Do Risk Managers Believe in Stress Testing outcomes?

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Abstract

When remembering the controversy between the “quants”, who base their predictions on a model developed under the concept that the future comes from the past and pass through the present, and the professionals who base their decisions on subjective degrees of personal beliefs... I convince myself that the question that entitles this job is a reality.

In order to provide an alternative answer for this question I propose to develop models that include the Risk Managers’ believes or the Macroeconomic Expert’s judgment. In this way I suggest to analyze several risk models that combine a quant model and the Risk Managers’ beliefs.

The risks we need to analyze with the proposed models are the following:

- **Credit Risk**: besides the traditional model PD*EAD*LGD, I recommend to include in the analysis the behavior of the default rate and its impact on the EAD, the incidence of Vasicek model and the relationships among default rate, GDP growth rate and unemployment rate.
- **Market Risk**: is necessary to develop a model that shows the static and dynamic behavior of each variable under study.
- **Liquidity Risk**: the model needs to contemplate both types of liquidity risks, the liquidity risk resulting from the market conditions and the liquidity risk derived from the Entity’s specific characteristics.
- **Interest Rate Risk**: the inclusion of the series’ behavior and volatility analysis and the relationship among them together with the macroeconomics series are necessary for the model to be developed. The term structure is the base of the interest risk model, given that it reflects the effect of inflation, market liquidity and GDP.
- **Operational Risk**: the model need to be based on the developments made in Insurance Business, including references to the credibility theory.

Finally, we cannot forget the last big crisis we have experienced in 2008 that is well shown in the film named “Margin call”. For this reason it is important to get on with Risk Managers and present them these credible models in order to convince both “quants” and “believers” to start using them.
Risk Management

In order to begin the explanation of this paper, let me please start with by the end. "I don't think the interest rate will rise to the level you say", "We are going to sell our position and be the only ones not making money", "Please, do not bring the risk analyst to the meeting any more, he only makes box score and algorithms but don’t know nothing about the market". Does it sound familiar? These phrases and many others like them have preceded the fall of Lehman Brothers.

If we look for a definition of Risk Management, we could say it may be: “A structured approach to handle the uncertainty concerning a threat”. Under this idea, the risk assessment is a sequence of strategies to be developed in order to handle the unknown and, immediately, derives the concept of risk aversion. What do we understand for risk aversion? Let me show you the facsimile of the telegram sent at the time of the Titanic’s sinking so we can better understand this concept.
“The story that I have to tell is marked all the way through by a persistent tension between those who assert that the best decisions are based on quantifications and numbers determined by patterns on the past, and those who base their decisions on more subjective degrees of belief about uncertain future. This controversy that has never been resolved”¹

Robert Engle, from New York University, observed a culture in which risk managers had no power and their pronouncements were dismissed as background noise. Joe Nocera and Bethany McLean described the financial crisis in the book named: “All the devils are here - The hidden history of the financial crisis”.

The dilemma between what is possible or probable is another issue that divides those people who work evaluating risks. In the meantime, while it is true that trees never reach the sky, Mother Nature punishes us with events that never happened.

Currently, the responsibility of Actuaries has been increased with the implementation of the International Financial Reporting Standards (IFRS) for the calculation for risks reserves. In this regard our colleagues, the Chartered Accountants, say today "We are already complicated and now come the Actuaries".

In order to build a bridge between risk managers and actuaries I will try develop some ideas that bring closer both positions, because when booking very large risk reserves we diminish the success of the business.

The Enterprise Risk Management process that every CRO (Chief Risk Officer) needs to consider involves the following 5 stages:

1. Risk Identification
2. Risk Valuation, from two perspectives:
   a) qualitative, analyzing the reasons that may cause this risk
   b) quantitative, developing models in order to measure the exposure


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3. **Development of Risk Mitigation Strategies**
   a) **Reduce**, determine the loss derived from avoiding certain operations
   b) **Minimize**, set a loss limit or risk appetite
   c) **Absorb**, add the risk component to the Company’s cost structure
   d) **Transfer**, evaluate what is more convenient among selling part of the position, hiring insurance or hedging operations with derivatives such as swaps or options
4. **Risk Mitigation Strategy Implementation**
5. **Risk Control**

The dream of every Risk Manager is to find a solution for every risk the Entity faces by using a matrix like the following:

![Risk Matrix](image)

<table>
<thead>
<tr>
<th>Probability of Threat</th>
<th>Volume of Damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Risk = probability of threat * value of damage

Values:
1 = without value
2 = low
3 = medium
4 = high

Whith the aim to standardize the Risk Management Process outcomes, regulations are leading us to an extreme complexity of models. Along this way we have passed through Basel I, Basel II, Basel III, Solvency I and Solvency II. But these standards are not alone, they are complemented by the rules issued by Local Regulators, the ICAAP calculation and the risks derived from size and specific characteristics of our organization and, finally, the Macroeconomic surrounding where we perform our business.
**Stress Testing**

The problems appeared in 2008, when Risk Managers failed in differentiating what is VaR (Value at Risk) from Stress Testing. Based on this confusion and adding the fact that quants were not listened during the “Subprime” crisis, the failure condemn to VaR was an unavoidable consequence.

The Value at Risk outcomes are very helpful in order to quantify the potential losses we may face under normal market conditions while Stress Testing could be considered as a statistical measure of risks, given that is not associated with any calculation of probabilities like VaR it is. In summary, VaR vs. Stress Testing is nothing but the fight between the words Probable and Possible.

Stress testing objective is to combine the risks, estimated with the help of some probabilistic models, with the losses that may occur if our analysis takes into consideration the occurrence of stress situations. There is no specific way to combine these tools but there are some cases in which imagination is baffled by the facts.

Some of the statistical models available in order to complement VaR calculations are as follows:

- Continuous, Discrete and Heavy Tails probability distributions
- Time Series Analysis
- Calculation of Conditional Volatility with EWMA and Garch models
- Trend, Spline Lines and DNS models
- Transitions Matrices and Binary Regression for Scoring models
- Gap Duration Analysis
- Distributions Convolution for operational risk events
Credit Risk

The Basel II capital adequacy framework for banking institutions, also known as A-IRB, proposes a set of techniques for measuring credit risk.

Credit risk estimation is based on the well-known formula where the expected loss (EL) is the result of PD * EAD * LGD.

- **LGD (Loss Given Default)**

Basel documents explain that, in theory, LGD is calculated in different ways but the most popular is the “Gross LGD”; where the total losses are divided by the Exposure at Default (EAD). Another method for LGD estimation is to divide the losses by the portion of a credit line without warranty (or where the warranty covers only a portion of the EAD). This is methodology is known as “White LGD”. Based on the mentioned, if collateral value is zero the “White LGD” is equivalent to the “Gross LGD”.

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Organizers:
• **EAD (Exposure at Default)**

Under F - IRB, the EAD is calculated by taking into account the underlying assets, their future assessment, details of the contracts types and commitments. On the other hand, under A-IRB the Banking Institution is allowed to determine the appropriate EAD to be applied to each exposure. A bank using internal EAD estimates for capital purposes might be able to differentiate EAD values on the basis of a wider set of characteristics of the transaction (e.g. product type) as well as the characteristics of the borrower.

Entities can also implement blind coefficients, and there are different types of statistical methods to do so. Under F - IRB approach, focused on retail portfolio analysis, the entity determines the corresponding loss given default applicable to each commercial exposure on the basis of data analyzed.

• **PD (Probability of Default)**

There are two models widely used for the calculation of the PD. Companies that have risk rating tools, usually apply transition matrices while retail credit institutions or entities without risk rating the generally use scoring models.

  a. **Transition Matrix**

This is a perfect example of Markov model. A transition matrix perfectly fits the concept that future comes from the past and passes through to the present. This model calculates, for a particular loan or group of loans, the probability of changing its state of nature in the following period, e.g. one year.
This is the transition matrix used by the Credit Metrics model:

<table>
<thead>
<tr>
<th>Initial qualification</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>Def.</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBB</td>
<td>0.02</td>
<td>0.33</td>
<td>5.95</td>
<td>86.93</td>
<td>5.30</td>
<td>1.17</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>BB</td>
<td>0.03</td>
<td>0.14</td>
<td>0.67</td>
<td>7.73</td>
<td>80.53</td>
<td>8.84</td>
<td>1.06</td>
<td>1.06</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0.11</td>
<td>0.24</td>
<td>0.43</td>
<td>6.48</td>
<td>83.46</td>
<td>4.07</td>
<td>5.20</td>
</tr>
<tr>
<td>CCC</td>
<td>0</td>
<td>0</td>
<td>0.22</td>
<td>1.30</td>
<td>2.38</td>
<td>11.24</td>
<td>64.86</td>
<td>19.79</td>
</tr>
</tbody>
</table>

In this model, grades are provided by rating agencies to each company or country that has issued debt instruments in the market or is planning to do so.

Among the major fails that rating agencies have had in the recent times we can mention Argentina 2001, Subprime Crisis 2008-2009 and the Madoff affair. If we look at Argentina default, one of the largest in world history, we can observe that in June 2001 the country’s debt was graded as BBB and, in consequence, it was offered to investors around the world. This is the reason why, up to today, there is a lot of holdouts from Europe and Japan claiming for their defaulted bonds to be paid.

If we take a look at the first line of the transition matrix exposed above, it shows that probability of a BBB debtor falling into default is 0.18%. But this would only have happened if we should have applied excess interest rate model as follows:

\[ P^* = \frac{VN}{(1 + i^*)} \]

Where

\( P^* \) is the price paid for the bond at the time of purchase
\( VN \) is the Nominal value of the bond
\( i^* \) is the resulting interest rate
If we call $\Pi$ to the probability of default, it is easy to break down the price of the bond in the following way:

$$P^* = \left[ \frac{VN}{(1+i)} \right] (1-\pi) + \left[ \frac{fVN}{(1+i^*)} \right] \pi$$

As you can see, the bond price is compound by two terms. The first one is the value of the bond at a rate without risk, weighted by the probability of no default $(1-\Pi)$. The second component is the receivable value in case the debtor falls into default. In this case, the value of $f$ is the rate that is recovered in a default situation. Up to this year, the recovery rate is approximately 70%.

After making the corresponding term passages, we can see that the probability of default is:

$$\pi = \frac{(i^* - i)}{(1 + i^*)(1 - f)}$$

Returning to our example, in 2001 the rate of the 10-year U.S. Treasury bond -that we take as risk-free rate- was 5% while the rate paid by the BBB Argentinean bonds was $i^* = 15\%$. Applying a recovery rate of 70%, as it was the case for the Russian default; the probability of default was 12.4%. As you can clearly observe, this was far away from the 0.18% shown in Credit Metrics transition matrix.
b. The Scoring Models or Binary Regression Models

Scoring models are based on the behavior of borrowers and have their source in the development of a binary equation that is composed of a dependent variable with binomial behavior where:

\[ Y_i = 1 \text{ (if there is default)} \]
\[ Y_i = 0 \text{ (if there is not default)} \]

For every debtor analyzed, the regression variables used in the model may be several and they must be evaluated according to the destinations test that applies to each one of them. These variables can be binomial, e.g., if it is the owner or not, if it is employed or independent, etc., or continuous like salaries, relationship between income quota, etc.

Then the regression model is:

\[ P(y_i = 1|X; \beta) = 1 - F(-X; \beta) \]

Where \( F(-X; \beta) \) is a continuous function that denotes the probability of default in our case. It is applicable to the coefficients found in the regressions for every explanatory variable.

This cumulative probability depends on the binomial model selected for the \( F(-X; \beta) \) distribution. The parameters of the model that only estimate a dependent variable that takes values of 1 or 0; result from a regression of least squares and applying the concept of maximum likelihood for the calculation of the coefficient of the matrix \( X; \beta \).

These binary regression models depart from the composition of the model, not only in the selection of explanatory variables but in the selection of the distribution of probabilities of model errors.

Next you can find three binary regression models, presented in order for you to make your estimations:
1) The Probit model

\[ P(y_i = 1 | x_i \beta) = 1 - \Phi(-x_i \beta) = \Phi(x_i \beta) \]

This model presents a normal distribution for the regression errors from \( X' \beta \) matrix.

2) The Logit model

\[ P(y_i = 1 | x_i \beta) = 1 - e^{-x_i \beta} \]

\[ = \frac{e^{x_i \beta}}{1 + e^{x_i \beta}} \]

In this model, the distribution of the regression errors follows a logistic probability distribution with heavy tails or extreme values on both sides of the curve.

3) The Gompit model

\[ P(y_i = 1 | x_i \beta) = 1 - (1 - \exp(-x_i \beta)) = \exp(-x_i \beta) \]

The last model is based on the cumulative distribution of probabilities for the type I distribution of extreme values with an asymmetric probability distribution and estimated direction of least tending to a Weibull distribution.

The statistical LR, used to test if all of the explanatory variables as a whole, is equal to zero except for the constant and the Mc Fadden R² coefficient is applied in order to determine which model is the most efficient.
The Credit Scoring models' methodology and some measures on their quality have been discussed in surveys carried out by Hand and Henley (1997), Thomas (2000) and Crook and others (2007). However, only ten years ago, general literature regarding the topic of Credit Scoring models quality was not substantial. Fortunately, the situation has improved in the last decade with the publication of the works of Anderson (2007), Crook et al (2007), Siddiqi (2006), Thomas et al. (2002) and Thomas (2009), all of them addressing the Credit Scoring issue.

Despite the existence of several recent books and journal articles, there is extensive work dedicated to the evaluation of Credit Scoring models quality in all its complexity. Due to this reason, I have decided to summarize the findings noted in this field; starting with the definition of good/bad customers and then considering each of the most popular indexes and their expressions for scores normally distributed, usually with unequal variances.

Some of the rates most used in practice are the Gini rate, widely used in Europe, and the K/S rate, widely used in North America despite its use may not be optimal. It is obvious that the best performance of certain Scoring models it is related to political or expected approval cutoff factors. Therefore, we have to evaluate the quality indexes from this point of view.

Suppose that score $S$ is available for every customer and we offer you the following conditions:

$$D_k = \begin{cases} 1, & \text{good} \\ 0, & \text{bad} \end{cases}$$

The functions of empirical cumulative distribution (Cumulative Distribution Function) of the good (bad) customers are given by the following relationships:

$$F_{n,GOOD}(\alpha) = \frac{1}{n} \sum_{i=1}^{n} I(s_i \leq \alpha \forall D_k = 1)$$
The distribution of the K/S function follows the following format:

\[ F_{n,BAD}(\alpha) = \frac{1}{m} \sum_{i=1}^{m} I(s_i \leq \alpha \forall D_k = 0) \quad \alpha \in [B, A] \]

Where \( s_i \) is the score of the \( i \)th customer, \( n \) is the number of good customers, \( m \) is the number of bad ones and \( I \) is the indicator function, where (True) \( I = 1 \) and (False) \( I = 0 \). \( B \) is the minimum value of a given score and \( A \) is the maximum.

Often used to describe the quality of the model, the Scoring function, is the statistic of Kolmogorov - Smirnov (K/S). This indicator is defined as follows:

\[ KS = \max_{\alpha \in [B, A]} |F_{(m, Bad)}(\alpha) - F_{(n, Good)}(\alpha)| \]

The distribution of the K/S function follows the following format:

In this particular case you can observe that the maximum difference is found where Score value is 900, indicating that at this point you accept 20% of customers as good and reject 55% of the bad credits, being this a cut off a measure.
The Lorenz curve (LC), generally confused with the ROC curve (Receiver Operating Characteristic), can be successfully used in order to determine if the scoring model correctly identifies the bad credit from the good ones. This indicator defines two functions:

\[
\begin{align*}
x &= F_{m,\text{Bad}}(a) \\
y &= F_{n,\text{Good}}(a) \quad a \in [B,A]
\end{align*}
\]

Each point of the curve represents a value for a given score. If we assume a cutoff point, we can read the proportion of customers rejected as bad and good. The following figure provides a good example of a Lorenz curve. In the graph we can see that by rejecting 20% of good customers, we reject between 50% and 60% of the bad ones at the same time.

In addition to the LC the Gini coefficient, considered as an index of the scoring model quality, can also be estimated with this calculation. This index describes the overall quality of a scoring function and takes values between -1 and 1. In an ideal model, i.e. a scoring function that perfectly separates good customers from bad ones has a Gini index equal to 1. On the other hand, a model that assigns a random score to the customers has a Gini index equal to 0. Negative values indicate that we are looking at a model with reversed meanings of scores.
The Gini index can be defined as:

$$Gini = 1 - \sum_{k=2}^{n-m} [(F_{m_k} - F_{m_{k-1}})(F_{n_k} + F_{n_{k-1}})]$$

Once the Gini coefficient has been determined, we are able to estimate c_stat or AUC coefficient in the following way:

$$AUC \cdot c_{\text{stat}} = \frac{1 + Gini}{2}$$

In order to qualify the scoring model, it is necessary to compare the AUC value with the table located on the right of the previous graph.

- **Basel II and its relationship with Macroeconomics - Vasicek K coefficient**

The methodology established in Basel II document for Economic Capital estimation sets a K parameter applicable to the EAD. This coefficient, also known as the adjustment factor of Vasicek’s Rho coefficients, regulates the correlation between Rho coefficients and macroeconomic factors considered.
The K coefficient has to be estimated by using this Vasicek’s formula:

\[
K\% = \left\{ \begin{array}{ll}
N \left[ \frac{N^{-1}(PD) + \sqrt{\rho \times N^{-1}(0.99)}}{\sqrt{1 - \rho}} \right] - PD 
\end{array} \right\} \times LGD
\]

Then \( CE = K \times EAD \)

In the same Basel II document, the Bank for International Settlement (BIS) sets the following values of Rho

- Mortgage loans: \( \rho = 0.15 \)
- Products Revolving: \( \rho = 0.04 \)
- Other retail exposures: \( \rho = 0.03 \left[ \frac{1 - e^{-\rho \cdot PD}}{1 - e^{-\rho}} \right] + 0.16 \left[ \frac{1 - e^{-\rho \cdot PD}}{1 - e^{-\rho}} \right] \)
- Commercial portfolio: \( \rho = 0.12 \left[ \frac{1 - e^{-\rho \cdot PD}}{1 - e^{-\rho}} \right] + 0.24 \left[ \frac{1 - e^{-\rho \cdot PD}}{1 - e^{-\rho}} \right] \)

Due to the convexity of the curve, when PD values used are high the Economic Capital (EC) calculated using this coefficient is lower than the EC calculated with the original methodology; as you can see in the next chart:

**PD Scoring and Vasicek**

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Please note that when using a Rho coefficient for other retail exposures, if the scoring result estimates a PD of 8.5% the PD adjusted with Vasicek’s is the same. But, if the PD estimated with the scoring model is 12% the PD adjusted with Vasicek’s is 10%. In this case, it is recommendable to use the major value observed.
Market Risk

It has been recognized by all of us that markets do not meet any of the classical hypotheses mentioned by several authors until the mid of the 90’s. We, therefore, should reject the following concepts:

- financial assets returns are not \textit{n.i.d.} (normal and identically distributed)
- returns do not represent a “white noise”
- volatility does not follow the \( t^{1/2} \) rule

My experience has shown me that if values analyzed presented important asymmetries and an elevated kurtosis, this denoted the presence of heteroscedasticity.

You may now ask how to solve the problem of estimating market risk without moving away from the basic principle that you want to calculate as market risk: “Maximum possible loss that can occur in a default period with a probability of occurrence set in a subjective way”.

The answer to this question can be summarized in two basic concepts, static and dynamic estimates.

According to what was originally estimated; the maximum possible loss is given by:

\[ EL = \bar{R} - z_{LC} \sigma \]

Where

- \( EL \) is the expected loss
- \( \bar{R} \) is the average return or percentage change in assets prices
- \( z_{LC} \) is the safety coefficient chosen in order to estimate the maximum possible loss (static calculation)
- \( \sigma \) is the asset volatility (dynamic calculation)
Based on the mentioned, the static part of the solution can be found in the determination of the coefficient of safety to be implemented. It depends exclusively on the probability distribution followed by the returns on the asset under analysis.

Discarded the normal distribution of probabilities, aligned to what was mentioned at the beginning of this section, heavy tails distributions come in our help in order to determine the values of the maximum possible loss, with greater accuracy and credibility. Some good examples of distributions that interpret in a better way the behavior of the returns series are:

1. The Logistic Distribution

\[ F(z) = \frac{1}{1 + e^{-z}} \text{ Where } z = e^{-\frac{x - m}{a}} \text{ and } a = \sqrt{\frac{2\sigma^2}{\pi}} \]

The values of \( m \) and \( \sigma \) are the well-known mean and dispersion.

2. The Extreme Values distribution

\[ F(z) = \exp \left\{ -\left( 1 + ez \right)^{-\frac{1}{\varepsilon}} \right\} \text{ Where } z = \frac{x - \mu}{\psi} \]

Under this approach, the coefficients to be used are:

- \( \varepsilon \), that represents the shape of the curve
- \( \psi \), which represents the measure of variability
- \( \mu \), that represents more likely value or mode

This distribution has the characteristic that, depending on the \( \varepsilon \) value, the curve presents a maximum if it is negative or minimum if it is positive. In this regard, is worth to mention that Hosking method allows us the calculation of \( \varepsilon \), \( \psi \), and \( \mu \) coefficients.
3. The Kupiec solution

Paul Kupiec work showed us that, based on a normal distribution; it is possible to extend the tails of a distribution in such a way that probability of catastrophe is also contemplated. The abscissa, called ZKupiec value, is calculated starting from the ends with a 5% probability of occurrence.

\[
Z_{Kupiec} = \sqrt{\frac{p(1-p)}{f(z)^2}}
\]

\[
f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}
\]

<table>
<thead>
<tr>
<th>p</th>
<th>z(normal)</th>
<th>z(Kupiec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>2.326</td>
<td>3.733</td>
</tr>
<tr>
<td>0.02</td>
<td>2.054</td>
<td>2.891</td>
</tr>
<tr>
<td>0.03</td>
<td>1.881</td>
<td>2.507</td>
</tr>
<tr>
<td>0.05</td>
<td>1.645</td>
<td>2.113</td>
</tr>
</tbody>
</table>

Regarding dynamic calculation the problem relates to the volatility behavior. In this sense; J. P. Morgan's 415 report -called in this way because it had to be presented at 4.15 pm- had set the replacement of $t^{1/2}$ rule by the calculation of conditional volatility also known as EWMA.

\[
\sigma_t = \left(1-\lambda\right)\sum_{j=1}^{t} \lambda^j \sigma_{t-j}^2
\]

In this case volatility depends on past events. The best contributions to this dynamic calculation came into use in the mid 90's, thanks to the contributions of Robert Engle and Tim Bollerslev through the Garch models.

In 1982, Engle presented the Arch (q) model where volatility depends on past events according to the following formula:

\[
\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + ... + \alpha_q \varepsilon_{t-q}^2 + \eta_t
\]
On the other hand, in 1986, Bollerslev presented the Garch \((p, q)\) model where he introduced a new equation, easy for market operators to understand but complex for Actuaries to implement:

\[
\varepsilon_i^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{i-1}^2 + \sum_{j=1}^{p} \beta_j h_{i-1}^2 + \nu_i
\]

In both cases \(\varepsilon_i^2\) represents the conditional volatility. They are the errors, to the square, of a regression of the returns on a constant that is the average of the values considered.

Having rejected the presence of a normal distribution of probabilities and recalling that at the beginning of the XIX Century Sir Francis Galton warned us that traditional volatility is not stable, we discarded the \(t^{1/2}\) rule and used the Garch \((1,1)\) model in order to calculate volatility:

\[
\sigma_i^2 = \sigma_{i-1}^2 + \alpha (\varepsilon_{i-1}^2 - \sigma_{i-1}^2) + \beta \sigma_{i-1}^2
\]

Based on the mentioned, if the model is stationary the sum of \(\alpha\) and \(\beta\) parameters must be less than the unit. This sum of \(\alpha + \beta\) is referred as the persistence of the model and is the basis of the Garch \((1,1)\) model that can predict a variation for a horizon of \(t\) days. When we estimate the VaR for an asset or rate, the presence of liquidity risk demands us to provide not only the variation during a day but the possible changes for more days also.

We can observe that, for a series of financial market volatilities, the persistence of a Garch \((1,1)\) model is close to the unit.

In order to represent the model equation we have two alternatives:

a. Conditional variance estimated by a Garch \((1,1)\) model can be written as follows:

\[
\varepsilon_i^2 = \sigma_i^2 \quad \Rightarrow \quad \sigma_i^2 = \varepsilon_i^2 - \nu_i
\]

\[
\varepsilon_i^2 - \nu_i = \omega + \alpha \varepsilon_{i-1}^2 + \beta (\varepsilon_{i-1}^2 - \nu_i)
\]

\[
\varepsilon_i^2 = \omega + (\alpha + \beta) \varepsilon_{i-1}^2 + \nu_i - \beta \nu_{i-1}
\]
The square error of a heterocedastic process resembles an ARMA \((1.1)\) model, where the auto regressive root governing the model is the sum of \(\alpha + \beta\).

b. It is possible to, recursively, replace the variation of previous periods on the right side of the equation and express the variation as the sum of a weighted average of all of the wastes to the square.

\[
\sigma_i^2 = \sigma^2 + \alpha \epsilon_{i-1}^2 + \beta \sigma_{i-1}^2 \\
\sigma_i^2 = \sigma^2 + \alpha \epsilon_{i-1}^2 + \beta (\sigma + \alpha \epsilon_{i-2}^2 + \beta \sigma_{i-2}^2) \\
\vdots \\
\sigma_i^2 = \frac{\omega}{1 - \beta} \frac{(1 - \beta^k)}{1 - \beta} + \alpha \sum_{j=1}^{k} \beta^{j-1} \epsilon_{i-j}^2 + \beta^k \sigma_{i-1}^2 \\
\sigma_i^2 = \frac{\omega}{1 - \beta} + \alpha \sum_{j=1}^{k} \beta^{j-1} \epsilon_{i-j}^2 
\]

Considering that a Garch \((1.1)\) model decays exponentially pondering recent volatility, if time delay setting is \((1-\beta k) = 1\), it gives greater importance to the nearest volatility and less importance to more distant values because \(\beta k = 0\) affects the last residue. Consequently the variation for the period \(t\), recursively, is as follows:

\[
\sigma_{t+r}^2 = \sigma_t^2 + \sigma_{t+1}^2 + \sigma_{t+2}^2 + \cdots + \sigma_r^2 \\
E_{t+r}(\sigma_{t+r}^2) = E_{t+1}(\sigma_t^2) + E_{t+2}(\sigma_{t+1}^2) + E_{t+3}(\sigma_{t+2}^2) + \cdots + E_r(\sigma_r^2) 
\]

As previously indicated, with a Garch model the variance of the returns can be modelled as a predictable process in the following way:

If it is calculated with a Garch \((1,1)\) model using conditional variance, the forecast for the period number 2 is calculated for the first period as follows:
\[ E_{t-1}(\sigma^2_{t+1}) = E_{t-1}(\sigma + \alpha \varepsilon^2_t + \beta \sigma^2_t) = \sigma + (\alpha + \beta)\sigma^2_t \]
\[ E_{t-1}(\sigma^2_{t+2}) = E_{t-1}(\sigma + \alpha \varepsilon^2_{t+1} + \beta \sigma^2_{t+1}) = \]
\[ = \sigma + (\alpha + \beta)\left[\sigma + (\alpha + \beta)\sigma^2_t\right] = \sigma + \sigma(\alpha + \beta) + (\alpha + \beta)^2 \sigma^2_t \]

Replacing \( T \) for the future period is the following formula that ensures the variation during \( T \) day if the series has a \text{Garch (1,1)} behavior. In the formula mentioned above if \( \omega \) tends to zero the decay factor is \((\alpha + \beta)k\). Due to this reason a strong impact in the series has an exponential fall and stress disappears in a few days tending to the non-conditional variance. The number of days is a function of the sum of \( (\alpha + \beta)k \) or persistence of the model.

In consequence, the sum that totals the variation for the \( T \) periods are as follows:

\[ E_{t-1}(\sigma^2_t) = \frac{\sigma}{1-(\alpha+\beta)} \left[ \left\{ \frac{1-(\alpha+\beta)^{T-1}}{1-(\alpha+\beta)} \right\} \frac{1-(\alpha+\beta)^T}{1-(\alpha+\beta)} \right] \sigma^2_t \]

Having started from these models, along time several modifications have appeared in order to calculate conditional volatility such as \text{Tarch, Egarch, Agarch, Igarch and P-Arch models}. The interesting thing about these models is that they allow us to set how long a strong impact is diluted in time or has a traditional volatility by using the coefficients \( \alpha \) and \( \beta \) in the following form:

\[ \Delta = \frac{1}{1-(\alpha+\beta)} \]

A back-testing of the mentioned shows the following for a \text{Garch (1.1) model} calculated using a logistic distribution:
Therefore, with this model we are able to calculate what happens in a business plan at some point if a possible or probable drop in prices takes place since it is estimated that it can occur.
**Interest rate Risk**

The study of interest rates structures has implications in terms of financial risks and macroeconomics. Their calculations are performed in order to meet the expectations regarding inflation and GDP growth. Financial risks estimation allows us to establish the cash flow composition and the risk of cost increases derived from its mismatch. Expectations are expressed in the structure found and the implicit interest rate for each one of the periods involved in the curve.

Therefore, bearing in mind that \((1 + i)^n\) is not true it has been necessary to perform the analysis of interest rates structures in order to better understand the interest rates behavior during the period under study.

\[
TNA_t(\lambda) = \theta(z_t) + \varepsilon_t \\
= \beta_{1t} + \beta_{2t} \left(\frac{1 - e^{-\lambda T}}{\lambda r_i} \right) + \beta_{3t} \left(\frac{1 - e^{-\lambda T}}{\lambda r_i} - e^{-\lambda t} \right) + \varepsilon_{it}
\]

Diebold and Li (2006) compare their two-stages forecast against univariate and multivariate time series methods. The different methods produced similar results, but the two-stages forecast approach obtained better predictions than the direct series of different interest rates, especially for the longer term.

- **\(\beta_1\)** is a variable that is independent of the time of maturity. It reflects the long-term yields that are at the end point of the curve.

- **\(\beta_2\)** influences in the beginning of the curve (also called short end) and is weighted by a function of the time of maturity. This function is 1 if \(T = 0\) and slightly exponential. It decreases to zero when \(T\) is large.

- **\(\beta_3\)** is also \(T\) function, but this function is zero for \(T = 0\). In this case, it decreases and then returns to zero when \(T\) grows. Therefore, the impact of \(\beta_3\) is the addition of a bump in the curve.
Organizers:

- $\lambda$ affects the weights of $\beta_2$ and $\beta_3$ functions. Therefore, it "determines the DNS model. The load of the $\lambda$ parameter determines the shape of the yield curve". In previous studies, by default was set a value for $\lambda$ without estimate. For example, Diebold and Li (2006) fixed $\lambda$ in 0.0609 while Diebold and Rudebusch Aruoba (2006) estimate that $\lambda$ is equal to 0.077.

Yu and Zivot (2007) adopted these values for $\lambda$ in their empirical study related to corporate bonds. They argue that the $(\lambda)_{ij}$ of $\lambda$ are not very sensitive to different values of $\lambda$. In addition they affirmed that (i) this could be graphically illustrated and (ii) that a fixed $\lambda$ could maximize the load on the component of curvature in the medium term (i.e. 30 months for $\lambda = 0.0609$ and 23.3 months for $\lambda = 0.077$).²

- **How to estimate the $\beta$ coefficients and set the value of $\lambda$**

It is known the formula:

$$TNA_t = \beta_{1t} + \beta_{2t} \left( \frac{1 - e^{-\lambda T_t}}{\lambda T_t} \right) + \beta_{3t} \left( \frac{1 - e^{-\lambda T_t}}{\lambda^2 T_t} - e^{-\lambda T_t} \right) + \epsilon_t \quad (DNS)$$

Where $TNA_t$ is the estimated value for each period $t$ according to the coefficients of the equation $\beta_1$, $\beta_2$, $\beta_3$ and $\lambda$.

It is also known that:

$$\epsilon_t^2 = (TNA_t - TNA_{t-1})^2$$

---

² Analysing the Term Structure of Interest Rates using the Dynamic Nelson-Siegel Model with Time-Varying Parameters. Siem Jan Koopman, Max I.P. Mallew, Michel van der Wel. Department of Econometrics, VU University - Amsterdam, Department of Finance, VU University – Amsterdam, Tinbergen Institute - Amsterdam

Organizers:
Consequently, if we change at least the $\beta_1$, $\beta_2$, $\beta_3$ coefficients but maintaining the $\lambda$ value chosen based on the logarithmic trend’s slope; the best values obtained can be used for the TIRS (Temporary Interest Rate Structure) calculation.

Given that the calculation of the $\beta_3$ maximum determines the shape of the curve, that has a maximum value equal to 0.30, we should look for a solution to the equation for each value of $\tau$ or number of months using the following:

$$\beta_3 = 0.3 = \left(1 - \frac{e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau}\right)$$

<table>
<thead>
<tr>
<th>Months</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.29888036</td>
</tr>
<tr>
<td>9</td>
<td>0.19925357</td>
</tr>
<tr>
<td>12</td>
<td>0.14944018</td>
</tr>
<tr>
<td>15</td>
<td>0.11955214</td>
</tr>
<tr>
<td>18</td>
<td>0.09962678</td>
</tr>
<tr>
<td>21</td>
<td>0.08539439</td>
</tr>
<tr>
<td>24</td>
<td>0.07472009</td>
</tr>
<tr>
<td>27</td>
<td>0.06641786</td>
</tr>
<tr>
<td>30</td>
<td>0.05977607</td>
</tr>
</tbody>
</table>

- What can we do with residuals?

The use of square residuals is interesting, mostly in order to calculate a parallel jump of the interest rate structure found. This is a more complicated issue that exceeds the basic financial calculation. However, due to its importance, it should be applied since the financial risk in a given interest-rate curve derived from the mismatch of cash flows causes heavy losses that must be covered by financial institutions.

The interest rate risk may be estimated by using the square root of the mean of square residuals multiplied by the coefficient of confidence given by the probability distribution that follows these variations. The value of the jump above mentioned would be:

$$\Delta TNA = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n}} K$$
Liquidity Risk

In terms of liquidity risk, we can say that the mismatch of cash flows affects the company’s portfolio in two ways:

a) Higher financing costs
b) Inability to comply with obligations

- Gap Duration

One possible way for financing is to liquidate the company’s portfolio in order to obtain the required resources to fulfill the entity’s obligations. Under this scenario, the value of the portfolio is given by the value of the discounted cash flow expected form those assets, according to the following basic principle:

\[ P = \frac{FF_1}{(1+i)} + \frac{FF_2}{(1+i)^2} + \cdots + \frac{FF_n}{(1+i)^n} \]

Where \( FF_n \) represents the expected value to be paid or collect by a business line or credit, \( i \) represent the interest rate used to actualize this cash flow, and \( P \) is the current value of the portfolio under analysis.

In order to know the interest rate to be considered for portfolio liquidation purposes, we will use the interest rate structure offered by the market to develop a model using the DNS model explained in the interest rate risk chapter.

Once the fee structure have been obtained, bearing in mind the interest rates previously found, we need to calculate the current values of assets and, at the same time, their duration in order to estimate the risk of those assets that could derived from a variation in the interest rate, according to the following development:
\[ MD = -\frac{1}{(1+i)} \sum_{t=1}^{n} \frac{tCF_t}{P} \]
\[ CV = \sum_{t=1}^{n} \frac{t(t+1)CF_t}{(1+i)^{t+2}} \]

Both effects can be graphically shown:

\[ P - P' \] is the net loss of the portfolio by the incidence of Duration and Convexity
\[ P - P'' \] is the net loss of the portfolio only by the incidence of Duration

So the variation percentage of the portfolio is given by:

\[ \Delta% P = MD \delta i + 0.5 CV \delta i^2 \]

After calculating the duration of applicable assets and liabilities, Gap Duration analysis needs to be performed according to the following tables:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Market Value</th>
<th>Duration</th>
<th>Value weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail Finance (P00)</td>
<td>VM P00</td>
<td>D P00</td>
<td>VM P00*(D P00/100)</td>
</tr>
<tr>
<td>Retail Lease (LSG)</td>
<td>VM LSG</td>
<td>D LSG</td>
<td>VM LSG*(D LSG/100)</td>
</tr>
<tr>
<td>Total Assets</td>
<td>TMVA</td>
<td>DA</td>
<td>TWA</td>
</tr>
</tbody>
</table>

1. The market value is the current value of the cash flow discounted with the interest rate structure found.
<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Market Value</th>
<th>Duration</th>
<th>Value weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Deposits (PF)</td>
<td>VM PLF</td>
<td>D PLF</td>
<td>VM PLF*(D PLF/100)</td>
</tr>
<tr>
<td>Bank Loans (CLT)</td>
<td>VM CLT</td>
<td>D CLT</td>
<td>VM CLT*(D CLT/100)</td>
</tr>
<tr>
<td>Bonds (ON)</td>
<td>VM ON</td>
<td>D ON</td>
<td>VM ON*(D ON/100)</td>
</tr>
<tr>
<td><strong>Total Liability</strong></td>
<td><strong>TMVL</strong></td>
<td><strong>DL</strong></td>
<td><strong>TWL</strong></td>
</tr>
</tbody>
</table>

1. The market value is the current value of the cash flow discounted with the interest rate structure found.

This information is calculated for the different durations of each group and is finally used in order to obtain the values of the last column, which represents the proportion in which each group participates in total the Gap Duration.

Calculations were performed as follows and using the following nomenclature:

\[
W = MV \times D, \quad DA = \left(\frac{TWA}{TMVA}\right) \times 100, \quad DL = \left(\frac{TWL}{TMVL}\right) \times 100
\]

- The effects of the cash flow mismatch

Cash flows mismatch is present at the time of assets valuation. Liquidity risks, in the event of market illiquidity, derive from the fact that these assets will have to be financed at higher interest rates and shorter tenors.

\[
GD = \frac{\Delta BP}{10000} \times \frac{DL}{(1 + Tir)}
\]
Operational Risk

The definition of Operational Risk (OR) adopted is based on four sources of risks: internal processes, human resources, systems and external events. Thus, the first definition agreed in the banking industry was published by the Basel Committee (2001) through the document named "Working paper on the Regulatory Treatment of Operational Risk", which defines operational risk as "the risk of loss due to the inadequacy or failure of processes, personnel and internal systems or well because of external events".

Operational risk losses continue putting on the spotlight financial institutions of international prestige. As an example, given its importance, we can mention the USD 2 billion losses suffered by JP Morgan Chase in 2012. The lack of proper internal controls did not allow the detection of an error in its operations with derivatives. The severity of the losses was reflected in the decision taken by Fitch rating agency which downgraded the bank note, a step up to A + with a negative Outlook.

It is evident that this type of losses can endanger the stability of financial systems. So, the need for an adequate and robust operational risk system of measurement and control should continue to be a concern for agents involved in its management and supervision.

Operational risk defined by Basel III does not differ from Basel II:
- inadequacy or failure of processes
- personnel and internal systems
- caused by external events

For operational risk estimation, Basel framework offers the following approaches:

1. Basic indicator Approach (BIA)
2. Standardized Approach (SA)
3. Advanced Measurement Approach (AMA)
Regarding the AMA, Basel II establishes some general guidelines allowing financial institutions to design its own models for operational risk measurement and management. There is a wide flexibility from the Committee which defined a series of key issues regarding the implementation of the Loss Distribution Approach (LDA), which in practice leads to very different results.

Based on the mentioned, banks with the same operational risk profile can face very different capital requirements depending on the choices made in terms at the time of the operational risk model development and implementation. According to this, capital requirements vary significantly depending on the chosen loss threshold, the severity distribution estimated and, to a lesser extent, the theoretical frequency distribution (adjusted).

The methodological diversity mentioned intends to bring the banks towards the advanced approach, in such a way that each entity specific operational risk profile is reflected in more detail.

This will help to quantify capital requirements more precisely. In general, they are expected to be less than the ones calculated using the basic or standard approaches.

The origin of the LDA is related to risk applications from the insurance business, being the following the requirements for the approval of internal OR models:

**General requirements**
- Active involvement of senior management in the OR management
- Integration between systems of measurement and management
- OR resources available in areas of control and audit

**Qualitative requirements**
- Unit independent of management responsible for methodology implementation
- OR model integrated to the institutional OR management process
- System to provide periodical OR information to senior management
- OR system sufficiently documented
- OR system internally and externally validated

Organizers:
Quantitative requirements

- Identify events that generate big losses and are located in the tails of the distribution
- Satisfy IRB criteria of robustness
- Capital requirement calculation should be the sum of the losses expected and unexpected (stress testing)

In order to model the amount of operational risk losses, Basel Committee proposes a list of sub exponential distributions. In order to perform the analysis, the Bank has considered Pareto and Log Normal distributions, since they are the most used in the international market.

<table>
<thead>
<tr>
<th>Severity function (X)</th>
<th>Severity Distribution (F(x))</th>
<th>Parameter conditions</th>
<th>Estimation using Poisson distribution as frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pareto ((\theta,\alpha))</td>
<td>(1 - \left(\frac{\theta}{\theta + \alpha}\right)^\alpha)</td>
<td>(\theta &gt; 0) (\alpha &gt; 0)</td>
<td>(\theta \left[\frac{\lambda - 1}{\lambda - 1}\right])</td>
</tr>
<tr>
<td>Log Normal ((\mu,\sigma))</td>
<td>(\Phi\left[\ln(x) - \mu\right]\sigma^{-1})</td>
<td>(\sigma &gt; 0)</td>
<td>(\left(1 - \frac{1}{\lambda}\right)\exp\left(\mu + \frac{\sigma^2}{2}\right)\exp\left[\mu + \sigma^2\left(1 - \frac{1}{\lambda}\right)\right])</td>
</tr>
</tbody>
</table>

Next, you will find a graph that synthesizes the process to follow in order to apply the LDA:
The frequency distributions most commonly used are as follows:
Severity distributions used for the operational risk calculation are as follows:
Conclusions

- Possible and probable are two words non reconcilable. Probable is part of VaR and Possible is part of Stress Testing.
- Possible stress situations have to be considered. An extreme value may occur and there are models that explain how a great negative impact dissolves along time.
- Credit Risk:
  - This risk has two parts: (i) Debtors with qualification provided by credit risk grading agency and (ii) debtors qualified by a credit scoring.
  - Together with the qualification provided by credit risk grading agencies is important to verify what happens with the excess of interest rate that an investment is offering.
  - Scoring models can be periodically revised. They have to be reformulated when test results are bad. And also needs to be revised the score limits set in order to accept a debtor (risk appetite).
- Market Risk:
  - This is where discussions between quants and believers are focused on. There are many professionals who believe in the classical hypotheses, but reality and back testing have shown that we have to believe in the models.
- Interest Risk and Liquidity Risk:
  - It is very difficult to divide these two risks. Interest rate risk directly affects of assets and liabilities durations. The main challenge is to insert a jump in the TIRS analysis. Parallel jumps do not take into consideration some possible stress situations.
- Operational Risk:
  - Here lies a big controversy with Basel framework. Institutions have not the same operating manual or control environments. Each entity has his own and particular risk profile. For that reason, is necessary to determine the frequency and severity distributions based on the Company’s history of operational risks events. Then, a recommended solution is to proceed with a convolution between both distributions.
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