DEFINING A POSTERIORI DISTRIBUTION OF A LONG-TERM RATE STRUCTURE

(A MIXED AND EMPIRICAL APPROACH BASED ON FISHER’S FORMULA)

OCTAVIO ROJAS¹ – ACTUARIAL CONSULTANT

ABSTRACT

As actuaries, our calculations are considered to be of public use. It is because of this that, within our processes, local and international regulations are in force. Some have been set within a multidisciplinary team that includes actuaries, while in others actuaries do not form part of the discussion, although the topics being covered do affect its practice. Another fact, is that some topics are left out (or not discussed in a thorough manner), given that the conditions under which these would apply correspond to a limited number of cases, which are commonly associated to economic environments. This creates loopholes that, unless properly discussed, can give place to multiple interpretations and/or practices. Thus, making it difficult for sound peer-reviewing processes to take place. The purpose of this paper is to provide the reader with a procedure for estimating a long-term rate structure based in a nominal rate of return (which we shall denote by $R$), assuming Fisher’s formula, disclosing the nominal rate in its main components: a real rate and an expected long term inflation. Its setting is in an economic environment for which a yield curved cannot be properly calculated, giving the lack of data and that Government Bonds are not emitted on a regular basis. The algorithm used is divided in two phases. During the first phase, the historical series of the country’s consumer price index (cpi) monthly variation is used in order to get the best estimate from a GARCH(p,q) model. The second, uses the results of the previous phase, in order to establish a series of Markov Chains, by calculating a random number from a normal distribution where its mean equals the result of the previous month and its standard deviation is the model’s (unconditional) standard deviation, conditioned in a way that the estimated future average annual cpi variation falls within the range $(0,R)$.

Keywords: Long-term rate structure, Yield Curve, Government Bonds, Treasury Bills, GARCH Models, Markov Chains, Hyperinflation, IAS 19, IAS 29.

¹ Act. Octavio Rojas, MAAA.
Member of IAA’s PBSS and AWB.
Tel.: (+58424) 110.5784 – Email: octagon64@gmail.com
1 INTRODUCTION

As actuaries, our calculations are of public use. It is because of this that, within our processes, local and international regulations are in force. Some have been set within a multidisciplinary team that includes actuaries, while in other actuaries do not form part of the discussion, although the topics being covered do affect its practice. Another fact, is that some topics are left out (or not discussed in a thorough manner), given that the conditions under which these would apply correspond to a limited number of cases, which are commonly associated to economic environments. This creates loopholes that, unless properly discussed, can give place to multiple interpretations and/or practices.

The above, which is part of the abstract, can be fully explained through an example.

IAS 19, states that:

"...

74 Actuarial assumptions are unbiased if they are neither imprudent nor excessively conservative.

75 Actuarial assumptions are mutually compatible if they reflect the economic relationships between factors such as inflation, rates of salary increase, the return on plan assets and discount rates. For example, all assumptions which depend on a particular inflation level (such as assumptions about interest rates and salary and benefit increases) in any given future period assume the same inflation level in that period.

76 An entity determines the discount rate and other financial assumptions in nominal (stated) terms, unless estimates in real (inflation-adjusted) terms are more reliable, for example, in a hyperinflationary economy (see IAS 29 Financial Reporting in Hyperinflationary Economies), or where the benefit is index-linked and there is a deep market in index-linked bonds of the same currency and term.

...

78 The rate used to discount post-employment benefit obligations (both funded and unfunded) shall be determined by reference to market yields at the end of the reporting period on high quality corporate bonds. In countries where there is no deep market in such bonds, the market yields (at the end of the reporting period) on government bonds shall be used. The currency and term of the corporate bonds or government bonds shall be consistent with the currency and estimated term of the post-employment benefit obligations.
One actuarial assumption which has a material effect is the discount rate. The discount rate reflects the time value of money but not the actuarial or investment risk. Furthermore, the discount rate does not reflect the entity-specific credit risk borne by the entity’s creditors, nor does it reflect the risk that future experience may differ from actuarial assumptions.

The discount rate reflects the estimated timing of benefit payments. In practice, an entity often achieves this by applying a single weighted average discount rate that reflects the estimated timing and amount of benefit payments and the currency in which the benefits are to be paid.

In some cases, there may be no deep market in bonds with a sufficiently long maturity to match the estimated maturity of all the benefit payments. In such cases, an entity uses current market rates of the appropriate term to discount shorter term payments, and estimates the discount rate for longer maturities by extrapolating current market rates along the yield curve. The total present value of a defined benefit obligation is unlikely to be particularly sensitive to the discount rate applied to the portion of benefits that is payable beyond the final maturity of the available corporate or government bonds.

...”

In an environment where:

a. There exists a poor stock market
b. Government Bonds are issued on a non-regular basis
c. It complies with the definition of a hyperinflationary environment
d. Information on long-term inflation does not exits

Based on the above, the only way that should enable us to define a real discount rate is by defining a long-term inflation rate, so rate can be derived from the average nominal government bonds.

The fact that the rule does not provide an explicit way to cope with these cases, gives places to multiple interpretations, including setting rates based on a “rule of thumb”. Thus, making it impossible for a proper peer-review to be made, or an International Standards of Actuarial Practice (“ISAP”s) on this issue to be published.

In the case of all other financial assumptions (e.g.; salary increases, pension increases, etc.), real rates can be defined in terms of current inflation (or lagged). Nevertheless, if we were to use long-term nominal salary increases, based on a calculated real salary increase rate, the same long-term inflation rate should be applied (in accordance with the standard).

[Escriba aquí]
Following is a procedure used to define a real discount rate, by estimating a long-term inflation, which attempts to deal with the loophole in the accounting standard referred above.

It is important to mention that the procedure has been developed to produce real rates, regardless of the economic environment, regardless of the ambiguity presented in paragraph 76, given that, it does not provide a way to measure when (and why) real rates can be more reliable than nominal rates. Thus, making that, to play safe, many practitioners solely use real rates when IAS 29 applies. When paragraph 76, states that countries where IAS 29 must apply real rates just as an example (without describing the means to identify under which other condition real rates should be used). We strongly believe that, nowadays in countries where controversies exist this could have been handled differently and in a more professional manner should real rates be used (for example: Ecuador). But, for this to happen, paragraph 76 should be amended.

More than a statement on how to cope with the problem, it is our intention for this paper to serve to open the debate on this subject. Being our main interest, to help raise quality standards within the actuarial community.

2 HYPERINFLATION CONCEPT

Hewitt, M. (2017), wrote an article entitled “These 21 countries have experienced hyperinflation in the las 25 years”.

In this article, he starts by stating that: “Hyperinflation is not an unusual phenomenon.32 countries have experienced hyperinflation over the las 100 years of which no less than 21 have experienced it in the past 25 years and 3 in the past 10 years.”

The countries mentioned are the following (presented in the same order as in the article):

- Argentina (1975-1991)
- Belarus (1994-2002)
- Bolivia (1984-1986)
- Brazil (1986-1994)
- Bosnia-Herzegovina (1993)
- Ecuador (2000)
- Georgia (1995)
- México (1994)
- Nicaragua (1987-1990)
- Perú (1984-1990)
- Poland (1990-1993)
- Romania (2000-2005)
- Russia (1992-1994)
- Turkey (1990’s)
- Zimbabwe (1999-2009)

[Escriba aquí]
The above countries, comply with the concept that hyperinflation occurs in a country when it reaches a point that its monthly inflation increases at a rate of at least 50.0%. Apart from this, each country has reached and left its hyperinflation condition by different means. Thus, suggesting that hyperinflation should be studied in a country specific basis. Even more, as seen from the countries listed above, Venezuela has never been under hyperinflation.

The above, is a subject shared by most economists. As an example, there are articles such as the one written by John Greenwood, Chief Economist at Investco Ltd. entitled: “Could hyperinflation be next for Venezuela?” [Greenwood J. (2017)]

In lieu of the above, we must present what is going to be our definition in line with the appliance of IAS 19. For this, we must submit ourselves to the definition within IAS 29 (Financial Reporting in Hyperinflationary Economies), which states the following:

“...”

3. This Standard does not establish an absolute rate at which hyperinflation is deemed to arise. It is a matter of judgement when restatement of financial statements in accordance with this Standard becomes necessary. Hyperinflation is indicated by characteristics of the economic environment of a country which include, but are not limited to, the following:

   a) the general population prefers to keep its wealth in non-monetary assets or in a relatively stable foreign currency. Amounts of local currency held are immediately invested to maintain purchasing power;
   
   b) the general population regards monetary amounts not in terms of the local currency but in terms of a relatively stable foreign currency. Prices may be quoted in that currency;
   
   c) sales and purchases on credit take place at prices that compensate for the expected loss of purchasing power during the credit period, even if the period is short;
   
   d) interest rates, wages and prices are linked to a price index; and
   
   e) the cumulative inflation rate over three years is approaching, or exceeds, 100%.

...”

Questions that rise from the above, should include:

- What would have been the time spam during which hyperinflation was present in the 21 countries under the above definition?
- How many would have to be added in lieu of IAS 29 definition?

Paragraph 41, of IAS 29 states that:” This Standard becomes operative for financial statements covering periods beginning on or after 1 January 1990.”

[Escreba aquí]
From IAS 29, hyperinflation definition, only the last is easy to track, given that it is the only one than is of a quantitative nature. Nevertheless, it is kind of ambiguous since it says, “is approaching”, raising the question: *At which point should we state that the economy is approaching?*

The answer to the above question is of great relevance because the lesser amount set as the breakpoint, the greater number of countries as well as the timing under which the country will be considered as being under hyperinflation.

As an example, based on the information published by the World Bank (economic indicators) and if all countries adopted IAS as their local standards when these were issued, between the years 1990-2016, the number of countries under hyperinflation should we use IAS 29 section 3e as our way to define hyperinflation, and setting as our breakpoint a 95.5%, would be 73, disclosed as follows (Annex C shows the complete list of countries per region):

<table>
<thead>
<tr>
<th>REGION</th>
<th>Countries That Have Complied</th>
<th>Complied (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>East Asia &amp; Pacific</td>
<td>32</td>
<td>5</td>
</tr>
<tr>
<td>Europe &amp; Central Asia</td>
<td>36</td>
<td>22</td>
</tr>
<tr>
<td>Latin America &amp; Caribbean</td>
<td>23</td>
<td>19</td>
</tr>
<tr>
<td>Middle East &amp; North Africa</td>
<td>15</td>
<td>6</td>
</tr>
<tr>
<td>North America</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>South Asia</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>Sub-Saharan Africa</td>
<td>28</td>
<td>20</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>144</td>
<td>73</td>
</tr>
</tbody>
</table>

*The above, is to be considered just a “rough estimate”. To determine in which of the 73 countries mentioned above, IAS 29 should apply, we must first study how many met the rest of the conditions set in paragraph 3.*

### 3 LITERATURE REVIEW

Amongst the literature on this subject, we find the paper Cruz, E., & Query, J. (2017), entitled: “Developing Actuarial Assumptions within the Framework of a Hyper-Inflationary Environment”

Their approach, assumed that” ...wage growth rates can be expressed as a function of inflation, which is true for the vast majority of the wage policies of Venezuelan companies, then it is possible to express the interest rate as a function also of wages...”. Later, they developed a formula based on Fisher’s, such that the interest rate to be used will be result of as a function based on the salary increase rate and the real discount rate.

Given that in their introduction, they state that these procedure does comply with IAS 19, a question arises: Should an actuarial valuation for a company require more than one salary scale,
would the use of more than one discount rate comply with IAS 19? The question is due to the fact a discount rate will result per each salary increase rate.

Ireland, P. (1996) when describing the Fisher formula states that: “Irving Fisher (1907) presents what is perhaps the most famous theory of nominal interest rate determination. According to Fisher’s theory of interest, movements in nominal bond yields originate in two sources: changes in real interest rates and changes in expected inflation. Thus, Fisher’s theory provides a guide for investigating the extent to which long-term bond yields serve as reliable indicators of long-term inflationary expectations.” In his paper, Ireland refers to the US market.

Unlike Ireland, the proposed methodology provided in this paper, uses Fisher’s theory to use long-term inflationary expectations to estimate real long-term bond yields. Our approach, uses the expected proportion over time of the long-term inflation within the average nominal rate on long-term bonds to estimate a real long-term bond yields curve.

Wu, M. (2016), wrote a paper analyzing “... the historical background and current conditions relating to Venezuela’s economic crisis, focusing on its monetary and financial aspects...” Its purpose is to:” ... assist in making recommendations for appropriate monetary reforms...” Although it does not mention Venezuela’s Treasury Bills market behavior (mentioned later in this paper, for explanation purposes), it is recommended as a complemental reading, to have a better understanding of the Venezuelan crisis.

4 IN VENEZUELA: ARE GOVERNMENT BOND YIELDS LINKED WITH INFLATION?

To provide an answer to the above question, we must first say that in Venezuela, Government Bond Yields (GBY) in local currency are commonly linked to Government Treasury Bill Yields (GTBY), presented in one of two ways, these are:

\[
(GBY) = \left\{ \begin{array}{c}
(GTBY) + \Delta f_{ixed}\% \\
\end{array} \right. \tag{4.1}
\]

It is because of the above, that we shall focus, primarily on the behavior of the (GTBY). It is because of this that, given the sharp drop in oil prices during 2009, and the over expenditure in social programs during 2010, we shall explain its behavior considering two periods:

- 01/2003 to 04/2010
- 05/2010 to 12/2015

4.1 FROM 01/2003 TO 05/2015

During these period, given that government expenditure was stable as well as oil prices (until 2009, when a sudden drop occurred), the behavior of the GTBY was very much in line with inflation, as shown below:

---


[Escriba aquí]
As seen in the above chart, both inflation and GTBY behave the same throughout the period. Thus, we can assure, given the link between GTBY and GBY, that GBY are linked with inflation during this period.

4.2 FROM 05/2010 TO 12/2015
As from 05/2010, government expenditure got to its highest level, changing the whole picture as shown in the following chart:
During this time, the strategy, given the sharp rise in oil prices and their expectancy that these would hold for a long period of time, their focus was on making the GBY attractive (not so the GTBY). Thus, they issued bonds, increasing their fixed amount (minimum rate of return, previously denoted by $\Delta\%$), as shown:

<table>
<thead>
<tr>
<th>Number of Emissions in Force</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var.</td>
<td>3</td>
<td>9</td>
<td>17</td>
<td>22</td>
<td>26</td>
<td>30</td>
</tr>
<tr>
<td>Fixed</td>
<td>4</td>
<td>4</td>
<td>9</td>
<td>14</td>
<td>18</td>
<td>23</td>
</tr>
<tr>
<td>Total</td>
<td>7</td>
<td>13</td>
<td>26</td>
<td>36</td>
<td>44</td>
<td>53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average Difference</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var.</td>
<td>5.60</td>
<td>7.36</td>
<td>8.15</td>
<td>8.44</td>
<td>8.63</td>
<td>8.77</td>
</tr>
<tr>
<td>Fixed</td>
<td>4.09</td>
<td>4.09</td>
<td>7.62</td>
<td>8.66</td>
<td>8.82</td>
<td>8.99</td>
</tr>
<tr>
<td>Total</td>
<td>4.69</td>
<td>6.35</td>
<td>8.03</td>
<td>8.53</td>
<td>8.71</td>
<td>8.86</td>
</tr>
</tbody>
</table>

The above rates, served as leverage, to keep track of inflation.

Bearing in mind that, the rates shown so far are nominal, we should be clear of the following:
• In real terms, nominal rates (variable or fixed) will be affected by the ratio between the purchase price and the price set by the Government (the lesser its purchase price, the greater the increase over the rate)
• During hyperinflation, given that uncertainty plays an important role. This is normally linked with the country’s credit rating (currently, Venezuela has the worst credit rating in the world), assuring that it also links with inflation

Based on the above facts, we can state that Venezuelan GBY are linked with inflation. 

5 THE ALGORITHM

Our algorithm comprises the following steps:

1. Defining the initial parameters:
   a. Define the best estimate for the cpi monthly series from a GARCH(p,q) model
   b. Define the average nominal government (or corporate) bond rate as of date ($R$)
   c. Define an initial confidence interval based on the results of the model and $R$
   d. Modify/change the model so it can serve for the purpose of forecasting

2. Calculations per chain:
   a. Define a confidence interval at time $t$, based on the cpi’s history as of the previous month, which will have information from the historical data and the estimated results up to the previous month
   b. Generate an estimate from the modified GARCH(p,q) model:
      i. If the estimated amount falls within the confidence interval, it will be added to the series
      ii. Otherwise, if:
         • The estimated amount is less than the lower limit, the difference will be added to the lower limit
         • The estimated amount is greater than the upper limit, the difference will be subtracted to the upper limit

---

3 It is important to keep in mind that linked does not necessarily means for rates to turn real positive.
4 The reason for this step is due to the condition that to comply with Fisher’s formula, annual average forecasts on long-term inflation rates shall never be above the value of $R$

[Escriba aquí]
In any case, the resulting amount shall be within the range of the interval and it will taken as the next value of the series.

A total amount of 5,000 iterations of length 420 future months where drawn (in order to comply with a future time-span of 35 years).

6 APPLING THE ALGORITHM: RESULTS

Following are the results of each phase of the algorithm, presented in the same order as in the previous section.

6.1 DEFINING THE INITIAL PARAMETERS

6.1.1 Define the best estimate for the cpi monthly series from a GARCH(p,q) model

The GARCH(p,q), introduced by Bollerslev T. (1986), is a generalized version of Engle (1982) ARCH(q) which is commonly used in financial time series analysis, given that these present the following characteristics:

- These are leptokurtic
- Volatility presents itself in terms of clusters
- It has a leverage effects (volatility is greater after a fall in the variable)

An important remark is that, on top of the above, the distribution of the series in hand (monthly cpi variations), is also positive skewed. Thus, it is seldom to find negative variations.

By using parts of the code presented by Zivot (2012), we got our first set of results. These are:

*---------------------------------------------------------------*
*                  GARCH Model Fit                          *
*---------------------------------------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1,0)
Mean Model  : ARFIMA(0,0,0)
Distribution : norm

Optimal Parameters
-------------------

[Escreba aquí]
According to the literature on the subject, the fact that the sum of omega plus alpha1 is less than 1, implies that the process is stationary. Nevertheless, the fact that the lagged square change has a trend that is monotonous decreasing, as shown in the following graph:

![Lagged Squared Change](image)

Suggests that if the model is used for forecasting (just as it is), its results will tend to be equal to zero very rapidly.

The above remark, shall be used in section 6.1.4 (Modify/change the model so it can serve for the purpose of forecasting).
6.1.2 Define the average nominal government (or corporate) bond rate as of date (R)

To be able to use Fisher’s formula, first we need to derive the average nominal long-term rate of interest. To do so, we shall make use of the information issued by the Central Bank of Venezuela as of 12/2015, in its “BONOS DE LA DEUDA PÚBLICA NACIONAL — CARACTERÍSTICAS DE LAS EMISIONES Y TASA DE INTERÉS APLICABLE”. This table provides the characteristics of the public debt emissions in terms of the amounts that were still in the market, the formulas used to calculate the rates that were to be paid and the year’s resulting nominal rates per emission. The information provided is as follows:

### TABLA 7

<table>
<thead>
<tr>
<th>Emisión Descr.</th>
<th>Código</th>
<th>Instrumento</th>
<th>Fecha Emisión</th>
<th>Vencimiento</th>
<th>Monto en Circulación (Bs. B)</th>
<th>Tasa Fija</th>
<th>Tasa Flotante</th>
<th>Tasa Prom.</th>
<th>Tasa de Referencia</th>
<th>B</th>
<th>Tipo Bono</th>
<th>Notas</th>
</tr>
</thead>
<tbody>
<tr>
<td>010</td>
<td>3601</td>
<td>2010-03-01</td>
<td>2010-03-01</td>
<td>18</td>
<td>1,454,164,035</td>
<td>L T 7.7%</td>
<td>15,569,105</td>
<td>65,373,610</td>
<td>9,705</td>
<td>9,705</td>
<td>15,569,105</td>
<td></td>
</tr>
<tr>
<td>020</td>
<td>3602</td>
<td>2010-03-01</td>
<td>2010-03-01</td>
<td>18</td>
<td>1,454,164,035</td>
<td>L T 7.7%</td>
<td>15,569,105</td>
<td>65,373,610</td>
<td>9,705</td>
<td>9,705</td>
<td>15,569,105</td>
<td></td>
</tr>
</tbody>
</table>

Note: The table provides the information related to the bonds issued by the Venezuelan government.

- **Fecha Emisión**: Date of issuance.
- **Vencimiento**: Maturity date.
- **Monto en Circulación**: Circulating amount in billions of Bolivars.
- **Tasa Fija**: Fixed rate.
- **Tasa Flotante**: Floating rate.
- **Tasa Prom.**: Average rate.
- **Tasa de Referencia**: Reference rate.
- **B**: Type of bond.
- **Tipo Bono**: Type of bond.

The table includes the following notes:
- **Notas**: Additional notes or comments related to the bond issuance.
By dividing the total amount of interest paid by the total circulating cash, it gives an average nominal long-term rate of 16.00%.

6.1.3 Define an initial confidence interval based on the results of the model and R
According to the historical data of cpi monthly variation, as of 12/2015, its average and standard deviation are as follows:

- Average: 0.027933
- Std. Deviation: 0.023573

If were to calculate an average rate based on an annual inflation rate of 16.00%, this would in a monthly rate of 0.012445. So, our first task would be to calculate a factor such that the average rate of the historical data matches this figure. The resulting factor is of approximately 44.55%. Thus, our confidence interval would be generated by multiplying the historical parameters shown above by the calculated factor, to define its upper and lower limits. Under the assumption of a normal distribution with mean equals to the adjusted mean and standard deviation (0.012445, 0.010503), a 90% confidence interval would be the following: (-0.010962, 0.066828).

6.1.4 Modify the model so it can serve for forecasting purposes
To start with, we should first focus on our task, which is for the long-term rate to comply with Fisher’s formula, which states that for a nominal long-term rate:

\[ \ln(1 + R) = \ln(1 + r) + \ln(1 + i) \] (6.1)

So, if \( R \) equals 16.00%, then \( i \in (0.00\%, 16.00\%) \).

Now, given that we are forecasting, our formula will have to be written as follows:

\[ \ln(1 + R) = \ln(1 + r_t) + \ln(1 + i_t) \] (6.2)

So that the condition should apply at any point \( t \in \{0,1,2, \ldots \} \)

From Section 6.1.1, we have deduced that, unless we change the model:

- There will come a time when the values will all be the same.
- All values will fall outside the boundaries of the model, given that the last known values will produce an annualized value greater that 16.00%

To avoid the above, the following steps were taken:

[Escriba aquí]
• In the original formula, for forecasting purposes we replaced $\varepsilon_t^2$ with $\bar{\varepsilon}_{(0,t)}^2$, where $\bar{\varepsilon}_{(0,t)}^2$ is the average value of $\varepsilon_t^2$, from 0 to t, multiplied by the absolute value of $v \sim N(0,1)$.

Thus, the above, gives place to the following:

$$h_t = \Omega + a \times h_{t-1} + b \times \bar{\varepsilon}_{(0,t-1)}^2$$ (6.3)

Where: $\bar{\varepsilon}_{(0,0)}^2 = \varepsilon_0^2$ and 0 the starting point

• Finally, to comply with the condition that at every point in time, the value $(1 + h_t)^{12} - 1 < 16.00\%$, the value of $h_t$ will be multiplied by:

$$\psi = \sqrt{((44.55\%) \times (1 - 44.55\%))} = 49.70\%$$ (6.4)

### 6.2 Calculations per chain

#### 6.2.1 Define a confidence interval at time t

After each iteration takes place, we calculate a new average and standard deviation by adding $h_{t-1}$, to the series. Then we proceed to calculate the new upper and lower limits of the interval using the same procedure described in Section 6.1.3.

#### 6.2.2 Generate an estimate from the modified GARCH(p,q) model

To generate each new value:

- We proceed to calculate $h_t$ by using formula (6.3)
- We multiply the above result by $\psi$
- We follow the procedure to define the value that shall be added to the series
After a total of 5,000 iterations of length 420 (equivalent to 35 years), the results were the following:

The above charts demonstrate how each year, the nominal rate can be disclosed by its components in accordance with Fisher’s formula.

At this point, it is of interest to analyze each component.

Given that our aim was to find a real rate, its estimated behavior across time, is of great importance.

The above, provides a trend which is very similar of that of a yield curve. Which is a very promising result given that the necessary information to generate a proper yield curve is not available.
Now, it is of great interest to see the behavior of the model in terms of the long-term inflation.

In accordance with the literature on financial series, as stated in Section 5.1.1, volatility is greater after a fall in the variable and it presents itself in terms of clusters. This explains the slope over the first four years and how it starts to stabilize as from the fifth.

We can conclude by stating that, from our point of view, the aim of the procedure was achieved and that the methodology used is both consistent with financial and economic principles. Nevertheless, in the following chapter we shall properly prove our statement.

7 GOODNESS OF THE VALUATION RESULTS AND NEXT STEPS

To value de goodness (or reasonableness) of the results, we first calculated the average real rate of the model as of 12/2015. This resulted in an approximate value of 4.13%.
To see whether such result is reasonable (meaning by this that it falls within a range of countries that have a stable economy), we shall make use of the information provided by Willis Towers Watson (2016). Within its report, they present the following table:

<table>
<thead>
<tr>
<th>Country</th>
<th>Averages – ASC 715</th>
<th>Averages – IAS 19</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2016*</td>
<td>2015*</td>
</tr>
<tr>
<td>Canada</td>
<td>3.89%</td>
<td>3.81%</td>
</tr>
<tr>
<td>Germany</td>
<td>2.18%</td>
<td>2.08%</td>
</tr>
<tr>
<td>Japan</td>
<td>0.90%</td>
<td>0.92%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>2.32%</td>
<td>2.28%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.91%</td>
<td>1.23%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>3.78%</td>
<td>3.67%</td>
</tr>
<tr>
<td>United States</td>
<td>4.39%</td>
<td>4.01%</td>
</tr>
</tbody>
</table>

*For the purposes of this table, 2016 represents the discount rate assumption used for pension obligations at the end of 2015.

As we can see from the above for IAS 19 purposes, our value as of 12/2015 (4.13%), falls between the rates used in Canada (3.96%) and United States (4.27%)\(^5\). Apart from this, Venezuela fully complies with IAS 29, paragraph 3, apart b which states that: "... the general population regards monetary amounts not in terms of the local currency but in terms of a relatively stable foreign currency...” In the Venezuelan case, such foreign currency is the US Dollar.

It is because of the above, that we can conclude that our goal of getting a rate that complies with the condition set in paragraph 76 of IAS 19 has been achieved.

Given the controversy that resulted from a dictamen made by the IFRS in the case of Ecuador it would be interest to use the procedure to see what the underlying real yield curve of their bonds looks like and see how it compares with the real GBY of Ecuador as a next step for this study [for further details, please refer to IFRS (2017)].

On top of the above, we are currently working on setting real rates for all other actuarial assumption (salary increase rates, rates of return, health costs, etc.), by using a similar procedure. Results will be presented on a future paper.

\(^5\) The rates for Canada and United states, provide a geometrical average of 4.11%.
1. As seen before, IAS 29 definition for hyperinflation, allows for a greater number of countries to be regarded as such. Thus, more emphasis should be made in defining real rates that comply with IAS 19, by the means of an open debate.

2. If by “reliable”, paragraph 76 of IAS 19, means that the real rate summarizes most of the conditions that define the economic environment (set under IAS 29), then the present procedure does fully comply with IAS 19 standards. Thus, it serves as a basis to start a proper debate on real rates calculations for IAS 19 purposes.

   In the case of all other actuarial assumptions, a similar procedure can serve to estimate real rates (as it shall be seen on a future paper).

3. To truly define the scope of countries where real rates should be used, IAS 19, paragraph 76 must be amended. This could serve to minimize controversies, such as in the case of Ecuador, where one of the most important points in its discussion was:

   **What should be considered more relevant?**
   a. *For the discount rate to be consistent with the currency and estimated term of the post-employment benefit obligations* (IAS 19, par. 78), or,
   b. *For all assumptions* which depend on a particular inflation level (such as assumptions about interest rates and salary and benefit increases) *in any given future period assume the same inflation level in that period* (IAS 19, par. 75).

   As it can be seen, should real rates could be applied in this case, such controversy would have never been raised.

4. From our analysis, we have reached to the conclusion that the model by Cruz, E., & Query, J., cannot be said to comply with IAS 19/29, due the following reasons:
   a. The way they define the discount rate cannot assure that reflects the time value of money, nor that its unbiased
   b. If such procedure is used for a company that requires two or more salary scales for their actuarial valuation (e.g., one for blue-collar workers and another for white collar workers), then the valuation will be made under as many discount rates as salary scales are used.

[Escreba aquí]
REFERENCES


The data used for calculations was that of the CPI variation series for the period (01/1988-12/2015), published by the Venezuelan Central Bank.

Now, to comply with GARCH(p,q) condition for the model to be stationary, a smoothed subset of the data (12/2005-12/2015) was used to obtain the parameters.

The characteristics of the smoothed subset were the following:

<table>
<thead>
<tr>
<th>STATISTICS</th>
<th>Value (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min,</td>
<td>-0.74%</td>
</tr>
<tr>
<td>Q1</td>
<td>1.55%</td>
</tr>
<tr>
<td>Q2</td>
<td>2.09%</td>
</tr>
<tr>
<td>Q3</td>
<td>3.39%</td>
</tr>
<tr>
<td>Max.</td>
<td>11.06%</td>
</tr>
<tr>
<td>Mean</td>
<td>2.85%</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2.25%</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.87</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.56</td>
</tr>
</tbody>
</table>
11 ANNEX B – GARCH MODEL STATISTICS

Box-Ljung test
data:  coredata(inf2.2005^2)
X-squared = 366.9086, df = 12, p-value < 2.2e-16

ARCH LM-test; Null hypothesis: no ARCH effects
data:  inf2.2005
Chi-squared = 96.8666, df = 12, p-value = 2.331e-15

*-----------------------------*
*  GARCH Model Fit  *
*-----------------------------*

Conditional Variance Dynamics
-------------------------------

GARCH Model : sGARCH(1,0)
Mean Model : ARFIMA(0,0,0)
Distribution : norm

Optimal Parameters
---------------------

| Estimate  | Std. Error | t value | Pr(>|t|) |
|-----------|------------|---------|---------|
| mu        | 0.018084   | 0.000967| 18.6999 | 0.00000 |
| omega     | 0.000042   | 0.000012| 3.5681  | 0.00036 |
| alpha1    | 0.999000   | 0.251344| 3.9746  | 0.00007 |

Robust Standard Errors:

| Estimate  | Std. Error | t value | Pr(>|t|) |
|-----------|------------|---------|---------|
| mu        | 0.018084   | 0.001376| 13.1455 | 0.000000 |
| omega     | 0.000042   | 0.000017| 2.4769  | 0.013252 |
| alpha1    | 0.999000   | 0.192055| 5.2016  | 0.000000 |

[Esriba aqui]
LogLikelihood : 357.87621

Information Criteria
--------------------------------------
Akaike      -5.8657
Bayes       -5.7964
Shibata     -5.8669
Hannan-Quinn -5.8376

Weighted Ljung-Box Test on Standardized Residuals
--------------------------------------

<table>
<thead>
<tr>
<th>Lag</th>
<th>statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag[1]</td>
<td>31.24</td>
<td>2.282e-08</td>
</tr>
<tr>
<td>Lag[2*(p+q)+(p+q)-1][2]</td>
<td>36.79</td>
<td>1.772e-10</td>
</tr>
<tr>
<td>Lag[4*(p+q)+(p+q)-1][5]</td>
<td>48.96</td>
<td>1.260e-13</td>
</tr>
</tbody>
</table>

d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
--------------------------------------

<table>
<thead>
<tr>
<th>Lag</th>
<th>statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag[1]</td>
<td>0.1702</td>
<td>0.6799</td>
</tr>
<tr>
<td>Lag[2*(p+q)+(p+q)-1][2]</td>
<td>0.5191</td>
<td>0.6847</td>
</tr>
<tr>
<td>Lag[4*(p+q)+(p+q)-1][5]</td>
<td>2.3338</td>
<td>0.5419</td>
</tr>
</tbody>
</table>

d.o.f=1

[Escriba aquí]
Weighted ARCH LM Tests

------------------------------------

Statistic Shape Scale P-Value

<table>
<thead>
<tr>
<th>ARCH Lag[2]</th>
<th>0.6751</th>
<th>0.500</th>
<th>2.000</th>
<th>0.4113</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH Lag[4]</td>
<td>2.5186</td>
<td>1.397</td>
<td>1.611</td>
<td>0.3378</td>
</tr>
<tr>
<td>ARCH Lag[6]</td>
<td>3.0884</td>
<td>2.222</td>
<td>1.500</td>
<td>0.4544</td>
</tr>
</tbody>
</table>

Nyblom stability test

------------------------------------

Joint Statistic: 0.4708

Individual Statistics:

mu  0.17513
omega  0.11633
alpha1  0.07596

Asymptotic Critical Values (10% 5% 1%)

<table>
<thead>
<tr>
<th>Joint Statistic:</th>
<th>0.846</th>
<th>1.01</th>
<th>1.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual Statistic:</td>
<td>0.350</td>
<td>0.47</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Sign Bias Test

------------------------------------

<table>
<thead>
<tr>
<th>t-value</th>
<th>prob sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sign Bias</td>
<td>1.0721</td>
</tr>
<tr>
<td>Negative Sign Bias</td>
<td>0.3737</td>
</tr>
<tr>
<td>Positive Sign Bias</td>
<td>0.6390</td>
</tr>
<tr>
<td>Joint Effect</td>
<td>2.4254</td>
</tr>
</tbody>
</table>

Adjusted Pearson Goodness-of-Fit Test:

------------------------------------

[Escriba aquí]
<table>
<thead>
<tr>
<th></th>
<th>arch1</th>
<th>arch2</th>
<th>arch3</th>
<th>arch4</th>
<th>arch5</th>
<th>garch1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-5.8625522</td>
<td>-5.8393231</td>
<td>-5.8308627</td>
<td>-5.8788453</td>
<td>-5.8584400</td>
<td>0.67271402</td>
</tr>
<tr>
<td>2</td>
<td>-5.7932351</td>
<td>-5.7469003</td>
<td>-5.7153342</td>
<td>-5.7402111</td>
<td>-5.6967000</td>
<td>0.67271402</td>
</tr>
<tr>
<td>3</td>
<td>-5.8637425</td>
<td>-5.8414170</td>
<td>-5.8341006</td>
<td>-5.8834603</td>
<td>-5.8646582</td>
<td>0.67271402</td>
</tr>
<tr>
<td>4</td>
<td>-5.8343999</td>
<td>-5.8017867</td>
<td>-5.7839422</td>
<td>-5.8225407</td>
<td>-5.7927512</td>
<td>0.67271402</td>
</tr>
</tbody>
</table>

Elapsed time: 0.25397992
11.1 GRAPHICAL INFORMATION
Following are the graphs provided to value the chosen model.

Simulated Path of Series

Simulated Path of Conditional Sigma
Simulated Path of Series

Simulated Path of Conditional Sigma

[Escriba aquí]
Raw inf2.2005 inflations

Cleaned inf2.2005 inflations
The countries with an average length of 1 (shown in red), is because of the breakpoint. As an example, in the case of Philippines, between the years 1995-1997, the three years accumulated inflation was of 82.33%, 103.63% and 87.20% respectively.