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Asset Liability Management

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Abstract

The Internal Capital Adequacy Assessment Process (ICAAP), among others risks, includes the evaluation of Interest Rate Risk and Liquidity Risk. In this sense, one of the most important tools we have for their analysis is the estimation of the interest term structure. This structure reflects the impacts of some important variables of the macroeconomics, like the GDP growth and inflation.

There are several papers explaining how to build this structure such us The spline lines, with the Nelson and Siegel model and The modifications of Diebold and Le, that focus on an important value like the coefficient that governs the form of this structure.

In this paper I develop several models that change the value in terms of the way we apply different structures, such as the normal structure used in developed and stable countries and the inverted curves that may appear in emerging countries during crisis period.

Every time we are in front of a robust estimation of the interest term structure, we can use these values in order to estimate the Liquidity Risk using the Gap Duration Analysis.

Finally, to complete this paper I have introduced a simple exercise in order to observe the proper way to use several interest term structures in a normal economy and in an economy that is facing a crisis situation.

Key words: ICAAP, Dynamic Nelson and Siegel, Interest Term Structure, Liquidity Risk

The Interest Term Structure and the Liquidity Risk estimation

The Finance Area, both in financial institutions and investment projects, is essential to the analysis of the interest rate structure that is applied in order to calculate the interest rate and liquidity risks that faces the first type of entities and to determine the internal return rate for investment projects.

To build an interest rate curve, or interest rate structure in our words, we should start from observable rates structures known as spot or market rates. These rates will be the basis for the calculation of our interest rate structure.

Before moving forward, please remember that an interest rate structure is an index that shows us the evolution of what "The Market" expects in terms of inflation rate and GDP growth from the point of view of macroeconomics.

The introduction of stress testing models for the calculation of ICAAP forced entities to perform a much deeper analysis of the behavior of the interest rates. It is well known that this variable directly affects the economic capital forecasts performed by financial institutions, given that any jump in interest rates has a straight impact on their financing costs.

Spot rates, on their side, arise from the market. Based on this, it is important to determine the market, place and credit risk taker they come from when calculating an interest rate structure. As an example and in order to explain my idea, I suggest we take into consideration the interest rate structures from two very different economies: the United States of America and the Argentine Republic. In this regard, we need to look at various times throughout the economy in order to appreciate the changes experienced by the interest rate structures.

U.S. Treasury Bonds Spot Rates

As you can see in the table below, the volatility of observations in terms maturities decline as tenors, measured in months, get longer. Based on this fact, the maximum values decreased during the last century but the behavior trend reversed in the last year.

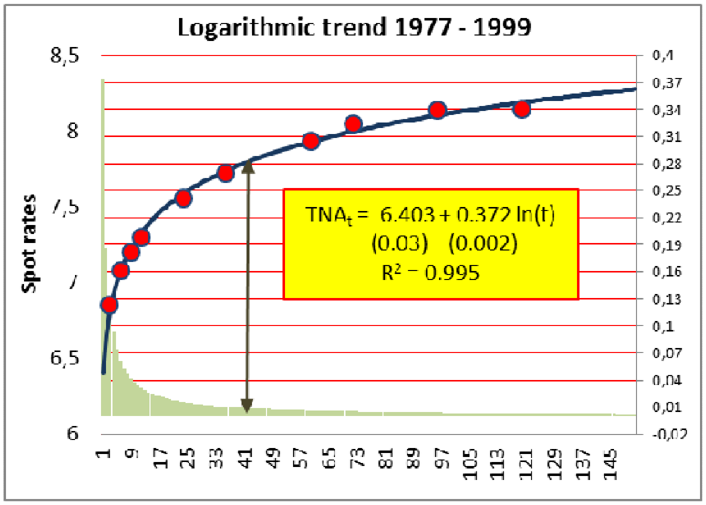
Period: 1977 - 1999					Period: 09/2015 - 06/2016				
Months	**Average**	**Minimum**	**Maximum**	**Volatility**	**Months**	**Average**	**Minimum**	**Maximum**	**Volatility**
3	6,851	2,732	16,020	2,695	**1**	0,157	0,010	0,260	0,092
6	7,079	2,891	16,481	2,702	**3**	0,196	0,020	0,310	0,103
9	7,201	2,984	16,394	2,679	**6**	0,348	0,110	0,500	0,113
12	7,302	3,107	15,822	2,602	**12**	0,498	0,260	0,650	0,105
24	7,558	3,777	15,650	2,474	**24**	0,775	0,640	0,980	0,097
36	7,724	4,204	15,765	2,375	**36**	0,996	0,790	1,280	0,139
60	7,933	4,347	15,005	2,282	**60**	1,383	1,070	1,700	0,193
72	8,047	4,384	14,979	2,259	**84**	1,708	1,330	2,040	0,223
84	8,079	4,352	14,975	2,215	**120**	1,946	1,500	2,260	0,241
96	8,142	4,433	14,936	2,201					
120	8,143	4,443	14,925	2,164					

Taking into consideration the average values of the previous observations[1], we can build an interest rate structure for the same maturities by calculating a logarithmic trend first that is then followed by a logarithmic regression. This can be expressed as:

$$\boldsymbol{TNA_t = a + b\,Ln(t)}$$

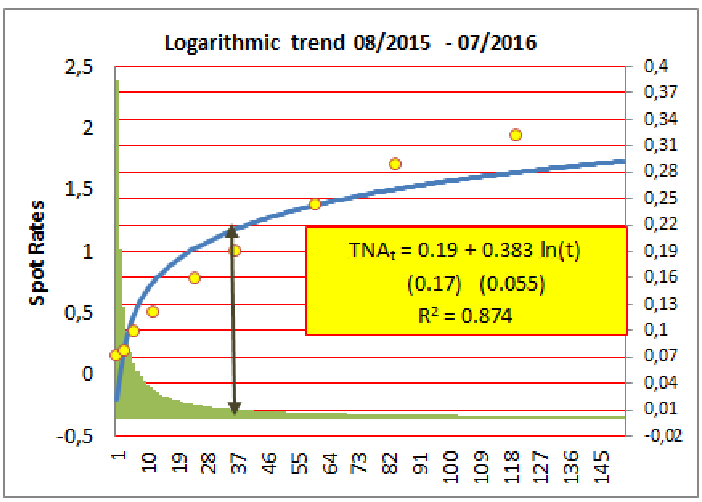
[1] The information for the period 1972-2000 has been obtained from the table reported in the working paper "Analysing the Term Structure of Interest Rates using the Dynamic Nelson-Siegel with Time-Varying Model Parameters" by Siem Jan Koopman, Max I.P. Mallee and Michel van der Wel.

Monthly information has been obtained from the website www.federalreserve.com



Observing the results from the log trend models of both series we can say that they mainly differ in the level of the equation and its dispersion.

Therefore, in the series from the past 12 months the intercept has a high probability of being null curvature or slope.



It is similar in both cases, marked with ↕, derivatives serves as an indicative of the period in which the tangent changes to values below 45°. This is important data to be used in the calculation of the DNS models.

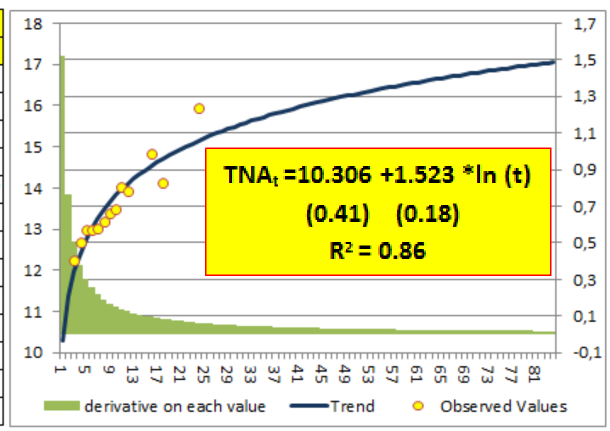
It is also important to note that the dispersion of both estimators indicate that they are not null

Argentine Republic Spot Rates

Spot Rates in Argentina's economy have a very different structure to the US or those in the European Union. As previously mentioned, from a macroeconomic point of view, the interest rate structure is an index that shows us the evolution of what "The Market" expects in terms of inflation rate and GDP growth.

In Argentina, the most updated official spot rates are the Lebac. These instruments are notes issued by the Local Central Bank (BCRA) in local currency and for several tenors. Maturities of these notes maturities are not standard and each tenor, not necessarily has a quotation every week. According to the mentioned, these instruments need to be standardized on the closest terms (e.g. 91 days tenor is considered as 90 days and so on). The BCRA's Lebac weekly auctions results in 2012 were as follows:

Period 2012				
month	interest rate	maximum	minimum	volatility
3	12,232	12,950	11,100	0,545
4	12,673	13,970	11,540	0,653
5	12,983	14,002	12,030	0,749
6	12,987	14,175	12,065	0,743
7	13,034	14,268	12,100	0,797
8	13,179	14,481	12,200	0,816
9	13,377	14,578	12,383	0,836
10	13,481	14,663	12,536	0,858
11	14,032	14,864	12,672	0,875
12	13,935	14,994	12,772	0,820
16	14,849	15,613	13,900	0,536
18	14,119	15,792	13,638	0,791
24	15,957	16,983	14,218	1,065

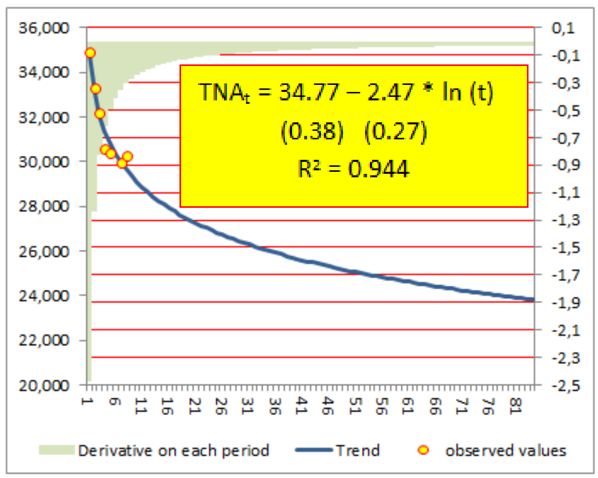


The behavior during calendar year 2014 is quite similar. The only difference is that intercept, or the level, is raised by macroeconomic effects. In this particular year, the applicable equation takes the following values:

$$TNA_t = 25.08 + 1.579 \ln(t)$$
$$(0.55) \quad (0.28)$$
$$R^2 = 0.821$$

When looking at the behavior of the Lebac during the period Dec 12th, 2015 to June 16th, 2016, due to the market expectations in terms of macroeconomics in place at that moment, is easy to see that the picture is completely different. At this point, we can observe that economy presents an interest rate structure in crisis situation. According to the definition made by Professor Van Horne[2] when referring to these structures… *"in the short term this is wrong but in the longer term, I hope that it will improve"*.

Period: 15/12/2015 al 16/06/2016				
month	interest rate	maximum	minimum	Volatility
1	34,885	30,250	38,003	3,064
2	33,290	29,249	37,100	2,496
3	32,196	28,500	37,500	2,357
4	30,602	27,600	33,250	1,631
5	30,413	27,000	37,000	2,077
7	29,976	27,250	32,250	1,692
8	30,257	27,500	35,000	1,943



The importance of the trend estimation

Once we have determined the spot rates to be used, what comes next is to concentrate on finding the interest rate structure applicable to the business cash flow. It is important to highlight that interest rate structure used in this model represents the economy expectation, while in cases of specific businesses spot rates would be the financing interest rates that companies are offered in the market. Despite the mentioned the Lebac spot rates, which can be easily accessed in the Argentinean market, in both cases represent the basis to be used in order to calculate the interest rate risks related the Company's cash flow.

[2] Financial Markets Rates & Flows, James C. Van Horne, 3rd Edition - Prentice Hall International, 1990, New Jersey

The trend is one of the statistical calculations required to infer the value of interest rates applicable to terms that are not shown by spot rates. As it has been seen in stable economies, terms available are the same over time. This is not the case in economies like the Argentine, where money supply covers one period of more when there is stability and space narrows when the uncertainty appears on the horizon.

The Nelson and Siegel spline lines and its modifications

A solution to the problems regarding trend calculation is the model proposed by Nelson and Siegel (1987). They suggest the use of approximation functions in order to model and forecast an interest rate curve. This approach is widely used in the world thanks to its convenient and parsimonious exponential approximation of three components. The BIS[3] (2005) reports that nine of thirteen Central Banks under its supervision have implemented models developed under the Nelson-Siegel's approach, or any of its amendments, for interest rates curves estimation. Nelson and Siegel models capture all the yield curve dynamics and their effectiveness has been then improved thanks to new enhancements proposed such as those outlined by Diebold and Li (2006).

The parametric model of Nelson and Siegel estimates the forward interest rate, named $R_{(t)}$ for the period *t,* by using the following model:

$$R_t = \beta_0 + \beta_1 e^{-t/\lambda_1} + \beta_2 e^{-t/\lambda_2}$$

Where B_0, β_1, β_2 are parameters to be calculated that are determined by their initial conditions, while λ_1 and λ_2 are temporary constant associated with the equation.

[3] Bank of International Settlements

In order to avoid the model to be over parameterized, the authors have proposed a solution to the differential equation which is as follows:

$$R_\tau = \beta_0 + \beta_1 e^{-\tau/\lambda} + \beta_2 \left[\left(\tau/_\lambda\right) e^{-\tau/\lambda}\right]$$

Continuing this path, Nelson and Siegel arrive to the following curve:

$$TNA_\tau = \beta_1 + (\beta_2 + \beta_3)\frac{1 - e^{\left(\frac{-\tau}{\lambda}\right)}}{\left(\frac{\tau}{\lambda}\right)} - \beta_3 e^{\left(\frac{-\tau}{\lambda}\right)}$$

Once the corresponding calculations have been made, we obtain:

$$TNA_\tau = \beta_1 + \beta_2 \frac{1 - e^{\left(\frac{-\tau}{\lambda}\right)}}{\left(\frac{\tau}{\lambda}\right)} + \beta_3 \left[\frac{1 - e^{\left(\frac{-\tau}{\lambda}\right)}}{\left(\frac{\tau}{\lambda}\right)} - e^{\left(\frac{-\tau}{\lambda}\right)}\right]$$

TNA_t (τ) denotes an interest rate at **t** time with **T** maturity. A parsimonious functional description of the yield curve is proposed by Nelson and Siegel (1987).

The formulation of Nelson-Siegel has been modified by Diebold and Li (2006) in order to reduce the coherence between the components of the yield curve. Under this new approach, for a given **t** time, the curve θ_t (τ) performs some smooth functions in order to represent the interest rate curve (yield) as a function of **T** maturity.

$$\boldsymbol{\theta_t(\tau) = \theta(\tau; \lambda; \beta_t) = \beta_{1t} + \beta_{2t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)}$$

Where $\beta_t = (\beta_{1t}, \beta_{2t}, \beta_{3t})$ and all of them apply to time t, with maturity T and λ-fixed coefficient that determines the exponential drop of the curvature of the second and third components of the above formula. In this model, the shape of the yield curve is determined by the three components and their β_t associated coefficients.

The first component takes the value 1 (constant) and can, therefore, be interpreted as a general level that equally affects both, the short and long-term interest rates.

The second component converges to one as T 0 ↓ and converges to zero as $T \to \infty$ for a period t. Based on the mentioned, this component especially influences the short-term interest rate.

The third component converges to zero as ↓ 0 T and $T \to \infty$, but is concave in T, for a given t. According to this, the component is therefore associated with medium-term interest rates.

Since the first component is the only one that is equal to one as $T \to \infty$, its corresponding coefficient β_{1t} is usually related to the long-term interest rate.

By definition the slope of the yield curve as $[\theta_t(\infty) - \theta_t(0)]$, is easy to verify that descent converge to $-\beta_{2t}$ for a given t. Finally, the form of the rate can be defined by $[(T*)\ (0)\ \theta_t - \theta_t] - [(\infty) - \theta_t\ \theta_t\ (\tau*)]$ for a medium ripening T∗, say two years, and for a given t. It can be shown that this form is equivalent to approximately β_{3t}.

If you observe a series of $\theta_t(T_i)$ interest rates for a set of N different terms T1 < ... < TN available at a given t time, the performance can be calculated by the simple regression model curve:

$$TNA_t(\lambda) = \theta_t(\tau_i) + \varepsilon_t$$

$$TNA_t(\lambda) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i}\right) + \varepsilon_{it}\quad For\ i = 1..N$$

Alterations ε_{1t} ... ε_{Nt} are supposed to be independent with zero mean and constant variance σ^2_t for a t period given. The least squares method provides estimates for the β_{jt} coefficients j = 1, 2, 3. These cross-sectional estimates can be found in the interest rates for different maturities available at the time t.

The series of regression estimates for β_t, for periods of time t = 1,..., T, appear strongly correlated with time. In other words, the coefficients are predictable and therefore the Nelson-Siegel framework can be used to forecast in this way.

This has been acknowledged by Diebold and Li (2006), who implemented the following two-step procedure:

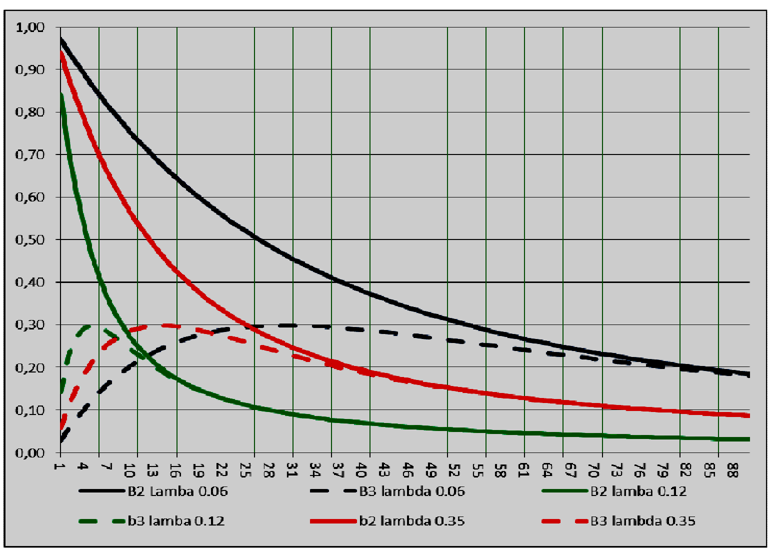
- First, estimate the β_t by using the least-squares cross section for each t;
- Secondly, estimates them as three series of time and apply methods for β_t forecasting time series and, therefore, the performance of the θ curve (τ; λ, βt).

Diebold and Li (2006) compared their two-stages forecast against univariate and multivariate time series methods. The different methods produced similar results, but the two-stages forecast approach made better predictions than the direct series of different interest rates, especially for the longer term.

- **β_1** is a variable that is independent of the time of maturity. It reflects the long-term yields that are the end point of the curve.
- **β_2** influences in the beginning of the curve (also called short end) and is weighted by a function of the time of maturity. This function is 1 if T = 0 and slightly exponential. It decreases to zero when T is large.
- **β_3** is also a T function, but this function is zero for T = 0. In this case, it decreases and then returns to zero when T grows. Therefore, the impact of β_s is the addition of a bump in the curve.

- **λ** affects the weight of β_2 and β_3 functions. Therefore, it "determines the DNS model, the load of the λ parameter determines the shape of the yield curve". In previous studies, by default was set a value for λ without estimate. For example, Diebold and Li (2006) fixed λ in 0.0609 while Diebold and Rudebusch Aruoba (2006) estimated that λ was equal to 0.077.

  Yu and Zivot (2007) adopted these values for λ in their empirical study related to corporate bonds. They argue that the Λij(λ) of each λ are not very sensitive to different values of λ. In addition they affirmed that (i) this could be graphically illustrated and (ii) that a fixed λ maximizes the load on the component of curvature in the medium term (i.e. 30 months for λ = 0.0609 and 23.3 months for λ = 0.077)[4]. However, reality showed us that this is not applicable to some emergent economies and much less in situations of inverse interest rate structures, like the one observed in Argentina. Based on the mentioned, we have to find a solution for the calculation of the λ parameter.



B$_2$ and β_3 values depend on the value that takes the λ parameter.

When taking a value of λ = 0.06, β_3 reaches its maximum in 31 months approximately and it converges in 100 months. With λ = 0.35 maximum is reached in 4-5 months and converges at 17 months.

[4] Analysing the Term Structure of Interest Rates using the Dynamic Nelson-Siegel Model with Time-Varying Parameters. Siem Jan Koopman, Max I.P. Mallee, Michel van der Wel. Department of Econometrics, VU University - Amsterdam, Department of Finance, VU University – Amsterdam, Tinbergen Institute - Amsterdam

Calculating the value of λ according to the logarithmic regression

Based on the same considerations made by Nelson and Siegel, Diebold and Li and Diebold, Rudebusch and Aruoba; the λ value would be the value of t (months) in which the logarithmic trend has a 45° slope.

Consequently, if we change at least the β_1, β_2, β_3 coefficients but keeping the λ value chosen based on the logarithmic trend's slope obtained; the best values can be used in the calculation of ETTI (Temporary Interest Rate Structure, by the acronym in Spanish).

Given that the calculation of β_3 maximum determines the shape of the curve, that has a maximum value equal to 0.30, we should look for a solution to the equation for each value of T (number of months) using the following:

$$\beta_3 = 0.3 = \left(\frac{1 - e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} \right)$$

Month	
τ	**λ**
2	0,89664
3	0,59776
4	0,44832
5	0,35866
6	0,29888
7	0,25618
8	0,22416
9	0,19925
10	0,17933
11	0,16303

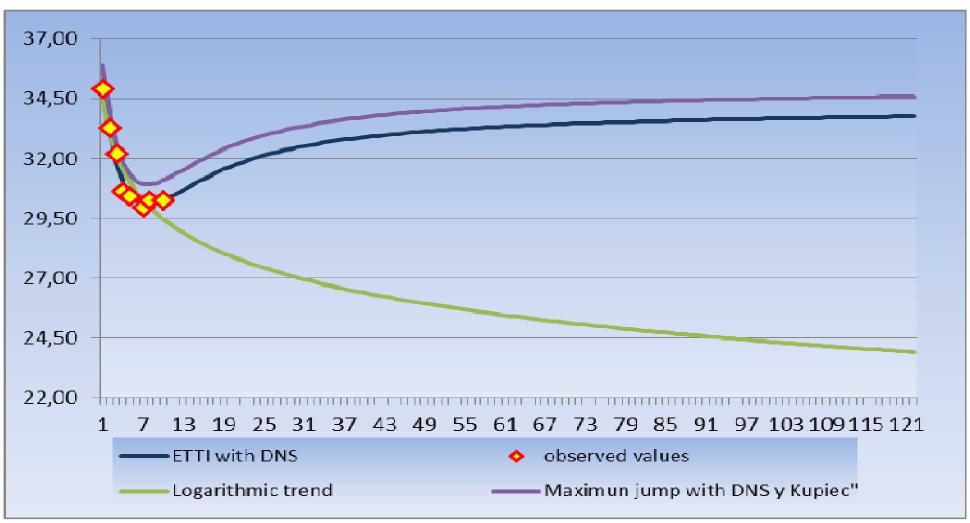
Month	
τ	**Λ**
12	0,14944
13	0,13794
14	0,12809
15	0,11955
16	0,11208
17	0,10549
27	0,06642
28	0,06405
29	0,06184
30	0,05978

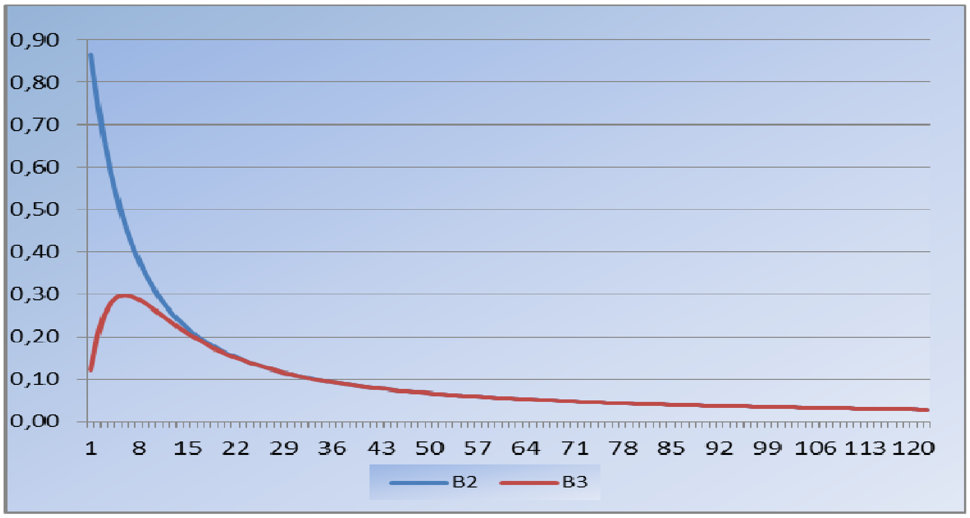
These values trend to the values used by Diebold and Li and Rudebusch and Aruoba

Applying this approach to the examples presented for the spot rates in the U.S. and the Argentine Republic, this last one can be consider as an extreme economic situation case, would be necessary to change the value of λ.

The result of implementing the mentioned solution to the interest rate curve in force in Argentina during the last week of calendar year 2015 is the following:

month	TNA	Logarithmic trend		DNS	Values
1	34,89	***a***	**34,56466**	**β1**	0,437362
2	33,29	***b***	**-2,21739**	**β2**	-0,153872
3	32,20	***R2***	**0,90560**	**β3**	-0,095604
4	30,60			**λ**	0,298880
5	30,41				
7	29,96				
8	30,26				
10	30,26				

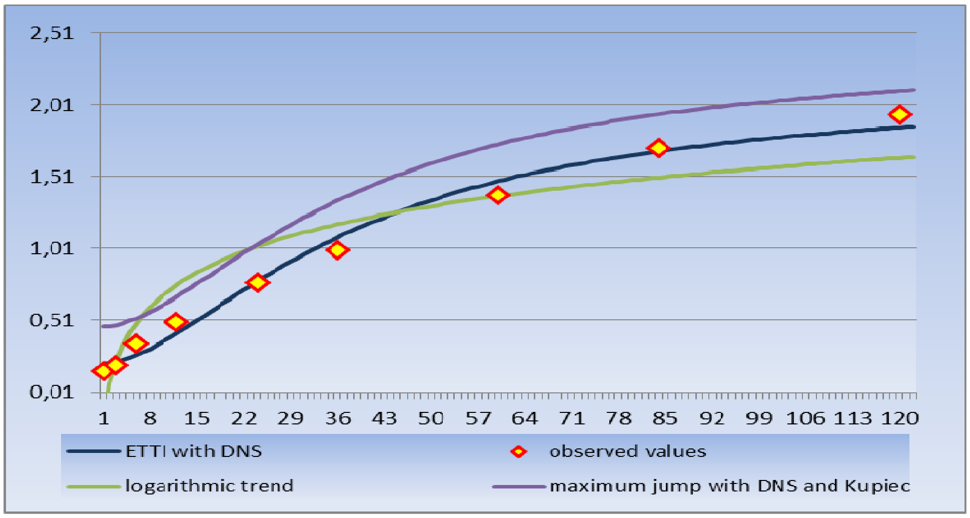




In this case, where DNS curve is shown as a variable by using the value of λ, we can observe that a change in the normality of the economy in a period of 1 year, based on the Lebac term quotation, is not reflected by the trend.

Applying the same concepts to the curves of U.S. bonds, we have the following table, calculated in the same way

month	TNA	logarthmic trend		DNS	values
1	0,16	***a***	**-0,19539**	**β1**	0,032973
3	0,20	***b***	**0,38371**	**β2**	-0,041465
6	0,35	***R2***	**0,87412**	**β3**	-0,010605
12	0,50			**λ**	0,089664
24	0,78				
36	1,00				
60	1,38				
84	1,71				
120	1,95				



Instead of using a value of λ = 0.0609, proposed by Diebold and Li, the value λ = 0.089664 used in this proposal allows an ETTI with minimum errors.

The parallel jump of the ETTI

The use of residuals is interesting, mostly in order to calculate a parallel jump of the interest rate structure found. This is a more complicated issue that exceeds the basic financial calculation. However, because of its importance, it should be applied given that in a given interest rate hike the financial risk derived from the mismatch of cash flows causes heavy losses that must be covered by financial institutions.

The most basic approach we can apply, and that you were able to see on the previous charts, is attach to each curve a parallel curve obtained after applying the average number of errors and multiplying them by the coefficient of variation given by the interest rate probability distribution function. In this case we have used the Kupiec factor, given that there is no distribution that represents these variations.

Based on the mentioned, the jump is obtained as follows:

$$\Delta\widehat{TNA_t} = \sqrt{\frac{\sum_{t=1}^{n} \varepsilon_t^2}{n}}\, K$$

Where: ΔTNA_t is the jump value of the parallel curve;

ε_t^2 is the sum of de square difference between the observed value and the DNS estimated value

K is the safety coefficient used.

An easy way to estimate β1, β2 and β3 parameters

Once we have determined the value for λ parameter, based on the logarithmic regression, we then use the Solver add inn from MS Excel in order to estimate the best β_1, β_2 and β_3 parameters. Here you have an example.

month	TNA	TNA/100	TNA estimated with DNS	ε^2 = (TNA-DNS)^2	
1	0,16	0,00157	0,00210	0,00000028	
3	0,20	0,00196	0,00222	0,00000007	
6	0,35	0,00348	0,00268	0,00000064	
12	0,50	0,00498	0,00420	0,00000061	
24	0,78	0,00775	0,00781	0,00000000	
36	1,00	0,00996	0,01087	0,00000083	
60	1,38	0,01383	0,01479	0,00000094	
84	1,71	0,01708	0,01688	0,00000004	
120	1,95	0,01946	0,01851	0,00000089	Apply Solver
			Σ ε^2	0,000004	Cell to be minimized
			β1	0,032973	Cells to be changed
			β2	-0,041465	
			β3	-0,010605	
			λ	0,089664	Value obtained by proposal

Note that the sum of the square errors must be used to estimate the jump curve of the term structure obtained. The cell that contains this sum is the one we use in order to apply the MS Excel Solver tool.

Conclusions

- Due to the fact that fee structure calculation is essential, it allowed us to continue thinking on the idea that says "$(1+i)^n$ does not exist". Every day we observe more economies facing situations of instability. In this context having on hand the fee structure allows us to take a series of decisions regarding interest rate risk on both directions, from the debtors' side and the creditors.

- The DNS model has been widely adopted by major banks across the world in order to set interest rate curves for different periods. It has been shown that these curves have much better detected the expectations of some interest rates level observed.

- The enhancement proposed in this work in order to modify the coefficient λ is a way to include the effect derived from variations of macroeconomic variables, especially in times of crisis.

- In stable economies, where λ coefficient varies between 0.08 and 0.07, the interest rate curve to be established is not much affected. This effects is observed when the lambda coefficient reaches values greater than 0.20.

BIBLIOGRAPHY

- Diebold, F X & C Li (2006).
  "Forecasting the Term Structure of Government Bond Yields."
  J. Econometrics 130, 337–364.
- Diebold, F X, C Li & V Yue (2007).
  "Global Yield Curve Dynamics and Interactions: A Generalized Nelson-Siegel Approach."
  J. Econometrics, forthcoming.
- Diebold, F X, S Rudebusch & S Aruoba (2006).
  "The Macroeconomy and the Yield Curve."
  J. Econometrics 131, 309–338.
- Nelson, C & A Siegel (1987).
  "Parsimonious Modelling of Yield Curves."
  Journal of Business 60-4, 473–489.
- Yu, W & E Zivot (2007).
  "Forecasting the Term Structures of Treasury and Corporate Yields: Dynamic Nelson-Siegel Models Evaluation."
  Working Paper.