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< Construction of a prospective life table based on the Algerian retired population mortality experience>

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1 Introduction

Life expectancy is still improving in developing countries; this improvement is almost different by sub-population. Mortality of the retirees is often lower compared to that of the global population. The use of dynamic life-tables based on global population data might distort all calculations when used for life annuities reserving and pension plan managing. The use of life tables adapted to the retirees' mortality experience is more suitable for this issue. Usually, the data of the insured population is not available for a long period allowing to do a robust forecast. Also, the fact that data is issue from reduced population simple compared to the global population can leads to important irregularities related to the reduced population at risk. In such a case, the direct use of the prospective mortality models such Lee-Carter (Lee and Carter, 1992) or Cairns-Blake-Dowd models (Cairns et al., 2006) to predict the future mortality trends is not practical at all. For this, some methods were proposed to consider the particularities of the insured population mortality while ensuring a good fitting quality and a strong forecasting capacity. These methods aim to position the experience life table to an external reference (Planchet, 2005; 2006). The main idea was to define a relationship regression between the specific death rates and the reference death rates. This process is principally based on the Brass Logit system (Brass, 1971). The use of the reference life table to estimate mortality schemes starting from incomplete or imperfect mortality data has become a common practice for experience life-tables construction both in developed and developing countries. Kamega (2011) and Kamega et Planchet (2011) used the same approach to estimate actuarial life tables for some sub-Saharan African countries with taking the French life tables as an external reference (TGH05 and TGF05). The main objective for the present work is to construct a prospective life table based on the mortality data of the Algerian retired population. The data is available for ten years (2004 to 2013) and for the ages [45, 95[arranged by five- age intervals. This data concerns the observed number of deaths and the survivals number by the end of each year of the observation period. In Flici (2015), we constructed a prospective life-table based on the global population mortality data. The length of the observed data allows doing a strong forecast. However, the forecasting results lack some coherence regarding the male-female coherence. For this, we suggested the use of coherent mortality models. In the present work, the global population mortality will be used as a reference to position and forecast the experience mortality.

2 brief presentation of the main formulas in the Algerian pension system

The Algerian public pension system for employees is managed by "La Caisse Nationale des Retraites CNR[1]. The pension benefits provided by CNR can be divided into two main categories: Direct Pension Benefits (DPB) and Survivors Pension Benefits (SPB).

The direct pensions feature 5 retirement formulas: Normal Retirement (NR); Early Retirement (ER); Partial Retirement (PR); Retirement without Age Condition (RAC) and Retirement Allowance (RA).

The classical retirement formula is the Normal Retirement. The regulatory age for retirement is 60 years. 32 years of contribution allow to benefit a replacement rate of 80% (full retirement) of the final wage. Some categories can benefit a retirement age bonus. Women can get retired at 55 years plus a bonus of 1 year for each birth with 3 in maxima. Other categories can also benefit some bonus: Moudjahidhine[2]; workers whose have been totally and permanently disabled in case if they cannot benefit a disability pension and having accumulated a minimum of 15 years of contributions; employees have been working in some particular nuisance conditions can, as the cases cited above, get retired at the age of 55.

Retirement Allowance: Retirement allowance is intended to the same categories like in the Normal Retirement but only when some conditions are not satisfied. However, 5 years of contributions are required. Starting from 1999, the legal age for this retirement category has passed from 65 to 60 years old.

The Early Retirement, proportional retirement and retirement without age conditions was introduced during 1990's.

Early Retirement: To benefit this formula, some conditions are required: being aged 50 and over (45 for women), having at least 20 years (15 for women) of contribution and have being working for the same employer for 3 years without interruption during the 10 last years. In addition, the employer must pay the contributions related to the anticipation period.

Retirement without age constraint: The only condition is having accumulated 32 years of contribution.

Proportional Retirement: Two principal conditions are required being 50 years and having accumulated 20 years of contributions (respectively 45 and 15 for women).

[1] National Pension Fund : www.CNR-dz.com
[2] Moudjahidine are the former combatants of the Algerian liberation war (1954-1962).

3 Data

The available data concern the retirees of “Caisse Nationale des Retraites - CNR”. For t = [2004, 2013], and x going from 45 years to the open age group [95 and +[, by five-ages, we have the observed number of deaths ($D_{x,x+5}^{t}$) and the survivals numbers by the end of each year ($l_{x,t}$). This data is arranged by sex but the numbers of males are greatly more important than those of females. We will give further details on this point below. This data concerns 5 different categories of direct retirement pensions: - Normal Retirement (NR), Retirement Allowance (RA), Early Retirement (ER), Proportional Retirement (PR) and Retirement without Age Condition (RAC). There is a sixth direct retirement formula (complementary retirement pension CRP) for which data is not much detailed.

The structure of the data depends on the nature of the retirement formula. For NR, the data are available starting from the age of 50/55; for RA, it is 60 years; the pension/ allowance starts to be served around the age of 40 and 55 years. Generally, women get retired 5-10 years before men in average. For this that the female’s data is relatively much available at lower ages. To insure data homogeneity, we prefer to start from the age of 50 for both sexes.

Table 1 presents the structure of retirees distributed by age, sex and retirement formula during all the period (2004 - 2013).

Table 1: Structure of the global retired population by sex and retirement formula (global: 2004 - 2013)

	NR			RA			ER			PR			RAC			
Age	Males	Females	Part. Sum	Males	Females	Part. Sum	Males	Females	Part. Sum	Males	Females	Part. Sum	Males	Females	Part. Sum	
40-44	0	0	0	0	0	0	0	0	0	0	18	18	219	50	269	
45-49	0	0	0	0	0	0	0	157	157	25 756	75 354	101 110	26 349	4 017	30 366	
50-54	84	14 180	14 264	0	0	0	3 453	1 177	4 630	602 621	114 875	717 496	189 338	23 583	212 921	
55-59	16 602	91 999	108 601	0	0	0	25 220	72	25 292	934 391	78 140	1 012 531	463 710	28 680	492 390	
60-64	1 071 454	191 028	1 262 482	241 330	28 199	269 529	588	4	592	607 297	31 785	639 082	397 623	17 659	415 282	
65-69	1 261 454	163 400	1 424 854	234 053	31 582	265 635	48	0	48	204 668	5 932	210 600	199 825	7 880	207 705	
70-74	1 202 953	133 044	1 335 997	169 740	27 891	197 631	22	0	22	36 254	1 079	37 333	55 260	2 393	57 653	
75-79	895 709	96 521	992 230	103 811	22 207	126 018	0	0	0	5	1	6	10	2	12	
80-84	541 684	60 723	602 407	60 303	17 871	78 174	0	0	0	2	0	2	4	0	4	
85-89	253 926	30 702	284 628	34 669	10 704	45 373	0	0	0	2	0	2	3	0	3	
90-94	106 789	16 336	123 125	16 436	5 314	21 750	0	0	0	5	0	5	4	0	4	
95 & +	73 028	15 951	88 979	14 539	3 605	18 144	0	0	0	12	0	12	11	2	13	Global sum
Sum	5 423 683	813 884	6 237 567	874 881	147 373	1 022 254	29 331	1 410	30 741	2 411 013	307 184	2 718 197	1 332 356	84 266	1 416 622	11 425 381

Source: CNR (2014)

We have in all more than 11 million person-years along the period 2004 - 2013, which represents an average of 1.4 million per year. The distribution of the corresponding numbers of deaths is presented in Table 2.

Table 2: Structure of global deaths by sex and retirement formula (global: 2004 - 2013)

	NR			RA			ER			PR			RAC			
Age	Males	Females	Part. Sum	Males	Females	Part. Sum	Males	Females	Part. Sum	Males	Females	Part. Sum	Males	Females	Part. Sum	
40-44	0	0	0	0	0	0	0	0	0	0	0	0	2	0	2	
45-49	0	0	0	0	0	0	0	0	0	1	110	111	66	11	77	
50-54	1	12	13	0	0	0	44	0	44	2 520	320	2 840	814	70	884	
55-59	167	414	581	0	0	0	364	0	364	6 388	289	6 677	2 894	111	3 005	
60-64	12 343	1 265	13 608	2 654	185	2 839	23	0	23	6 042	115	6 157	4 016	97	4 113	
65-69	26 022	1 724	27 746	4 104	303	4 407	0	0	0	2 789	3	2 792	2 914	34	2 948	
70-74	37 541	2 477	40 018	4 873	517	5 390	0	0	0	448	1	449	997	0	997	
75-79	43 634	2 932	46 566	4 708	577	5 285	0	0	0	0	0	0	0	0	0	
80-84	40 211	2 769	42 980	4 637	741	5 378	0	0	0	0	0	0	0	0	0	
85-89	29 559	1 973	31 532	3 447	620	4 067	0	0	0	0	0	0	0	0	0	
90-94	15 233	1 216	16 449	1 914	314	2 228	0	0	0	1	0	1	0	0	0	
95 & +	5 798	673	6 471	834	148	982	0	0	0	0	0	0	0	0	0	Global sum
	210 509	15 455	225 964	27 171	3 405	30 576	431	0	431	18 189	838	19 027	11 703	323	12 026	288 024

Source: CNR (2014)

For deaths, we observe in Table 2 that the column corresponding for females in PR is empty. That means that any death has been recorded during the observed period among the exposed population (1410) aged 45 - 65. There are two possible explanations for this. In first, this can be explained by the weakness of the observed population, or by a misreporting of deaths due to a confusion of these last between the different retirement categories.

4 Estimation of the Age Specific Death Rates m_{xt}

The objective of the present part is to estimate the mortality surface for males and females on the basis of the available data. Since the detailed age data is not available, we will estimate the mortality indicators following the five-age description. The first indicator that we can estimate in such a case is the observed death rate for age groups $(x, x+5)$ during year (t) noted $M_{x,x+5}^{t}$ which represents the number of death observed at age group $(x, x+5)$ and year (t) noted $D_{x,x+5}^{t}$ divided by the population at risk during the observation year $L_{x,x+5}^{t}$. The calculation formula can be written: $M_{x,x+5}^{t} = D_{x,x+5}^{t} / L_{x,x+5}^{t}$.

The estimation of the population at risk cannot be currently estimated in the current case. Data is not individualized but aggregated by five-age group and we cannot estimate the current exposure period during a year for each individual.

4.1 Estimation of the exposure to the death risk

In the absence of detailed information about the different dates of events occurred in the observed sample: date of entrance/exit in the sample, date of birth/death, the population at risk cannot be currently estimated. The first information allows estimating the current duration where the individuals has been exposed to the death risk and the second allows to estimate the exact age at the different events. The use of aggregated data leads to less accurate results. The estimation of the exposure to the death risk with grouped data must be done by imposing some simplest assumptions.

Since we do not have any information about the flow of individuals into/from the considered portfolio, we prefer to estimate the exposure to the death risk $L^t_{x,x+5}$ simply by the population number by the mid-year t which may be approximated by the average between the surviving population at the beginning and the end of the year by using the following formula:

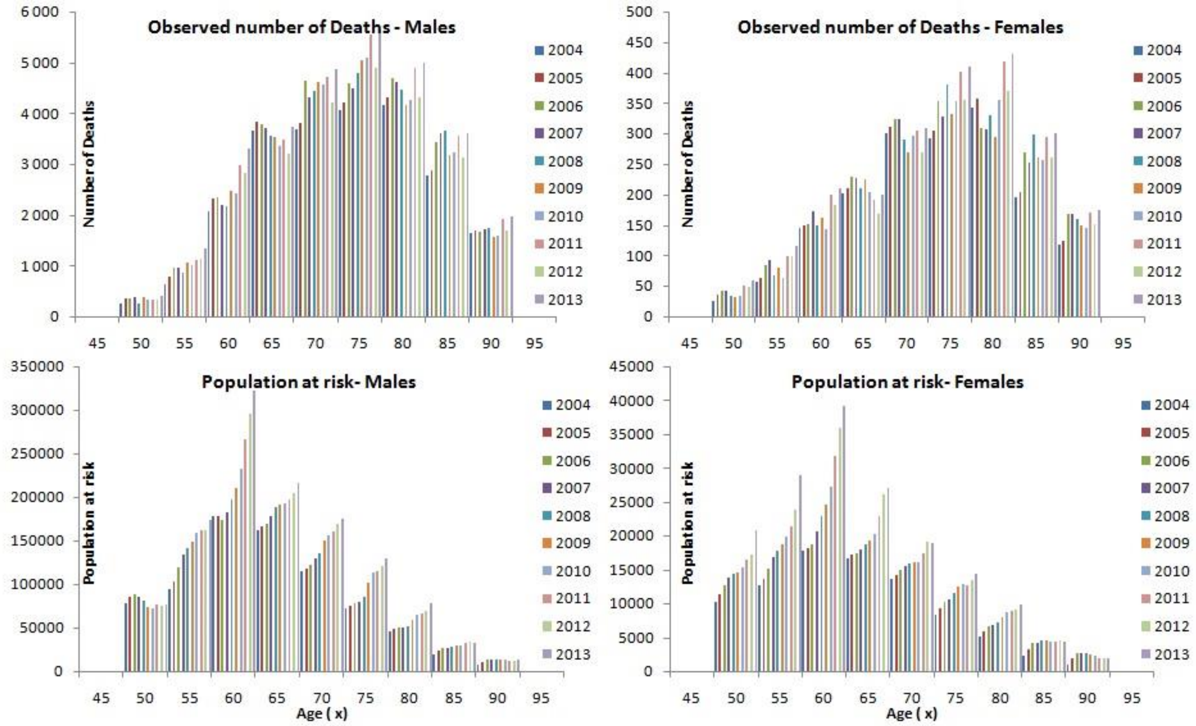
$$L^t_{x,x+5} = \frac{l^{t-1}_{x,x+5} + l^t_{x,x+5}}{2}$$

We remind that the available information is the deaths numbers for age $x \in [45-95[$ and time t = 2004, 2005, ...2013 and also the survival number by age at the end of each year of the considered period. To estimate the population at risk by five age groups in 2004, the population number at the beginning was needed. For simplification issue, we deduced it from the population number at the end of the year by assuming a similarity of the reports of population numbers at the end and the beginning in 2004 compared to 2005. The observed number of deaths and the population at risk for each age group $[x, x+5[$ and year t are represented in Figure 1.

The number of death has grown following a linear trend from 45 to 80 years for men and from 45 to 75 for women. Then, it decreases slightly beyond this age. In concern of the distribution of retirees number by age, we observe that the age category [60-65[comprises the most important number of retirees men and women. The obtained mortality surface is given by Figure 2.

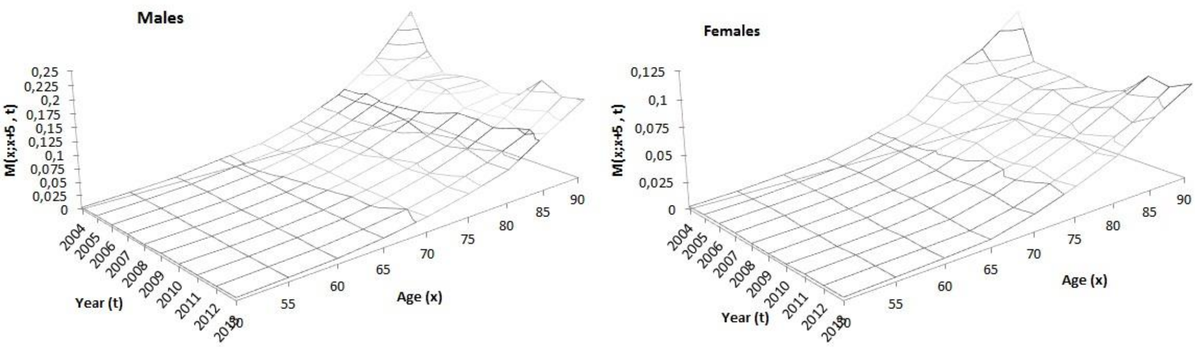
We observe that the obtained mortality surfaces show a relative stability until the age 80-85 years. Beyond this age, some apparent irregularities can be observed because of the reduction of the population exposed to the death risk.

Figure 1: Deaths and population at risk by age



Source: CNR (2014)

Figure 2 Crude mortality surfaces ($M^{t}_{x,x+5}$)



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4.2 Detailed ages mortality surfaces

The next step is to construct a detailed age life table. The idea is to suppose that the central death rate of the age group [x, x+5[noted $m_{x+2,5,t}$ to be equal to the observed death rate corresponding to the same age and time points and that we previously noted $M^t_{x,x+5}$. We can write: $m_{x+2,5,t} = M^t_{x,x+5}$. Then, we interpolate the detailed age central death rates $m_{x+0,5;t}$ for $x = 50, 51, 52, ..., 84$ by any smoothing function. The use of mortality models is more suitable to fit the curve of $m_{x+2,5}$ separately for each year t. The quadratic model seems the most suitable for this issue.

$$ln(\widehat{m}_{x+2,5,t}) = a + b(x+2,5) + c(x+2,5)^2$$

The fitting process will be oriented to minimize the weighed squared errors (WSE). The observed deaths can be used as a weight. The minimization problem can be written as follows:

$$minWSE = \sum_{x=50,55}^{75} D^t_{x,x+5} * (m_{x+2,5,t} - \hat{a} - \hat{b}(x+2,5) - \hat{c}(x+2,5)^2)$$

The fitting of the crude annual mortality curves is shown in Figure 3.

The objective of the quadratic fitting already done on the crude death rates aims in first to interpolate the single ages death rates and by the same to reduce the irregularities caused by the reduced sample size for some age categories. As a result of this fitting, the annual life tables for the years were interpolated and smoothed. However, specific death rates at each age x as a time series evolution are still marked by some fluctuations as shown in Figure 4.

Figure 3: Annual life-table fitting

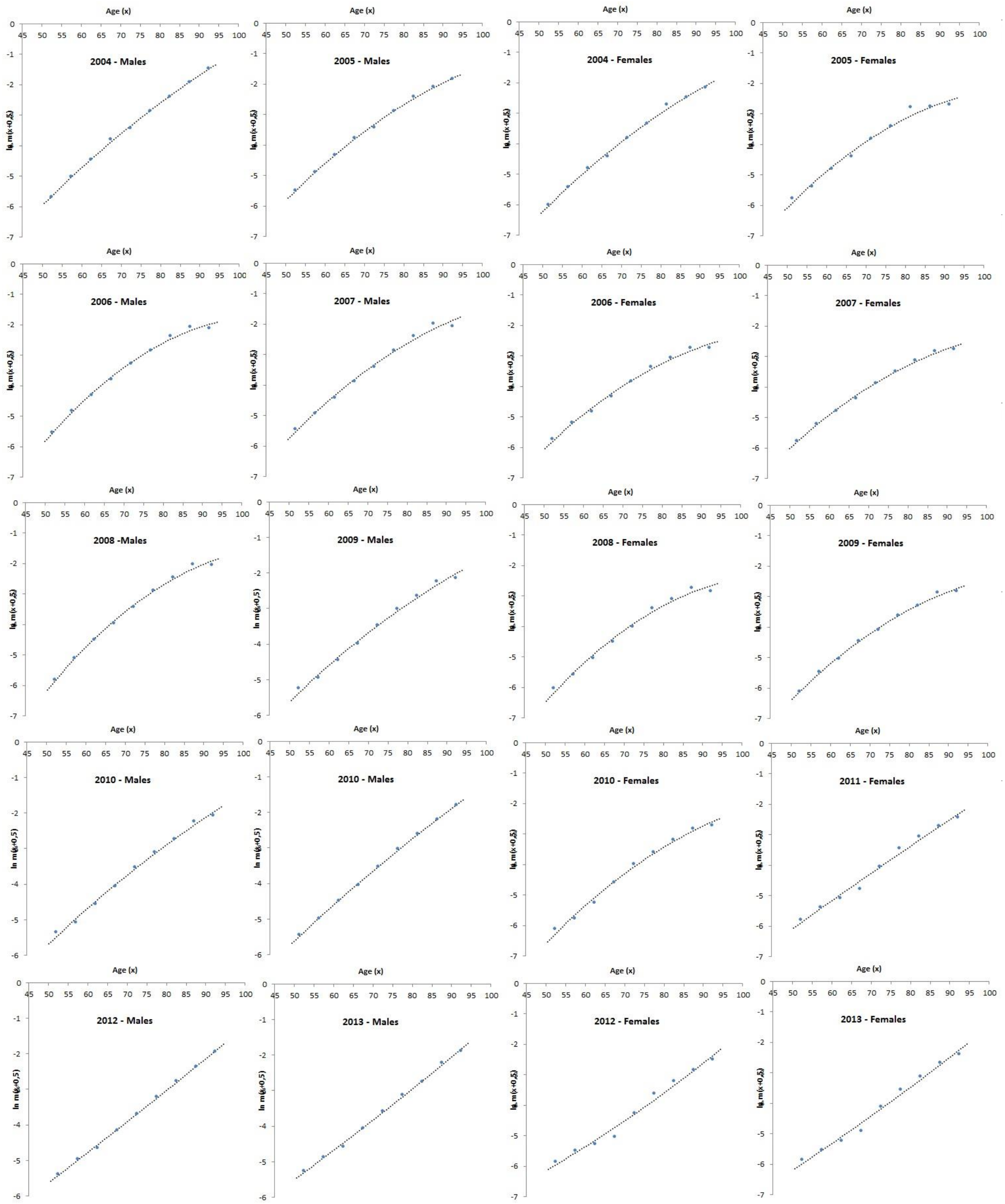
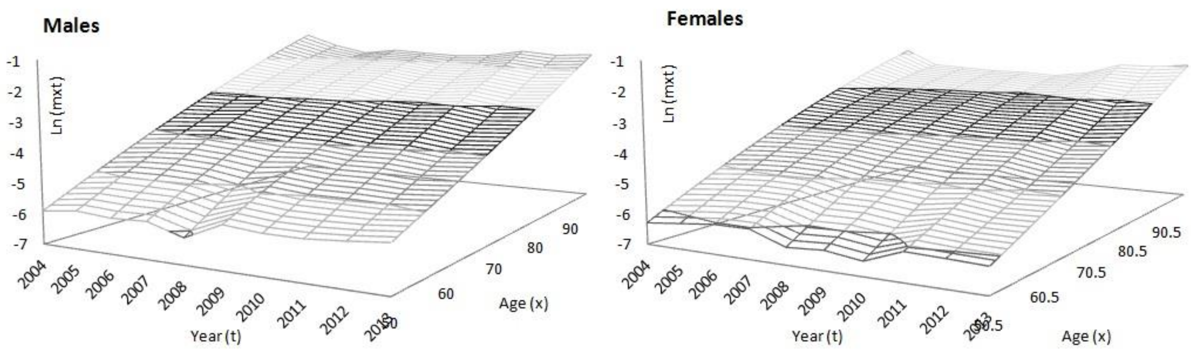


Figure 4: Detailed age mortality surface for males and females $ln(m_{xt})$



In overall, the experience mortality surfaces presented in Figure 4 do not present any kind of aberrant perturbations either in age or time series evolution.

5 External reference mortality

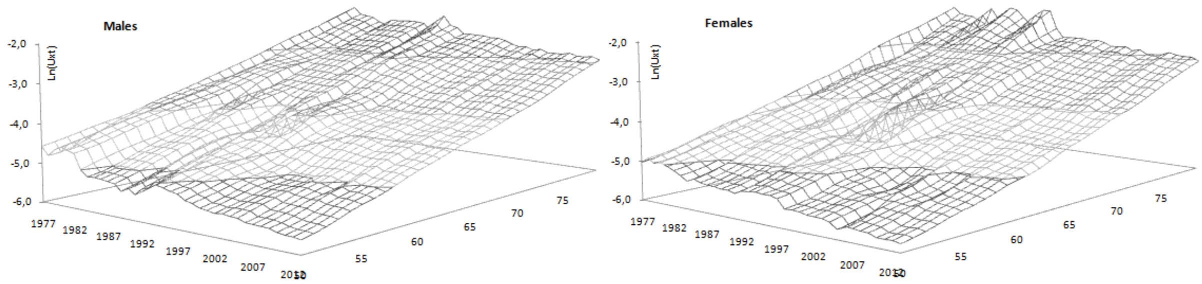
The length of the obtained mortality surface is limited and does not allow doing a robust forecast when the prospective mortality models are directly applied. For this, we need a consistent mortality reference to position the experience mortality. This positioning is traduced by a kind of a regression allowing to pass in any condition from the reference mortality rate to the experience ones corresponding to the same age x at the same years t. Here, we propose to use the Algerian global population mortality as a reference.

5.1 Historical mortality surface

The Algerian life tables for the global population are published by the Office of National Statistics (ONS) starting from 1977. Mortality rates are given by five age groups starting from 0 to around 75-80 for males, females and both sexes population. During the period prior to 1998, these life tables were not continuously published. For some years of the cited period, no life table has been published. An annual publication frequency was ensured only starting from 1998. By the same, the closure age varied between the open age group [70 and +] to [85 and +[in an irregular way. For mortality forecasting issues, the availability of a complete historical mortality surface is required. Flici (2014) estimated the missing five-age mortality rates in the Algerian historical mortality surface.

Actuarial calculations are usually based on single age's description of mortality rates. Because we are dealing with a population of pensioners, we will focus only on the mortality of the population aged 50 years and over. The mortality rates at our disposal until now need to be interpolated in order to suit the requirement of our calculations. The Karup-King method can be used to break-out the five age mortality rates into detailed age mortality rates. This interpolation method was explained in further details in Flici (2016-a). The obtained single age's mortality rates for the global population aged 50 to 80 are shown in Figure 5.

Figure 5: Single age global population mortality surfaces - ONS (1977-2014)



Source: Annual publications of ONS / Missing data: Flici (2014)/ Single age data: Interpolated by using the Karup King method.

The aim of positioning of the experience mortality on a reference mortality surface is to ensure more robustness in forecasting. The main idea is to allow deducing the projected experience mortality rates from the projected reference mortality. So, reference mortality rates must be projected. We have shown in Flici (2016-c), that the use of the prospective mortality models in an independent way does not guarantee coherent results regarding the male female mortality ratio. For this, the use of the coherent mortality models is highly recommended (Li and Lee, 2005; Hyndman et al., 2013). In Flici (2016-b), we conducted a comparison between these two models in the intention to project mortality for ages going from 0 to 80 years. It turned out from this comparison that the model proposed by Hyndman et al. (2013) leads to better results regarding the goodness-of-fit and the male female coherence. Here, we present shortly the fitting and the forecasting process proposed by Hyndman et al. (2013) well-known under The Product-Ratio Method.

5.2 The Product Ratio Method

The coherent mortality forecasting method proposed by Hyndman is based on the decomposition of the male female mortality surfaces into two new components: a joint mortality function and a differential mortality function. The joint mortality function represents a common averaged surface for both males and females. It can be calculated by the geometric average of the death rates at each year (t) and age (x) by the following formula:

$$m_{x,t}^{B} = \sqrt{m_{x,t}^{m} . m_{x,t}^{f}}$$

with B refers to Both sex population; m to males and f to females.

The differential mortality function is represented by the rooted male female mortality ratio. It can be calculated by:

$$\mathrm{R}_{x,t} = \sqrt{m_{x,t}^{m} / m_{x,t}^{f}}$$

The introducing of the root on the male female mortality ratio is supposed to facilitate the reconstruction of the fitted male and female age specific death rates by using the following formulas : $m_{x,t}^{m} = m_{x,t}^{B} . R_{x,t}$ and $m_{x,t}^{f} = m_{x,t}^{B} / R_{x,t}$.

The joint mortality surface and the differential mortality functions issued from the Algerian male and female mortality surfaces are shown in Figure 6.

Figure 6: ln($m_{x,t}^{B}$) and $R_{x,t}$ - Observed 1977-2014

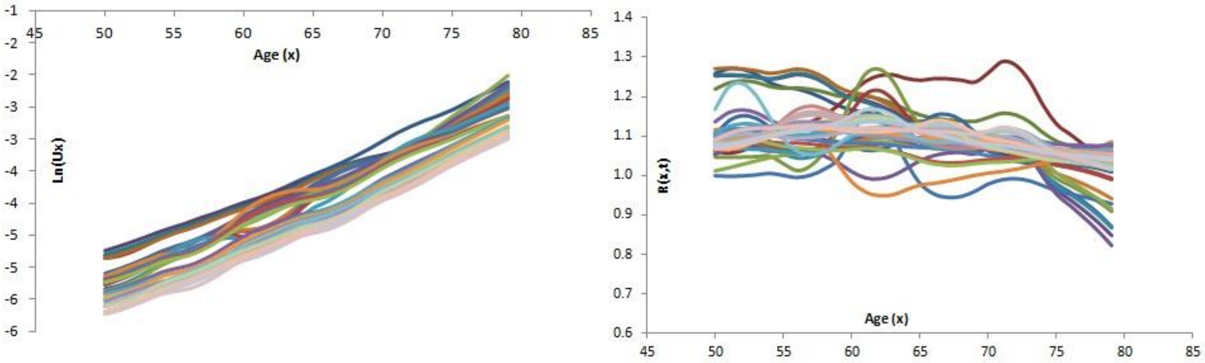


Figure left shows the annual mortality curves in log scale. Even if some differences can be easily identified when comparing different years, a relative stability is observed for the recent years [1998-2014] and an overall decrease can be observed regularly at all ages. The figure right showed the age pattern of the male female mortality ratio for the different years of the period [1977-2014].

5.3 Reference mortality coherent forecasting

The next step will be the forecasting of each on these 2 components by using the classical approach of Lee-Carter (Lee and Carter, 1992). For this, two models are to be estimated: the joint mortality surface and the differential mortality function. The log of the joint death rate at age (x) and time (t) is decomposed into three components:

$$ln(m^B_{x,t}) = \alpha_x + \beta_x * \kappa_t + \xi_{x,t}$$

The estimation process will be oriented to minimize the Weighed sum of the squared errors (WSSE):

$$MinWSSE(1) = \sum_{x=50}^{70} \sum_{t=1977}^{2014} W_{x,t}[ln(m_{x\,t}) - \hat{\alpha}_x - \hat{\beta}_x * \hat{\kappa}_t]^2$$

The weight $W_{x,t}$ is supposed to be the total deaths occurred at time t and age (x) for both males and Females. While respecting the identifiability constraints: $\sum\hat{\beta}_x = 1$ and $\sum\hat{\kappa}_t = 0$.

For the Sex ratio, we use:

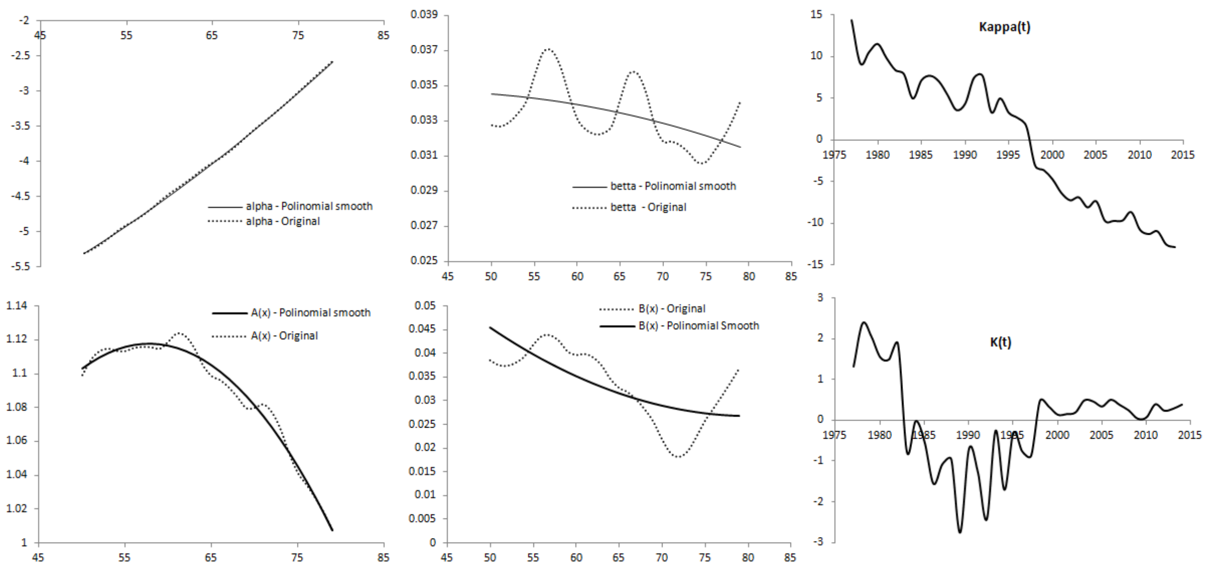
$$R_{x,t} = A_x + B_x * K_t + \varepsilon_{x,t}$$

The estimation process will be oriented to minimize the Weighed sum of the squared errors (WSSE):

$$MinWSSE(2) = \sum_{x=50}^{70} \sum_{t=1977}^{2014} W_{x,t}[ln(m_{x\,t}) - \hat{A}_x - \hat{B}_x * \hat{K}_t]^2$$

While respecting the identifiability constraints: $\sum\hat{B}_x = 1$ and $\sum\hat{K}_t = 0$. The decomposition process led to the results presented in Figure 7.

Figure 7: Coherent model parameters estimation



In order to improve the regularity of the projected mortality surfaces, the age parameters were smoothed by using 2nd order polynomial function.

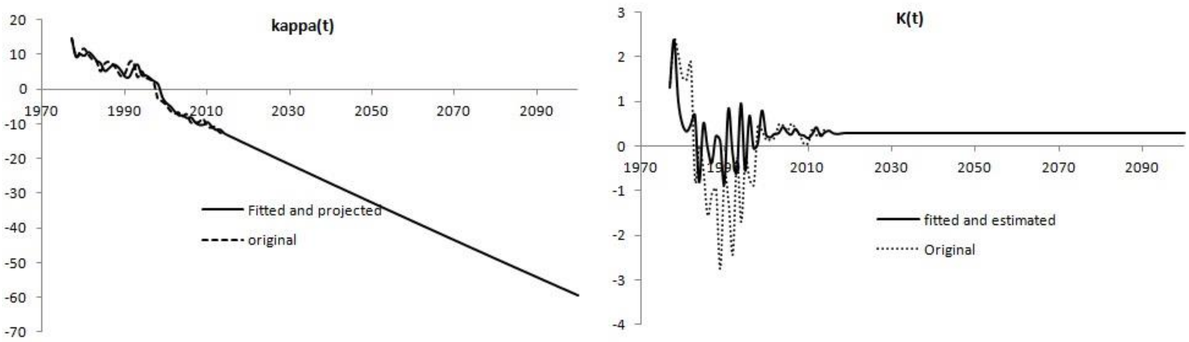
Time indexes forecasting

To forecast the mortality time index κ_t and the sex ratio variation index K_t, many time series models can be used. A general review about modeling κ_t is presented in Flici (2016-c). Here, we propose to compare three models: ARIMA(0,1,0), AR(1) and ARIMA(1,1,0). A random walk with drift is the most commonly used model to forecast mortality time index in Lee-Carter model (Lee and Carter, 1992; Li and Lee, 2005). The use of such a model requires that the historical evolution of the modeled parameter has globally a linear trend. This condition is verified in our case; except the bump observed during 90's which is due to the important mortality level caused by terrorism events that have known Algeria during 90's. The model can be written as: $\kappa_t = \kappa_{t-1} + u + \varepsilon_t$ with u as a drift and ε_t as an error term. The second model to be compared is the 1st order Auto Regressive model AR(1) with a constant which can be written as: $\kappa_t = \lambda.\kappa_{t-1} + u + \varepsilon_t$. In final, either to model κ_t itself, we model the differentiated series $\kappa_t - \kappa_{t-1}$ by using AR(1) with a drift. That leads to an ARIMA(1,1,0) for which we add a constant term and we can write: $\kappa_t - \kappa_{t-1} = \lambda.(\kappa_{t-1} - \kappa_{t-2}) + u + \varepsilon_t$ which can be simply expressed by: :$\kappa_t - \kappa_{t-1} = (1+\lambda).\kappa_{t-1} - \lambda.\kappa_{t-2} + u + \varepsilon_t$. The three models are compared on the basis of the Mean Squared Errors (MSE) and the Bayesian Information Criteria (BIC).

These three models can also be applied to forecast the Sex Ration time index (K_t). The historical series of K_t shows a high irregularity during the period prior to 1994.

The selection of the best forecasting model is not itself sufficient to guarantee a strong forecasting; the time rang to be used has also a great impact in this sense. We observed in models calibration that the suppression of the years [1992-1998] does not have any effect on the forecasting results under the three compared models. The use of the recent trend only (1998-2014) led however to a significant difference in this sense. We prefer to use the whole period as a basis for the forecasting. In adverse, the only way to obtain an acceptable result with the Sex ratio time index is the use of the recent observed trend only (1998-2014). The stability of the series K_t allows to obtain reasonable results. The final results are shown in Figure 8.

Figure 8: Time components forecast



The extrapolated times indexes κ_t and K_t combined with the age parameters allow to reconstruct the projected mortality rate and sex ratio surfaces. According to this, the projected death rates can be obtained by $ln(\widehat{m}^B_{x,t}) = \hat{\alpha}x + \hat{\beta}x * \hat{\kappa}t$ with $\hat{\alpha}x$ and $\hat{\beta}x$ the parameters of LC model estimated and fitted in the previous element and $\hat{\kappa}t$ the projected mortality time index. Similarly, the projected age specific sex ratio $\widehat{R}_{x,t}$ can be estimated by the formula: $\widehat{R}_{x,t} = \hat{A}_x + \widehat{B}_x * \widehat{K}_t$ with $\hat{A}_x$, $\widehat{B}_x$ the age parameters estimated and fitted in the previous points and $\widehat{K}_t$ the projected time index.

Then, and using these two extrapolated parameters, males and females mortality surfaces can be deduced by $m^m_{x,t} = m^B_{x,t}.R_{x,t}$ and $m^f_{x,t} = m^B_{x,t}/R_{x,t}$.

Old age mortality extrapolation

The singles mortality surfaces that we have until now are those projected until 2100 and extended only until the age of 79. To extend the death rates to the older ages, we use the Coale-Kisker model (Coale and Kisker, 1990). The Coale-Kisker method is based on a quadratic formulation of the growth of death rates beyond the age of 80, which implies a linear deceleration rate. Beyond the age of 80, passing from an age specific death rate to the next one can be done by the formula:

$$\hat{m}_x = \hat{m}_{x-1} exp(P_{80} + s.(x-80)) \quad x = 80, 81, 82, ...$$

P_{80} is the average growth rate of death rates between 65 and 80, and s is the slope of mortality growth rate deceleration which is supposed to decline following a linear trend. To estimate this slope s, authors imposed arbitrary a closure constraint: $m_{110}{}^m = 1$ and $m_{110}{}^f = 0.8$. This constraint does not imply that 110 years is defined as an ultimate surviving age as it has been done in Denuit and Goderniaux (2005) where the age of 130 years was defined as an ultimate age. The constraint imposed by Coale and Kisker (1990) meant just to orient the old age mortality trend and escape an eventual crossover between the male and female mortality curves (Flici, 2016-a).

For our application, we directly use the quadratic form of the Coale-Kisker method: $ln(m_x) = a + b(x-79) + c(x-79)^2 + \xi_x$. The model was calibrated to fit the death rates of the age range [60-79] and with imposing a constraint about the survival age limit which is supposed to be 120. This method is well detailed in Flici (2016-a). The projected mortality surface for males and females are presented in Figure 9.

The projected mortality surfaces for males and females show a high regularity. On the basis of the extrapolated mortality rates, the evolution of the life expectancy at age 50 can be deduced. Figure 10 shows a comparison between males and females and illustrate also the expected evolution of the gap in life expectancy at age 50.

Figure 9: Coherent mortality projected surfaces for males and females

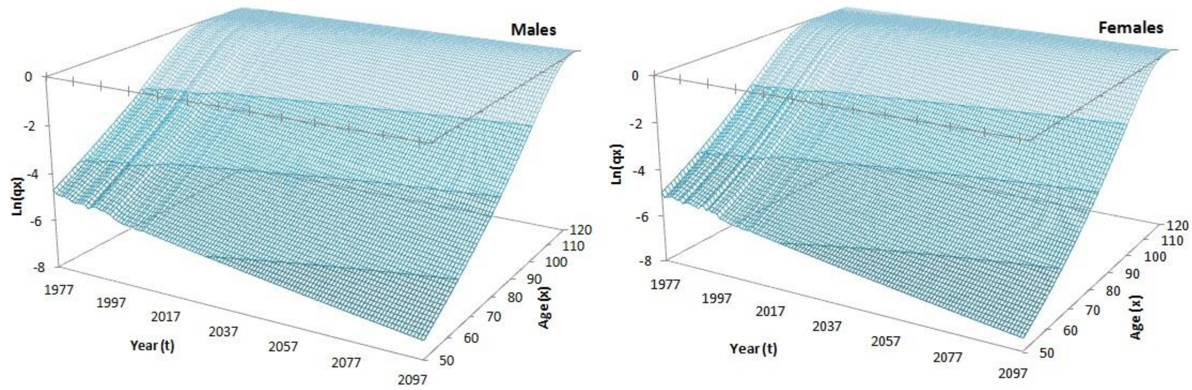
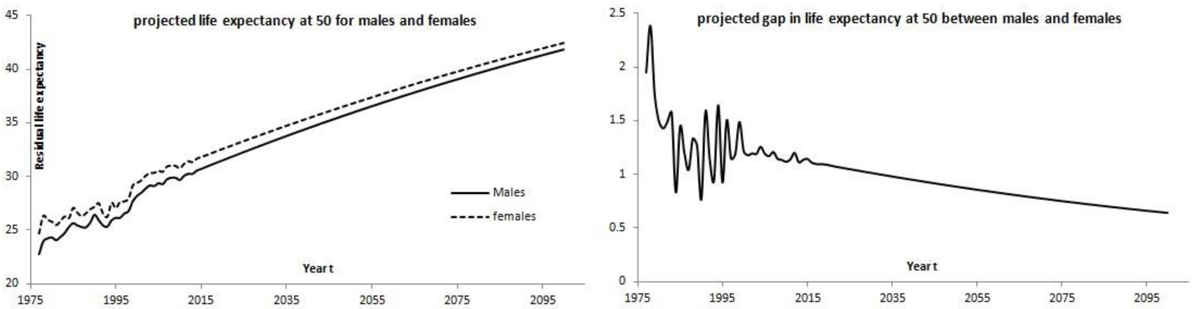


Figure 10: Projected life expectancy at 50 for the global population by sex



The residual life expectancy at age 50 for males is expected to increase from 30.5 in 2014 to 35.9 in 2050 and 41.8 by 2100. For females, it is expected to go from 31.7 in 2014 to 36.8 in 2050 and 42.5 by 2100. The gap in life expectancy between males and females will decrease from 1.14 to 0.65 year between 2014 and 2100.

6 Mortality positioning

The construction of experience life tables based on an external reference needs to define a well-known relationship between the experience mortality and the reference mortality. Then, the projected experience mortality can be deduced from the projected reference mortality.

Let $m_{x,t}^{ref}$ be the fitted Age Specific Death Rate for age x at time t for the reference population and $m_{x,t}^{exp}$ the fitted Age Specific Death Rate related to the retired population. We remind that, until now, $m_{x,t}^{ref}$ were projected until 2100. $m_{x,t}^{exp}$ are only available for the period

2004-2013. This relationship should be defined by a comparison between the mortality of the two populations for a common age range and period. Here, the common surface is defined by the age range [50-79] and the period [2004-2013]. In order to improve the goodness of the regression, a comparison will be conducted between the linear and the quadratic regression.

In first, we will test the linear regression proposed by Brass (1971). This method is based on the expression of the Logit of the experience mortality rates in function of the Logit of the reference mortality rates. This can be written as:

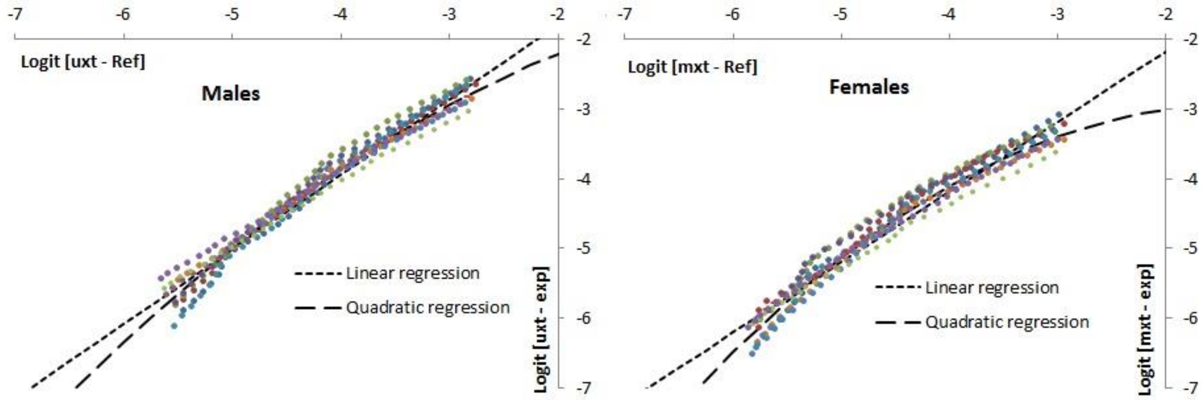
$$Logit(m_{x,t}^{exp}) = \gamma + \delta Logit(m_{x,t}^{ref}) + \zeta_{x,\,t}$$

With γ and δ to be the parameters of the linear regression, and $\zeta_{x,t}$is an error term normally distributed. The estimation process will be oriented to minimize the sum of squared errors (SSE) between the original and the predicted experience death rates:

$$MinSSE = \sum_{x=50}^{90} \sum_{t=2004}^{2013} [Logit(\widehat{m}_{x,t}^{exp}) - \gamma - \delta Logit(\widehat{m}_{x,t}^{ref})]^2$$

Figure 11 shows the obtained results.

Figure 11: Linear regression of $Logit(m_{x,t}^{exp})$ in function of $Logit(m_{x,t}^{ref})$



We observe that the linear equation does not fit well the regression of the experience mortality rates on the reference ones. The regression seems to have a quadratic shape more accentuated in the case of females. To evaluate the quality of the linear regression and its capacity to predict the experience death rates starting from the reference ones, we compare the observed deaths to those expected by the linear model. For this, the expected rates must be considered by admitting an errors threshold. That returns to the estimation of the confidence interval of the predicted death rates and the numbers of deaths.

We remind that the crude data that we used in our application concerning the experience mortality was available by 5 age groups from 50 to 95. The regression has been done for a common age range between the global and the retired populations. This age range was [50-80[. For this, the quality of the regression must be evaluated on the basis of the observed and the expected number of deaths for the age groups [50-55[, [55-60[, ... , [75-80].

If we note $D_{x,x+n}$ to be the average number of deaths observed in the age group $[x, x+n[$during the whole period 2004-2013. $L_{x,x+n}$ is the corresponding population at risk. The Crude Death rates averaged on the period [2004-2013] can be calculated as $M_{x,x+n} = D_{x,x+n}/L_{x,x+n}$. The predicted number of deaths can be calculated by : $\widehat{D}_{x,x+n} = \widehat{M}_{x,x+n}.L_{x,x+n}$ With $\widehat{M}_{x,\ x+n}$ is the expected death rates corresponding to the age group $[x, x+n[$ an calculated by the average of the Age specific mortality rates at the ages $x, x+1, x+2, ..., x+4$ by the following formula:

$$\widehat{M}_{x,x+n} = \prod_{z=x}^{x+n-1} m_z$$

The confidence bound must be calculated for 6 successive five age death rates. Here, the estimation of the confidence bound is different from the case of confidence interval estimated independently for each age and also from the case of a survival function. For this, we follow simply the methodology proposed by Planchet et Kamega (2013) in confidence bound estimation for single age death rates. Further details about the estimation process can be found in the same article. Here, we propose to adapt the proposed formula to suit the case of five-age death rates.

Construct a confidence interval at a confidence level of $1-\alpha$ for $\widehat{M}_{x,x+n}$ which is supposed to be normally distributed aims that there is a probability $1-\alpha$ that the real death rate is comprised in the confidence interval:

$$P(M_{x,x+n} \in [\widehat{M}_{x,x+n} \pm \mu_{\alpha/2}\sqrt{\widehat{M}_{x,x+n}(1-\widehat{M}_{x,x+n})\hat{L}_{x,x+n}}], x = x_1) = 1-\alpha$$

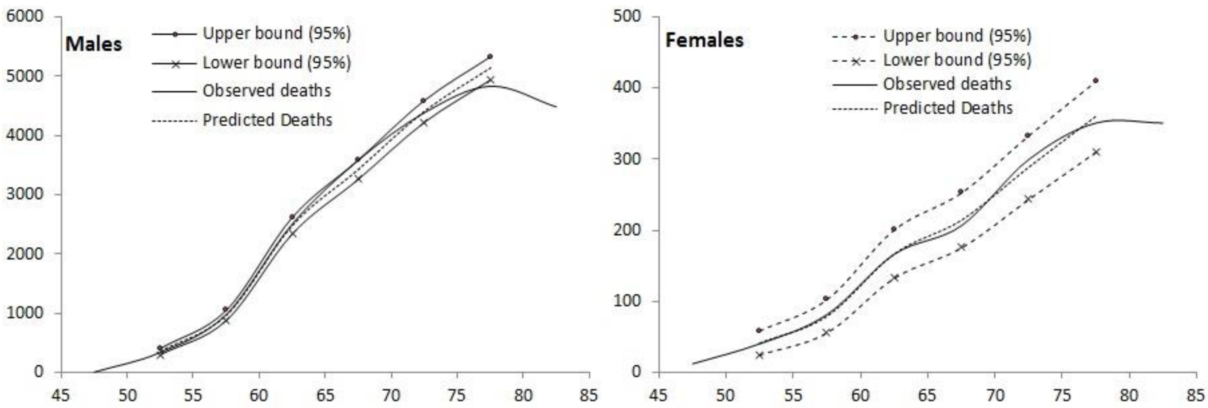
With $\mu_{\alpha/2}$ represents the $\alpha/2$ quantile of the reduced centered normal distribution.

In adverse to the confidence interval which allow to obtain a variation interval of a punctual estimates, the confidence bound tries to takes into account the probability that all the considered points will be simultaneously situated in the considered interval. According to this, if there is a probability $1-\alpha$ that $\widehat{M}_{x,x+n}$ will be situated in a well-defined confidence interval, there will be a probability $1-\beta$ that a G number of independent observations will be simultaneously situated in their respective confidence intervals with $1-\beta=(1-\alpha)^G$. The confidence bound can be then calculated by:

$$P(M_{x,x+n} \in [\widehat{M}_{x,x+n} \pm \mu_{\beta/2}\sqrt{\widehat{M}_{x,x+n}(1-\widehat{M}_{x,x+n}) * L_{x,x+n}}], x \in [x_1, x_G]) = 1-\alpha$$

The confidence bounds calculated on the number of deaths (averaged on the period 2004-2013) predicted by the linear regression are represented in Figure 12 compared to the observed numbers.

Figure 12: Confidence bounds of the number of deaths predicted by the linear regression



For females, we observe that the observed number of deaths is situated inside the confidence bound for all the age groups. For males, the predicted number of death is situated out of the confidence bound for the age groups [65-70[and [75-80[. This allows to conclude that the linear regression does not capture a sufficient part of the information about the experience mortality scheme from the reference.

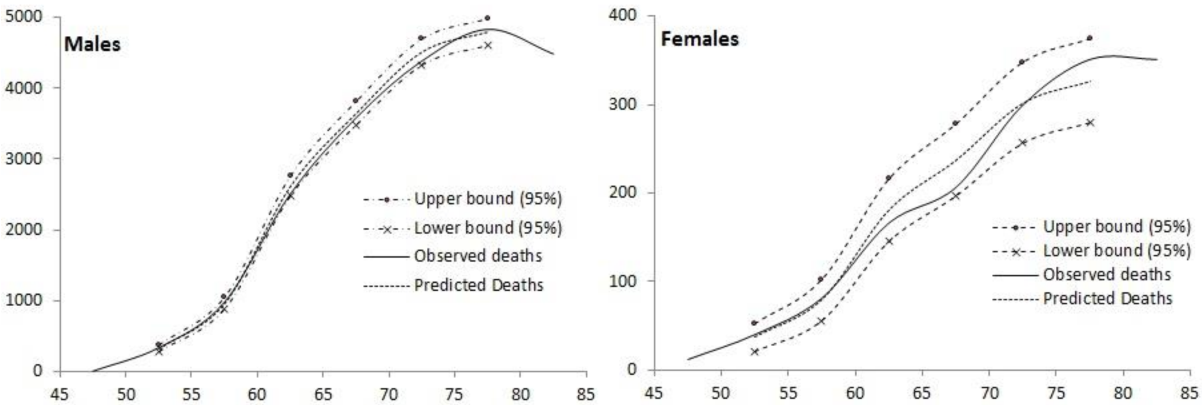
In order to reduce the part of the information loss due to the use of the linear regression, we propose to modify slightly the regression equation in order to capture the curvature of the cloud points. The regression of the experience mortality rates on the reference rates is supposed to be expressed by a 2nd degree polynomial function:

$$Logit(\widehat{m}_{x,t}^{exp}) = \eta + \chi Logit(\widehat{m}_{x,t}^{ref}) + \varphi Logit(\widehat{m}_{x,t}^{ref})^2 + \zeta_{x,t}$$

With η, χ and φ are the regression parameters and ζ an error term.

The re-calculated confidence bounds are presented in Figure 13.

Figure 13: Confidence bounds of the number of deaths predicted by the quadratic regression

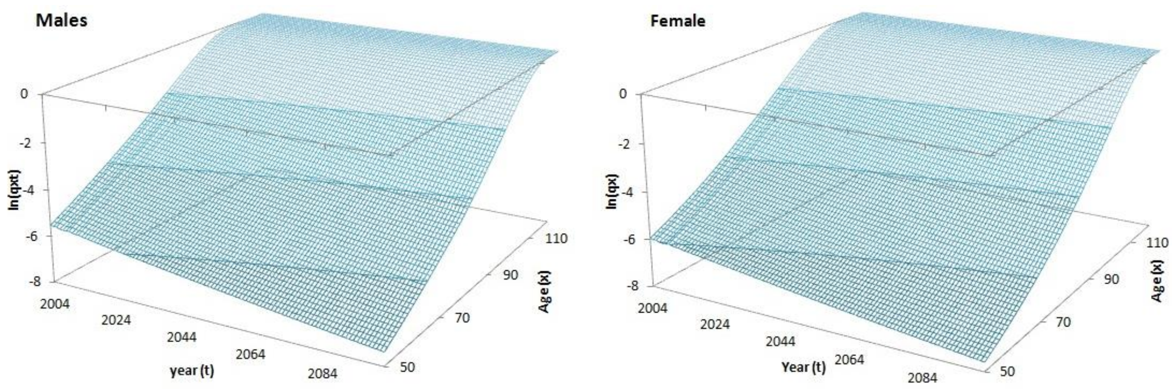


We observe that the quadratic function fits well the regression of the experience mortality rates on the reference mortality rates better than the linear regression. The observed number of deaths is situated inside the confidence bound for all ages and for males and females. The experience mortality rates will then be positioned on the reference mortality by using a quadratic regression rather than a linear regression.

Retired population dynamic life tables

On the basis of the estimated coefficients of the linear regression, the Logit of the retired population can be deduced from those of the reference population projected until 2100 for ages between 50 and 79. The age specific death rates for the ages beyond 80 are extrapolated by using the Coale-Kisker model. The extrapolated mortality surface is shown in Figure 14.

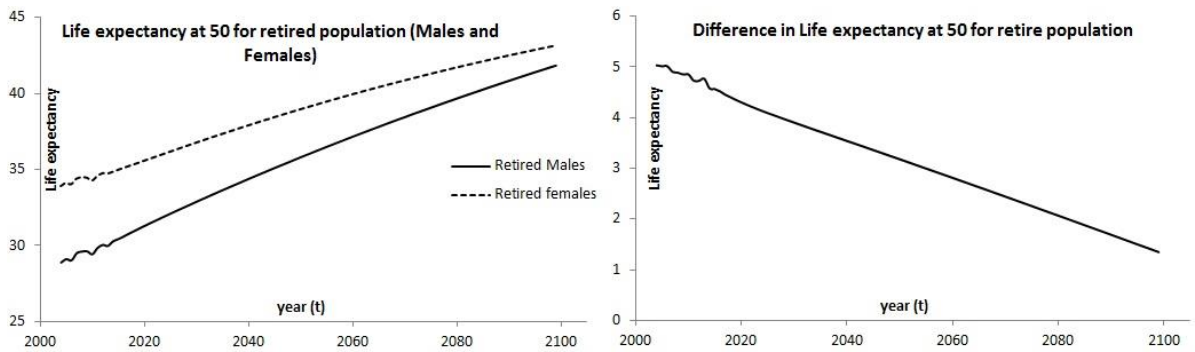
Figure 14: Experience mortality surface



The projected life expectancy at 50 years for the retired population

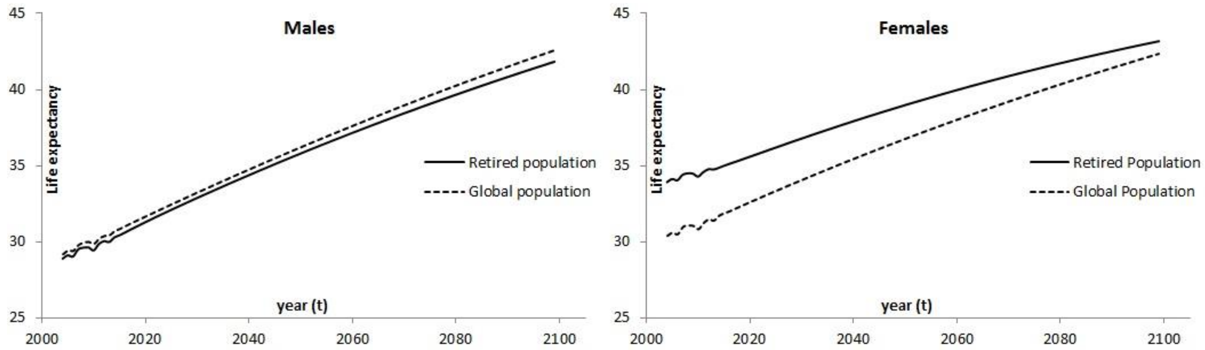
According to the forecasting results, the female life expectancy at age 50 is supposed to increase from 34.8 in 2014 to 39 in 2050 and to 40.9 in 2070. For males, this value is expected to grow from 30.4 in 2014 to 35.8 and 38.5 respectively in 2050 and 2070. The gap in life expectancy at age 50 which is at a level of 4.4 in 2014 is expected to decrease continuously until reaching a value of 2.4 by 2070. Figure 15 shows the obtained results in more details.

Figure 15: Projected life expectancy at 50 for the retired population



When we compare these results obtained on the retired population to the global population, significant differences can be observed when comparing males and females. Figure 16 gives a comparison in this sense.

Figure 16: Life expectancy at 50 - a comparison by sex



According to Figure 16, for males, there is not a significant difference between the global and the retired population in terms of life expectancy at age 50. The average gap is around 5 months during the whole observation period in favor of the global population. For females, the life expectancy of the retired population is 3.16 years higher than the life expectancy of the global population in 2014. This gap is expected to decrease in time and it will pass to 2.2 in 2050 and 1.7 by 2070.

There are two possible explanations for this last finding. The first explanation assumes that working conditions have an effect only on the female population. For men, life conditions are not different for the insured compared to the rest of the population. The insured and non-insured, the working and non-working men are approximately exposed to a similar life conditions. In adverse, working women have a better life and healthy and un-risky behavior than women of the rest of the population. Compared to men, working women are more concentrated in un-risky activity sectors (services & administration) (ONS, 2014). So, work gives women a particular life style and social appartenance and better life conditions compared to working men and women of the rest of the population. The second alternative supports the assumption that the ONS underestimates the life expectancy of the global female population. This second assumption is supported by the results obtained by Flici et Hammouda (2016). When analyzing the data issued from the health survey MICS-2012 which focused on the period [2008-2012], they found a gap of 3.9 years between males and females in the global population. This result is near to the gap and the life expectancy obtained on the retired population. That lets suppose that the retired population (men and women) does not presents any particularity in term of residual life expectancy compared to the global population, and the difference mentioned above resulted from a defection of the ONS estimates in concern of the female life expectancy. To affirm such assumption, we need to compare some additional sources of data (insurance companies, social security, self-employed pension regime) and to conduct a deep investigation in the methodology and the crude data used by the ONS to estimate female mortality.

7 Conclusion

When the mortality scheme of the insured population is different from the mortality of the rest of the population, it is recommended to adapt the actuarial life tables to the experience of the concerned population. Usually, the mortality experience data is available for a short observation periods and that reduces the robustness of the projections. Also, the fact that the experience life tables are based on a reduced population compared to the size of the global population leads to important fluctuations regarding the estimated rates. To ensure a good forecasting capacity while taking the retirees mortality experience into account, the experience mortality must be positioned on a reference allowing to do a robust forecast. A regression function must be calculated to allow deducing the experience death rates from the reference rates (Planchet, 2005; 2006, Thomas et Planchet, 2014). In this sense, the Brass logit system (Brass, 1971) is used to do such positioning.

Our objective in this chapter was to construct a prospective life tables adapted to the mortality experience of the Algerian retired population. For this, we used the mortality data issued from the portfolio of direct retirement of the Algerian National Retirement Fund (Caisse Nationale des Retraites) observed during the period [2004-2013] separately for men and women aged 50 years and +[. Firstly, the five age mortality rates were calculated. Then, the regression equation of the experience rates on the reference rates is calculated. The reference mortality here, was supposed to be the global population mortality. The projection of the global population male and female mortality has been projected by using the coherent mortality model proposed by Hyndman and al. (2013). To improve the quality regression which is firstly supposed to be linear, we proposed to use a quadratic regression to represent the experience mortality rates in function of the reference ones. By using this regression, the experience death rates were projected in function of the projected death rates of the global population.

According to the final results, the life expectancy at 50 years of the retired population will evolve from 30.4 and 34.8 years respectively for males and females in 2014 to 35.8 and 39 years in 2050. The gap in life expectancy at 50 years is expected to fall gradually from 4.4 to 3.2 years during the same period. If we observe the difference between the global population and the retired population mortality projections by sex we observe an important similarity for males but a significant difference for males. The life expectancy at 50 for the global male population is 0.27 year higher than the life expectancy of the retired male population in 2014 which is supposed to fall to nearly 0 by 2050. In adverse, the gap is very significant in the case of female populations. The retired women hope survive 3.6 years more than women of the global population beyond the age of 50 years in 2014. This gap is supposed to fall down to a value of 2.2 in 2050.

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