Valuation of embedded options by closed formulas in the framework of an ORSA model

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An alternative to a classical ALM model for ORSA

- Scope : Savings contracts
- ORSA framework requires obtaining information about the distribution of coverage ratio on the horizon of the strategic plan of the company

- Complexity:

1. Interactions between assets and liabilities
2. Embedded options (guaranteed minimum rate, profit sharing, surrender, etc.)

- Why don't we think of new classes of models ?

The major idea

Marriage between a french model and a Norwegian method of valuation

How it works

- Approximation of the initial Best Estimate:

$$\textit{Best Estimate}(0, \textit{Maturity}) = \alpha \times \sum_{j \geq 1} \Omega(0,j) \times \mathbb{E}^{\mathbb{P}^a}[\textit{benefit}] - \sum_{j \geq 1} P_n(0,j) \times \mathbb{E}^{\mathbb{P}^a}[\textit{premium}]$$

- α is a coefficient representing the cost of the surrender option
- $\Omega(0,j)$ the cost of revaluation option
- $P_n(0,j)$ the price of nominal zero-coupon bond at the initial time maturing at j
- $\mathbb{P}^a$ the historical probability

Focus

Proposition of a method for estimating factors $\Omega(0,j)$

Cost of revaluation option

- The proposed extension/valuation:

$$\Omega(0,j) = \mathbb{E}^{\mathbb{Q}}\left[\frac{e^{max(TMG_1,return_1)}}{e^{Discount_1}} \times \frac{e^{max(TMG_2,retunr_2)}}{e^{Discount_2}} \cdots \frac{e^{max\left(TMG_j,retunr_j\right)}}{e^{Discount_j}}\right]$$

- $\mathbb{Q}$ the risk-free probability used for valuation
- Our approach relies on the work of Lindset [2001] and Persson and Aase [1996]
- Using a Vasicek [1977] model, we have a closed formula :

$$\Omega(0,t) = \sum_{j=1}^{2^{m_t}} e^{{}^t\bar{c}_j\bar{G} - {}^t\bar{c}_j\bar{\Lambda} + \frac{1}{2}{}^t\bar{c}_j\bar{\Sigma}\bar{c}_j}\,\Phi\left(\bar{\alpha}_j^{\beta}, \hat{c}_j\underline{\bar{\Sigma}}_{\beta}\hat{c}_j\right)$$

- $\Phi(a,V)$ is the cumulative multivariate normal distribution evaluated at the points determined by the vector a and with variance covariance matrix V

Study of the cost of revaluation option

- Calibrate the model parameters on the curve published by EIOPA April 30, 2014
- Consider two types of returns, $\Omega_1(0,j)$ for a return equal to the discount process ($return_i = Discount_i$) and $\Omega_2(0,j)$ for a return of a Black & Scholes [1973] Gaussian asset, negatively correlated with the discount process :

Maturity	$\Omega_1(0, Maurity)$	$\Omega_2(0, Maurity)$
1	1.014604	1.080652
2	1.025127	1.180592
3	1.031960	1.278935
4	1.035486	1.383176
5	1.036076	1.493779

Table: Comparaison of the price of the revaluation between two return profiles

Using the specific parameters of our projection:

- $\Omega_1(0, Maturity)$ is more sensitive to volatility rate, compared to the sensitivity to minimum guaranteed rate or the initial nominal rate.
- $\Omega_2(0, Maturity)$ is very sensitive to the guaranteed annual minimum rate.

Using direct Monte Carlo methods

- Total error can be decomposed into a systematic error, and a statistic error:

$$\begin{aligned}
\varepsilon(N, M) &= \hat{\Omega}_{N,M}(0,t) - \Omega(0,t) \\
&= \mathbb{E}^{\mathbb{Q}}\left[\hat{\Omega}_{N,M}(0,t)\right] - \Omega(0,t) + \hat{\Omega}_{N,M}(0,t) - \mathbb{E}^{\mathbb{Q}}\left[F_{N,M}(0,t)\right] \\
&= \varepsilon_{sys} + \varepsilon_{stat}
\end{aligned}$$

where N is the number of simulations and M number of discretizations.

- ε_{sys} the systematic error characterizes the discretization scheme, the numerical method to approximate the integral and the number of discretization
- ε_{stat} depends on the number of simulations

Focus

Can we find a tradeoff between increasing N and M given a budget of time ?

Using direct Monte Carlo methods

- Duffie and Glynn [1995] theorem for $\Omega_1(0, t)$:

N	M	$MSE(N, M)$ Exact	Reduction	$MSE(N, M)$ *Euler*	Reduction
4	16	3.1787×10^{-5}	2.2997	2.7033×10^{-4}	2.0765
8	256	1.3822×10^{-5}	1.9861	1.3018×10^{-4}	1.5168
32	1024	6.9593×10^{-6}	1.1436	8.5829×10^{-5}	1.9491
64	4096	6.0853×10^{-6}	3.3430	4.4035×10^{-5}	3.1607
128	16 384	1.8202×10^{-6}	1.3609	1.3931×10^{-5}	1.1718
256	65 536	1.3375×10^{-6}	1.7584	1.1888×10^{-6}	3.3272
512	262 144	$7.6064 \times \times 10^{-7}$		3.5732×10^{-6}	
			Avg reduction $=1.98$		Avg reduction $=2.20$

Table: Estimation of Mean squared error reduction

Conclusion

Summary :

- The model enables stochastic projection and the valuation of the profit-share option
- Given a limited budget of time for simulations, the error using Monte Carlo simulation was estimated
- Considering embedded options by closed formulas leads to consistent operational results for ORSA
- The model avoids complications of an excess of parametrization

Limits

- The model concerns only the financial risks
- The ability to use the closed formulas intimately depends on the form of revaluation rate
- The calibration of certain parameters of the model can be complicated

Objectives

- This class of models is a promising research path
- Study the calculation of the distribution function of a multivariate normal distribution (used to calculate Ω)
- Optimize the calculation of factors Ω using a different tool than R
- Generalize the approach using Partial Differential Equation