model seeks to find those customer groups, or which characteristics of these clients can be used to identify clusters, where there is greater average potential value.

A portfolio of 57,246 clients of car insurers, of which some had household products or compulsory civil liability insurance (SOAT), was taken in the following proportion[3]:

Chart 2. Proportion of Insurance Products Acquired in Client Portfolio



The result of this section is the estimation of the potential value per client insured. Initially, the customer retention function was estimated by means of a survival function with information on car customers (current and cancelled) for the last 7 years.

[3] Some of the variables that are included in the built database have vehicle class, vehicle brand, customer assigned broker, bonus-malus discount assigned, branch or city, insured's gender, number of years in the company, among other categories.

Table 1. Customer Survival Function

Interval		Beg. Total	Deaths	Lost	Survival
0	1	57246	5161	119	0.9098
1	2	51966	15743	6092	0.6170
2	3	30131	7592	3410	0.4522
3	4	19129	4225	2672	0.3448
4	5	12232	2245	2901	0.2730
5	6	7086	1032	1744	0.2277
6	7	4310	160	4150	0.2114

The second step was to estimate the probability of purchasing additional products to car insurance using a LOGIT MULTINOMIAL model.

Table 2. Estimated Odds of Cross Selling

CATEGORY	P(Y=1/X)	P(Y=2/X)	P(Y=3/X)	P(Y=4/X)
1	77%	1%	22%	1%
2	47%	21%	20%	12%
3	45%	1%	53%	2%
4	26%	11%	39%	24%

Graph 3. Average Probability of Cross-Selling by Broker

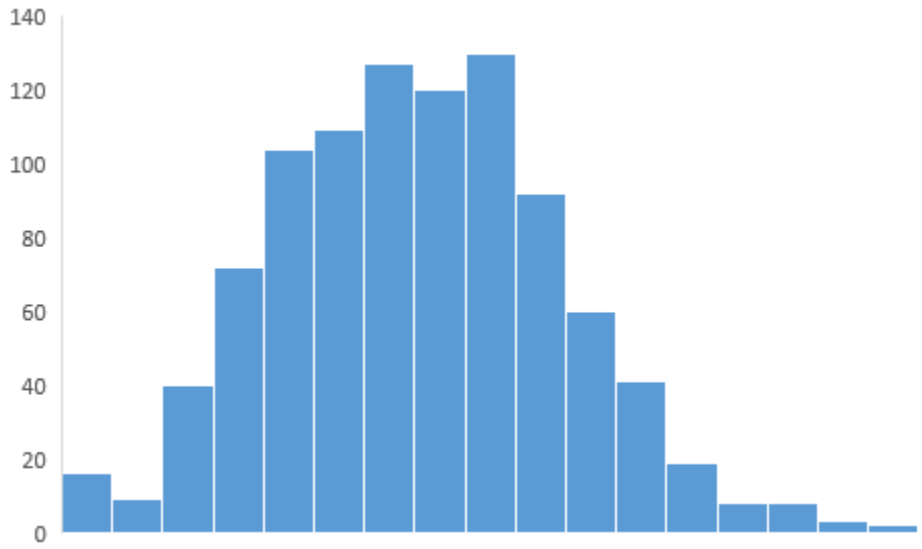
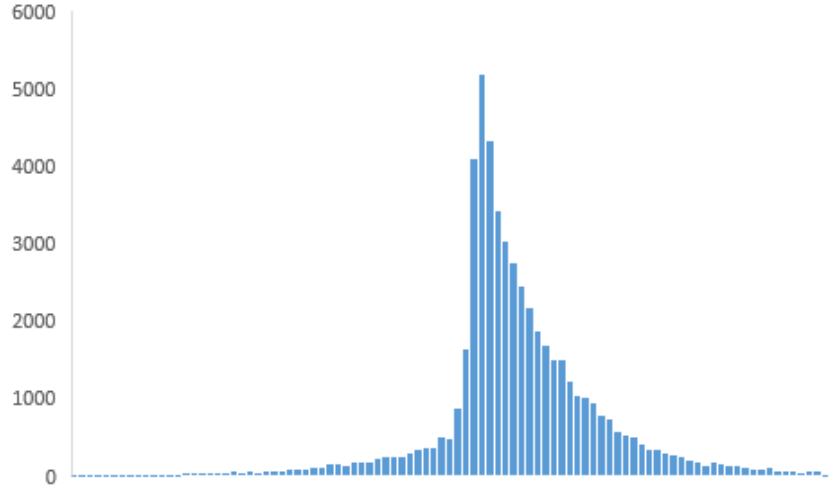


Table 3. Average Probability of Cross Selling - By Vehicle Mark

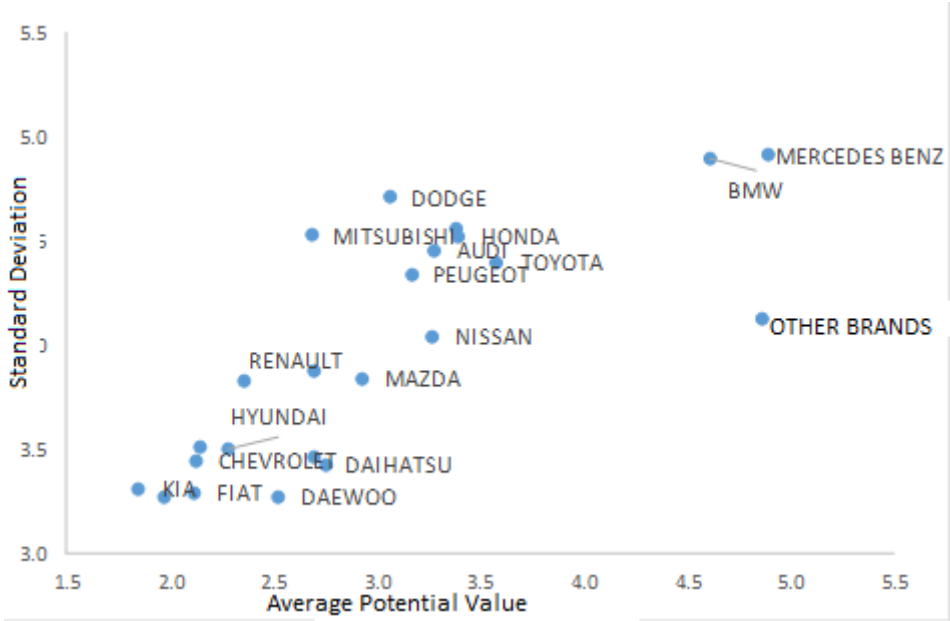
BRAND	P(Y=1/X)	P(Y=2/X)	P(Y=3/X)	P(Y=4/X)
MERCEDES BENZ	44%	2%	48%	5%
BMW	45%	2%	48%	5%
NISSAN	46%	2%	49%	4%
CITROEN	50%	1%	44%	4%
HONDA	50%	1%	44%	4%
TOYOTA	50%	1%	44%	4%
AUDI	51%	1%	45%	3%
SUBARU	52%	3%	39%	6%
MITSUBISHI	54%	3%	40%	3%
DAIHATSU	55%	1%	42%	2%
MAZDA	56%	1%	40%	3%
FORD	58%	1%	39%	2%
PEUGEOT	58%	1%	38%	3%
DAEWOO	59%	2%	35%	4%
RENAULT	59%	2%	37%	3%
VOLKSWAGEN	60%	1%	36%	3%
FIAT	60%	1%	37%	2%
HYUNDAI	60%	1%	36%	2%
SKODA	63%	2%	34%	2%
CHEVROLET	64%	1%	32%	2%
KIA	66%	1%	30%	2%
DODGE	67%	1%	31%	1%
OTHER BRANDS	74%	1%	25%	0%

The third step was to estimate the potential value for each current customer. Once the customer's value was estimated, they were clustered into groups of high, medium and low potential value.

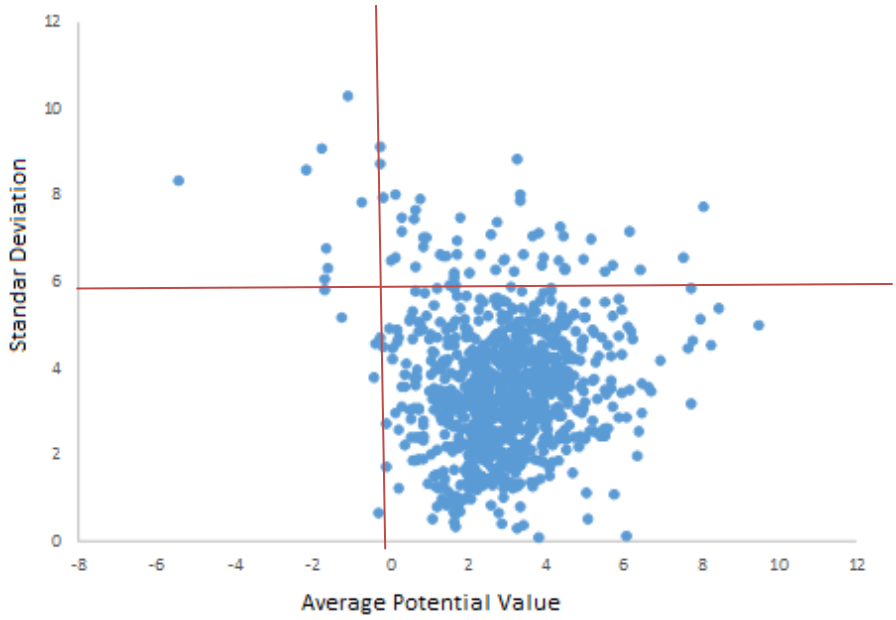
Graph 4. Customer Potential Value Histogram



Graph 5. Average and Standard Deviation of Potential Value of Customers - By Vehicle Brand



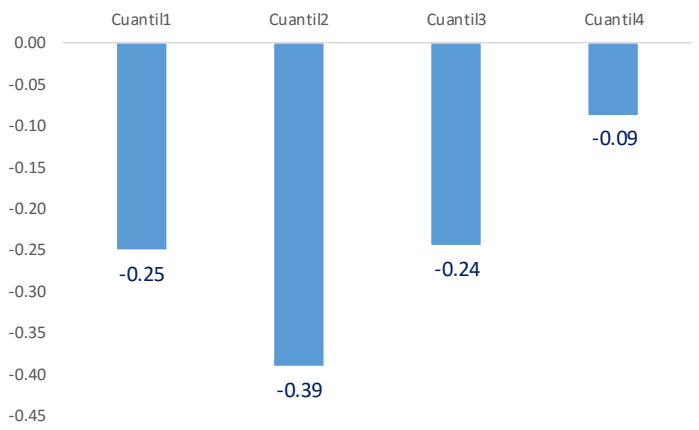
Graph 6. Average and Standard Deviation of Potential Customer Value - By Broker



The clients were separated into four clusters of analysis, according to the quartiles of the empirical distribution of the estimated potential value. When estimating the demand function using a GAM-LOGIT model, where the dependent variable is 1: if the customer is in effect and 0: if the customer has cancelled the policy in the last 24 months, and one of the independent variables is the premium of his or her last renewal.

The results show that the cluster of customers with the highest potential value proved to be those with lower price elasticity of demand function, while the segment of customers in the second quartile of potential value proved to be the ones with the highest elasticity.

Graph 7. Estimated Elasticity of Retention (GAM-LOGIT Model Coefficient)



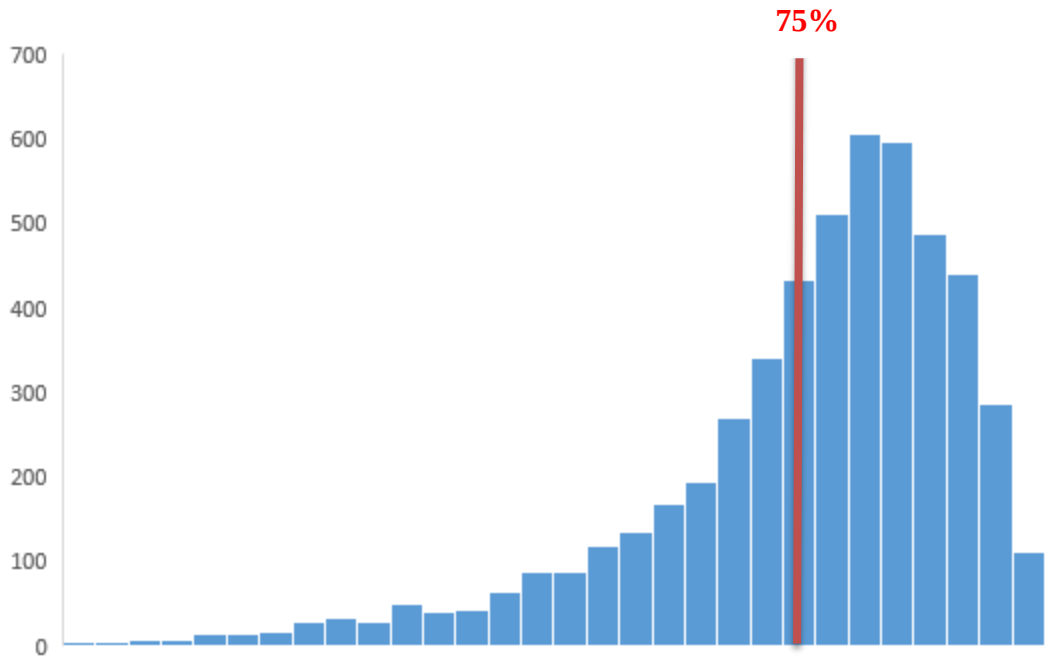
Finally, the rate optimization algorithm was applied for each segment of potential customer value, based on the following parameters:

The applied algorithm allowed estimates of the optimal increase for each customer cluster. The algorithm was restricted to generate optimal increments between 3% and 13% and a renewal rate that was at least 75%. The optimization algorithm by simulation was constructed to allocate the largest increase in the renewal premium to customers of lower potential value (first quartile).

Table 4. Optimum Rate Increase by Clients Cluster

		Probability of renewal	
Segment	Optimal Increase	Average	Standard Deviation
Quartile 1	12.4%	0.76	0.16
Quartile 2	9.1%	0.75	0.16
Quartile 3	8.2%	0.76	0.16
Quartile 4	5.8%	0.76	0.17

Figure 8. Probability of renewal - Optimized customer portfolio



By construction, the retention rate is maintained constant in customer clusters and the optimal growth in the premium is decreasing for the different clusters chosen.

VI. CONCLUSIONS

Models of client potential value estimation and rate optimization are becoming standard models in car insurance pricing. Based on the applications described, a methodology for estimating the potential value of customers and premium optimization is presented. The methodology accounts for the likelihood that customers will maintain the current relationship with the company and the cases where there are cross-selling options and, through the estimation of a demand function for different client clusters, estimates the optimal increase of the renewal premium issued for these clusters.

Through simulation optimization tools it is possible to generate different optimization algorithms for client clusters that are correlated with the potential value of the customer or that maintain the expected renewal rate of the portfolio under analysis.

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