Controlling a demographic wave in defined contribution pension systems

M. Angrisani[1] C. Di Palo[2]

[1]Sapienza University of Rome, Italy
Department MEMOTEF

[2]University of Cassino and Southern Lazio, Italy
Department of Economics and Law

IAA Colloquia
Oslo 7-10 June 2015

Outline

Purpose

The paper purpose is to provide an operating method that allows a DC pension system, in a state of stable sustainability, to overcome the issues of sustainability and, nevertheless, of intergenerational equity, arising when a demographic wave disrupts the system equilibria.

The method is developed on the basis of a general principle, referred to as the Separation Principle.

The method for controlling a demographic wave

We develop an actuarial-mathematical model, in the framework of the logical sustainability approach to manage the problem.
We provide a theorem, based on the general Separation Principle, which outlines the operating method and, in particular, states a rule on the rate of return on the pension liability.
Numerical illustrations of the theorem are also provided.

Remarks

In this study, we do not consider the longevity problem and we make use of deterministic rates: these facts do not affect the validity of the theoretical principle.

A first extension of our model in the case that the financial rate and the growth rate of productivity are both modelled by stochastic processes was presented at MIC Conference 2015, Portoroz 28-31 May.

Results

We prove that in order to tackle the demographic wave problem it is not possible to exclusively follow a PAYG scheme neither is it necessary to shift to a fully funded scheme.

The separation principle proposed asserts that it is necessary to fund all that is PAYG unmanageable, in this way overcoming the classical juxtaposition between PAYG and fully funded schemes.

The logical sustainability model

The logical sustainability model deals with:

- Formalization of pension systems
- Study of the conditions that imply the pension system sustainability

Regarding to our approach, the only previous reference, although "weak" and "primitive", in the literature for pension systems is provided by

- the Aaron's theorem (1966)

To our knowledge, all other "results" in this field are based on numerical simulations...

The logical sustainability model

It is based on the following papers

Main References

Angrisani, M. (2006, May). Funded and unfunded systems: two ends of the same stick. In 28th International Congress of Actuaries, Paris, France.
Angrisani, M. (2008). The logical sustainability of the pension system. Pure Mathematics and Applications, 19(1), 67-81.

The model functions

For each t in T, we denote with:

- $\alpha(t)$ the contribution rate
- $C(t)$ and $W(t)$ are the instantaneous flows of contributions and wages, respectively, with $C(t) \geq 0$, $W(t) > 0$, and $C(t) = \alpha(t)W(t)$;
- $P(t)$ is the instantaneous flow of pension expenditure at time t, with $P(t) > 0$;
- $L^T(t)$ is the total pension liability, with $L^T(t) > 0$;
- $F(t)$ is the pension system fund;
- $r(t)$ is the instantaneous rate of return on the fund;
- $r_L(t)$ is the instantaneous rate of return on the pension liability.

Basic Definitions

Definition 1. *A pension system is sustainable in time interval* T
if and only if

$$F(t) \geq 0 \qquad \textit{for each } t \textit{ in } T.$$

Definition 2. *For each* t *in* T*, the unfunded pension liability is*

$$L^{UN}(t) = L^{T}(t) - F(t).$$

It is assumed that $L^{T}(t) \geq F(t)$ for each t in T.

BASIC CONCEPT

$$L^{T}(t) = L^{UN}(t) + F(t)$$

DC pension systems with a structural funded component

Features:

- the key role of a specific indicator, the level of the unfunded pension liability with respect to wages denoted by $\beta(t)$
- the pension system has available two active rates, the financial rate on the fund or the growth rate of productivity
- the pension system can recognize the rate of return on the pension liability on the basis of the two previous rates according to the rule for the stabilization of $\beta(t)$

The model Indicators (1)

Function $\nu(t)$ is the *divisor of the total pension liability in the current pension liability*; for each t in T, it is defined as

$$\nu(t) = \frac{L^T(t)}{L^P(t)}, \quad \textit{with} \quad \nu(t) \geq 1.$$

Function $\gamma(t)$ is the *divisor of the current pension liability in the pension expenditure*; for each t in T, it is defined as

$$\gamma(t) = \frac{L^P(t)}{P(t)}.$$

Function $\gamma(t)\nu(t)$ is the *divisor of the total pension liability in the pension expenditure*; for each t in T, it is defined as

$$\gamma(t)\nu(t) = \frac{L^T(t)}{P(t)}.$$

The model Indicators (2)

Function $\beta(t)$ is the *level of the unfunded pension liability with respect to wages;* for each t in T, it is defined as

$$\beta(t) = \frac{L^{UN}(t)}{W(t)}$$

Function $D_c(t)$ is the *degree of funding of the pension liability*; for each t in T, it is defined as

$$D_c(t) = \frac{F(t)}{L^T(t)}, \text{ with condition } 0 \leq D_c(t) \leq 1.$$

Function $\alpha^{UN}(t)$ is the *level of the unfunded contribution rate*; for each t in T, it is defined as

$$\alpha^{UN}(t) = \frac{\beta(t)}{\gamma(t)\nu(t)}$$

The rule for the stabilization of indicator $\beta(t)$

It is assumed that $0 \leq F(t_*) < L^T(t_*)$.

For each t in T, it results

$$\dot{\beta}(t) = 0 \text{ and hence } \beta(t) = \beta(t_*)$$

if and only if

$$r_L(t) = r(t)\frac{F(t)}{L^T(t)} + \frac{\dot{W}(t)}{W(t)}\frac{L^T(t) - F(t)}{L^T(t)}$$

The rule for the stabilization of indicator $\beta(t)$ in order to the sustainability

Sufficient condition for the sustainability

It is assumed that $0 \leq F(t_*)$.
If for each t in T

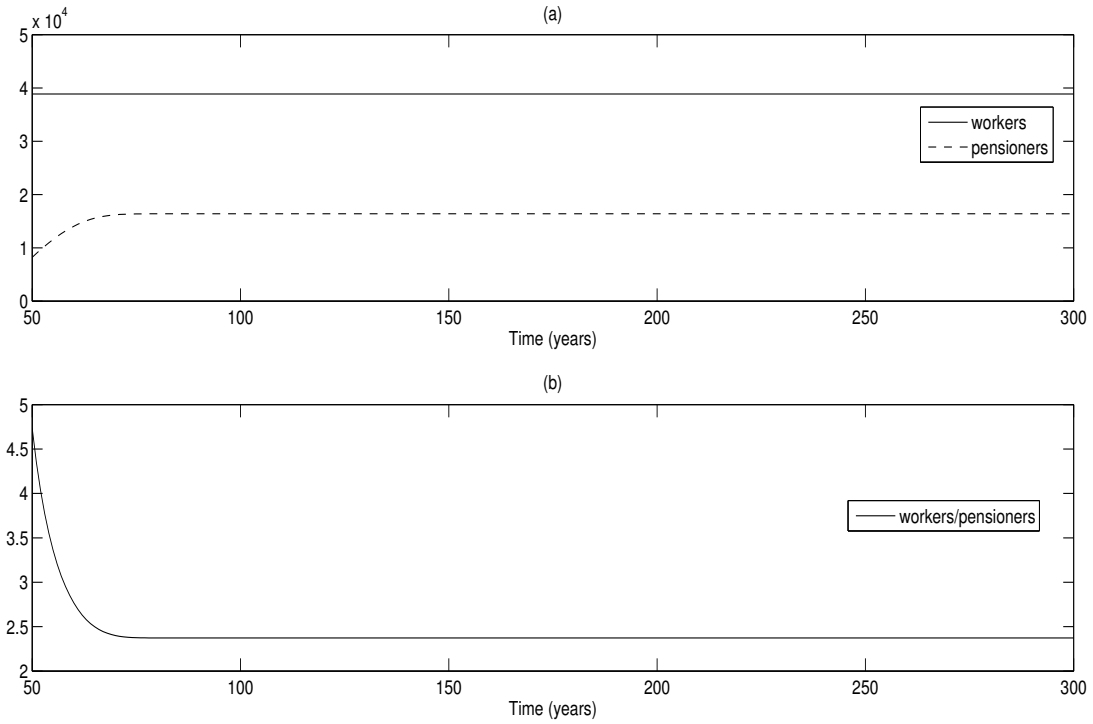
$$\alpha(t) \geq \frac{\beta(t)}{\gamma(t)\nu(t)} = \alpha^{UN}(t)$$

(where $\alpha(t)$ is the contribution rate and $\gamma(t)\nu(t)$ is the *divisor of the total pension liability* $L(t)$ *in the pension expenditure* $P(t)$)
then for each t in T

$$F(t) \geq 0$$

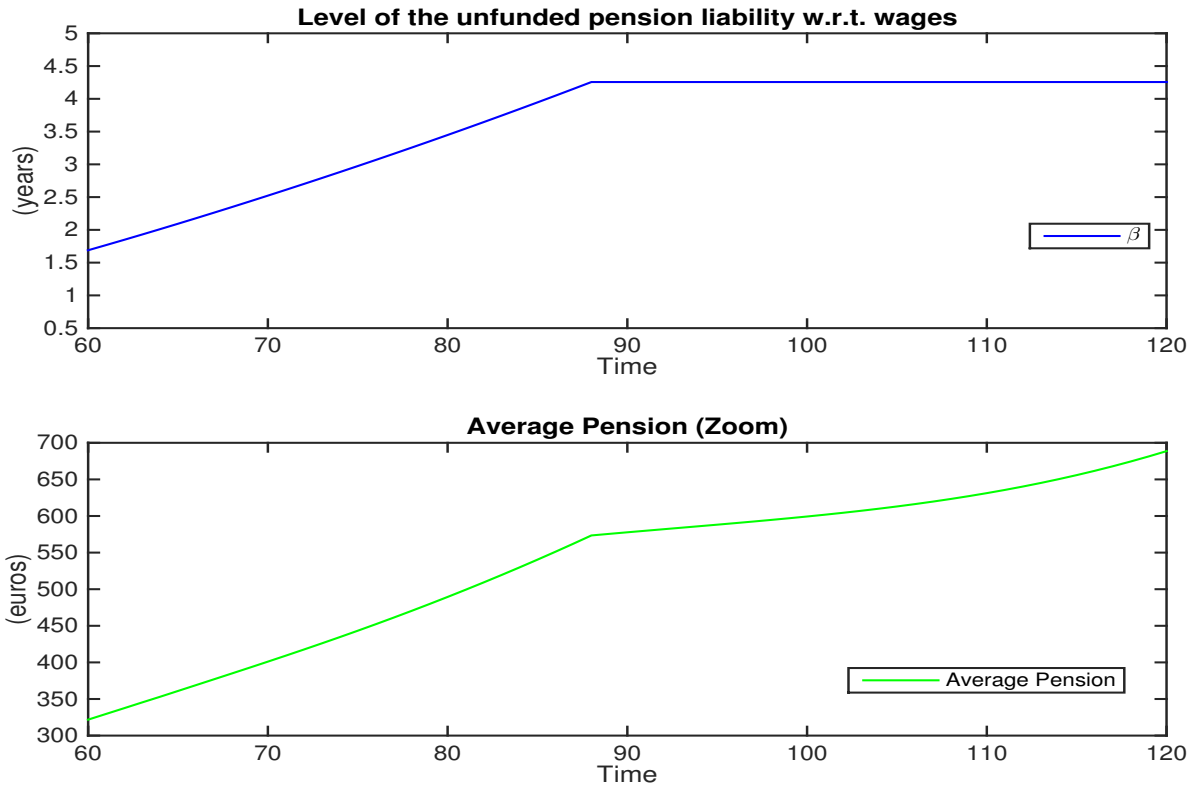
Numerical illustration of the rule for the $\beta(t)$ stabilization

We consider a stable pension system by means of an actuarially consistent model.



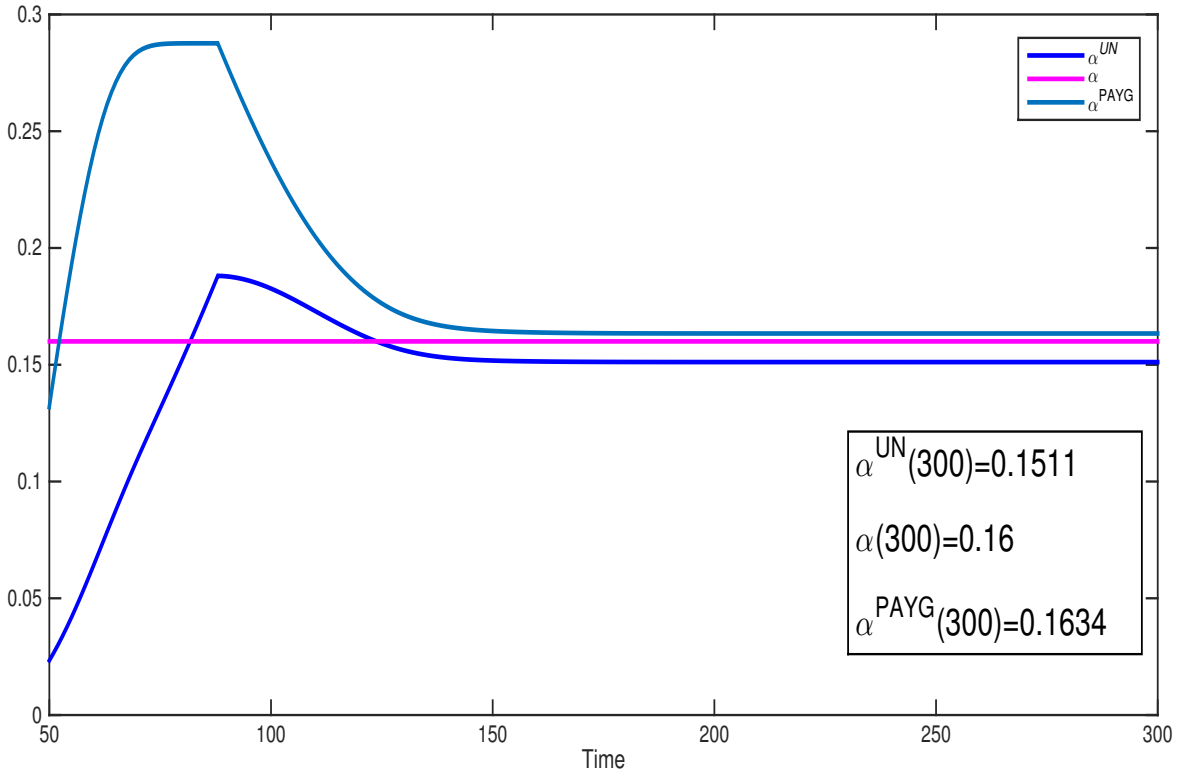
The application of the rule

We apply the rule for the $\beta(t)$ stabilization when the degree of funding of the pension liability falls under the level of 35%.

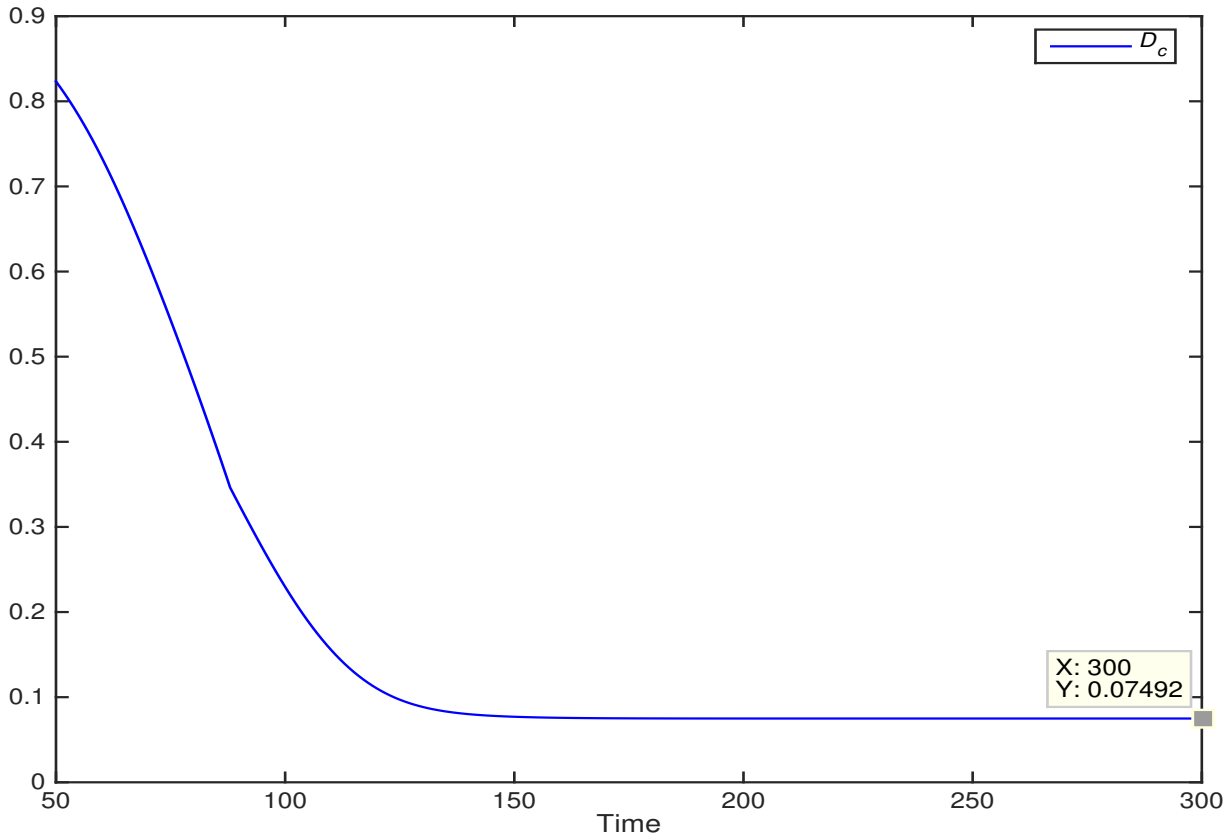


The application of the rule (cont.)

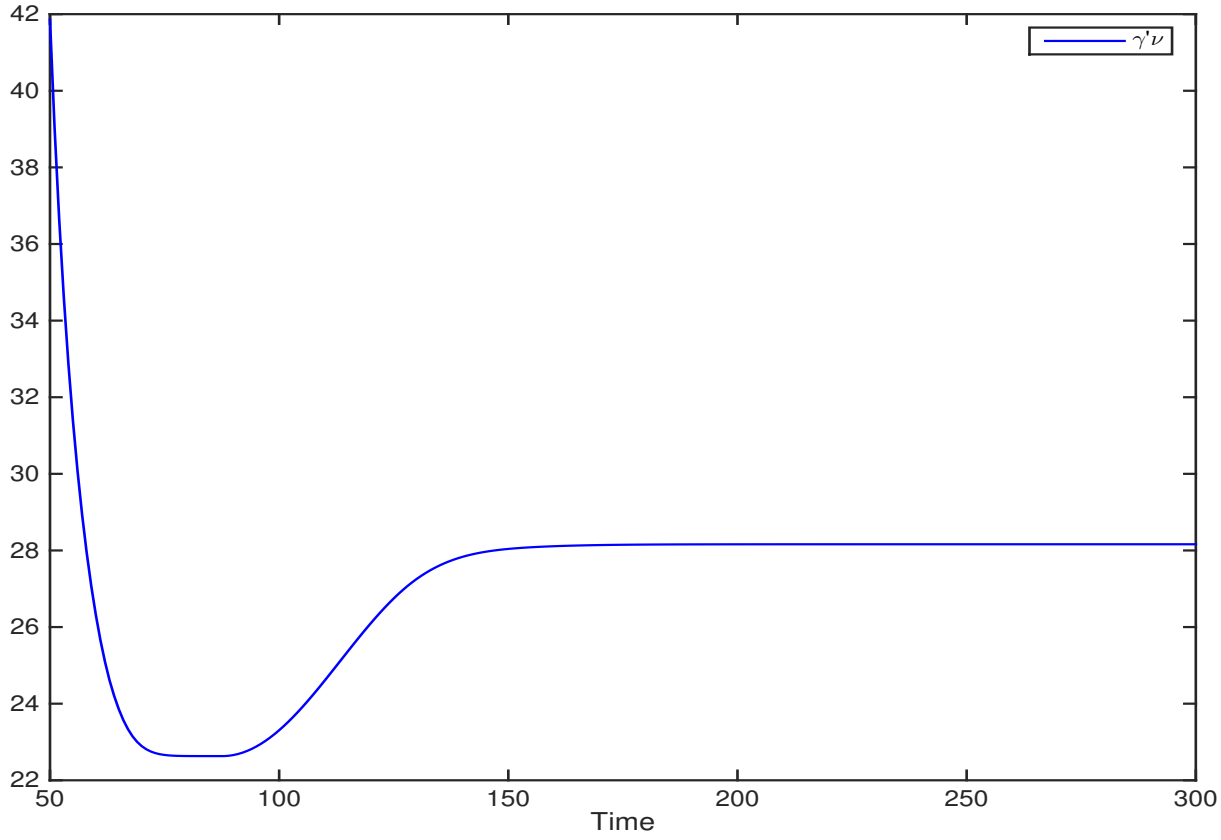
By the sufficient condition of sustainability, it follows that the pension system is sustainable starting from $t = 124$.



The degree of funding of the pension liability



The ratio between the pension liability and the pension expenditure



Pension systems in a stable state perturbed by a demographic wave

Demographic Wave: demographic phenomenon with the features of a strong but temporary increase in the birth rates followed by a subsequent decrease.

Which method should a DC pension system apply in order to face with a demographic wave?

the Separation Principle

it is necessary "to fund" the system component that cannot be managed according to the PAYG scheme.

1. no actuarial projections
2. no stochastic simulations

Generalities to control the demographic wave

Let us consider a DC pension system in a state of stable sustainability, namely the pension system:

- is DC type with a funded component and with a constant contribution rate;
- applies the $\beta(t)$ indicator stabilization rule;
- is in a state of general stability, namely $\gamma(t)\nu(t)$ is stable around a constant value.

To represent the demographic wave, we assume that an extra-number of new active individuals, over and above the stability value, enters the pension system for a limited time interval.

How to control the demographic wave

What should be done in order to maintain the sustainability and the intergenerational equity?

Our approach: all that causes disequilibrium, hence active people who outnumber the stability value of the new entrants, has to be placed in a separate part of the pension system, which has to be financially managed according to the fully-funded scheme.

How to control the demographic wave (cont.)

But

The two parts of the pension system, both the pre-existing stable part and the part linked to the demographic wave, have to be equivalent under the pension profile.
Namely, they have to share the same rules and, in particular, the same rate of return on the pension liability whereby it is indifferent for an individual whether he/she joins the first or the second subsystem.

The Separation Theorem

We consider a DC Pension System that is in a state of a stable sustainability:

- it has a constant contribution rate;
- it recognises the rate of return on the pension liability according to the rule for the $\beta(t)$ stabilization;
- there is a constant ratio between pension liability and pension expenditure.

Let t_i be the time in T starting from which the demographic wave enters into the Pension System. Starting from t_i, the Total Pension System is separated in the two subsystems, the Pivot Pension System, PPS_1, and the Auxiliary Pension System, APS_2, each one with the same constant contribution rate.

The Separation Theorem (cont.)

Furthermore, it is assumed that:

[A1] the instantaneous rate of return on the funds and the instantaneous growth rate of wages are both constant, r and $\overline{\sigma}_1$ respectively, and such that $r \geq \overline{\sigma}_1$;

[A2] the two subsystems recognise the same instantaneous rate of return on the pension liability, namely $r_{L1}(t) = r_{L2}(t)$, and both are equal to $r_L(t)$ that follows the rule

$$r_L(t) = rD_c(t) + \overline{\sigma}_1(1 - D_c(t));$$

[A3] the instantaneous flow $F_2(t)(r - r_L(t))$, named compensation flow, is transferred from APS_2 to PPS_1.

The Separation Theorem (cont.)

Then for each $t \geq t_i$, we have:

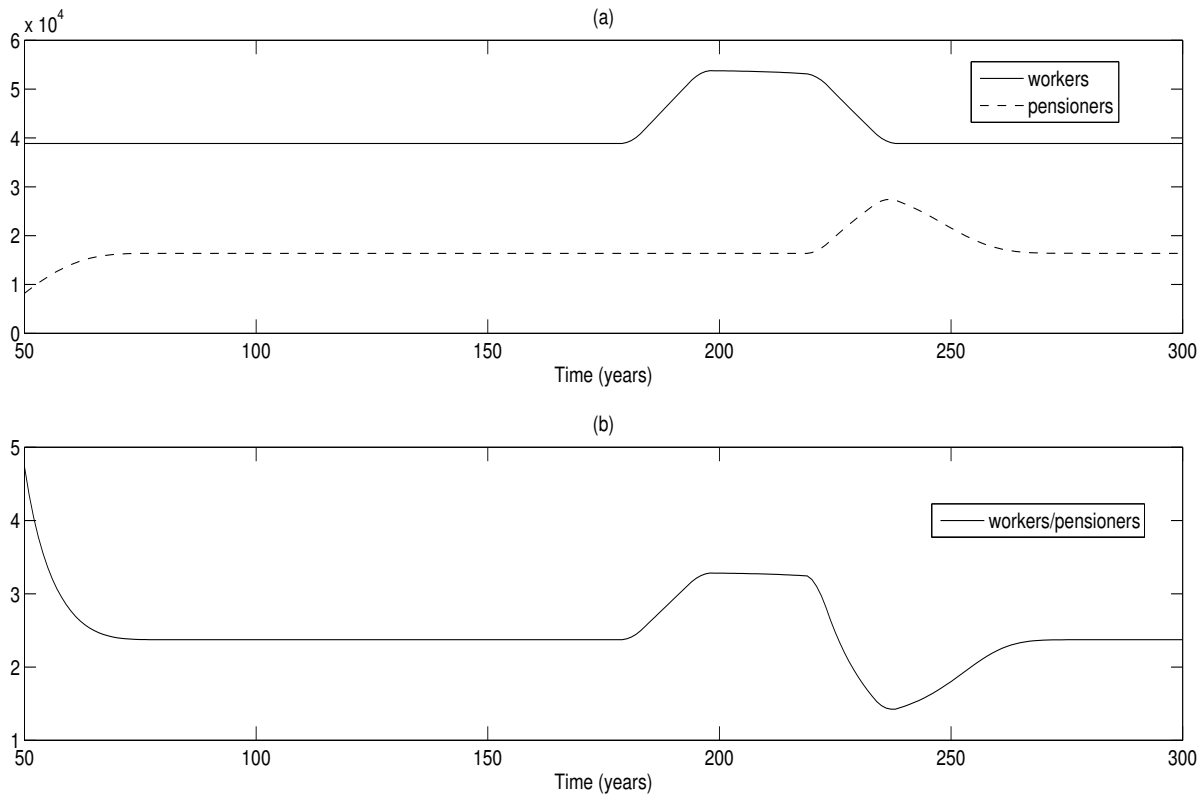
[T1] APS_2 is fully-funded;

[T2] $\dot{\beta}_1(t) = 0$, and hence $\beta_1(t) = \overline{\beta}_1$ with $\overline{\beta}_1 = \overline{\beta}_1(t_i)$;

[T3] PPS_1 is sustainable.

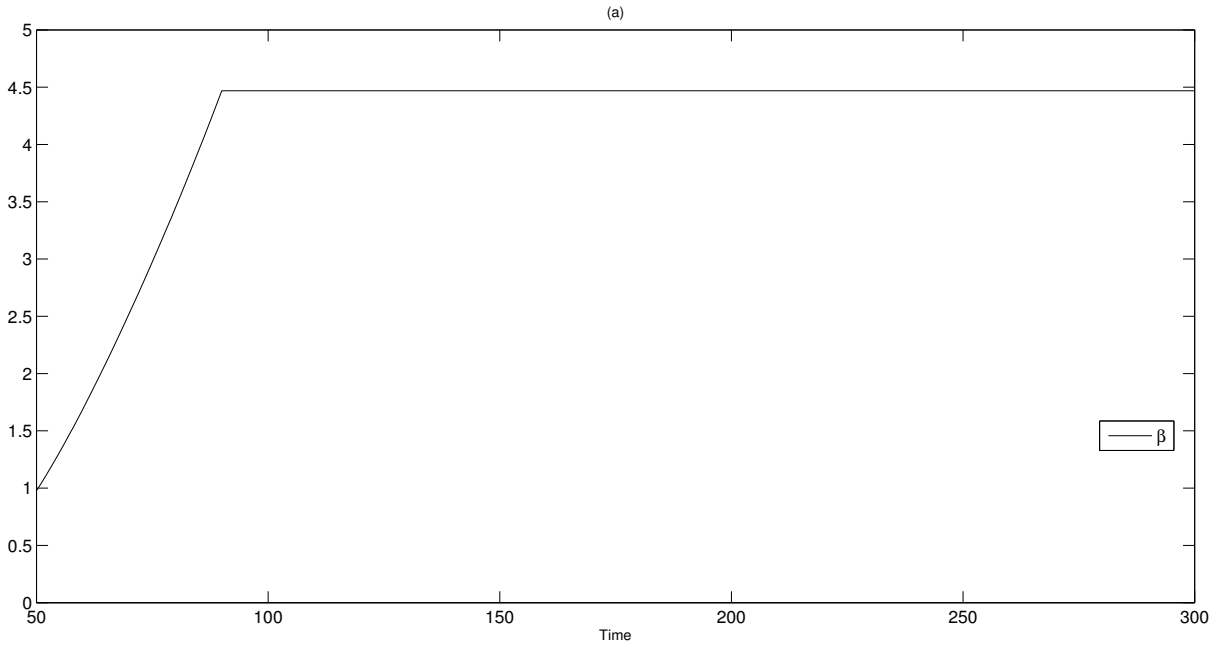
Effects of the demographic wave

We implement a consistent actuarial model to illustrate the control procedure described in the separation theorem.



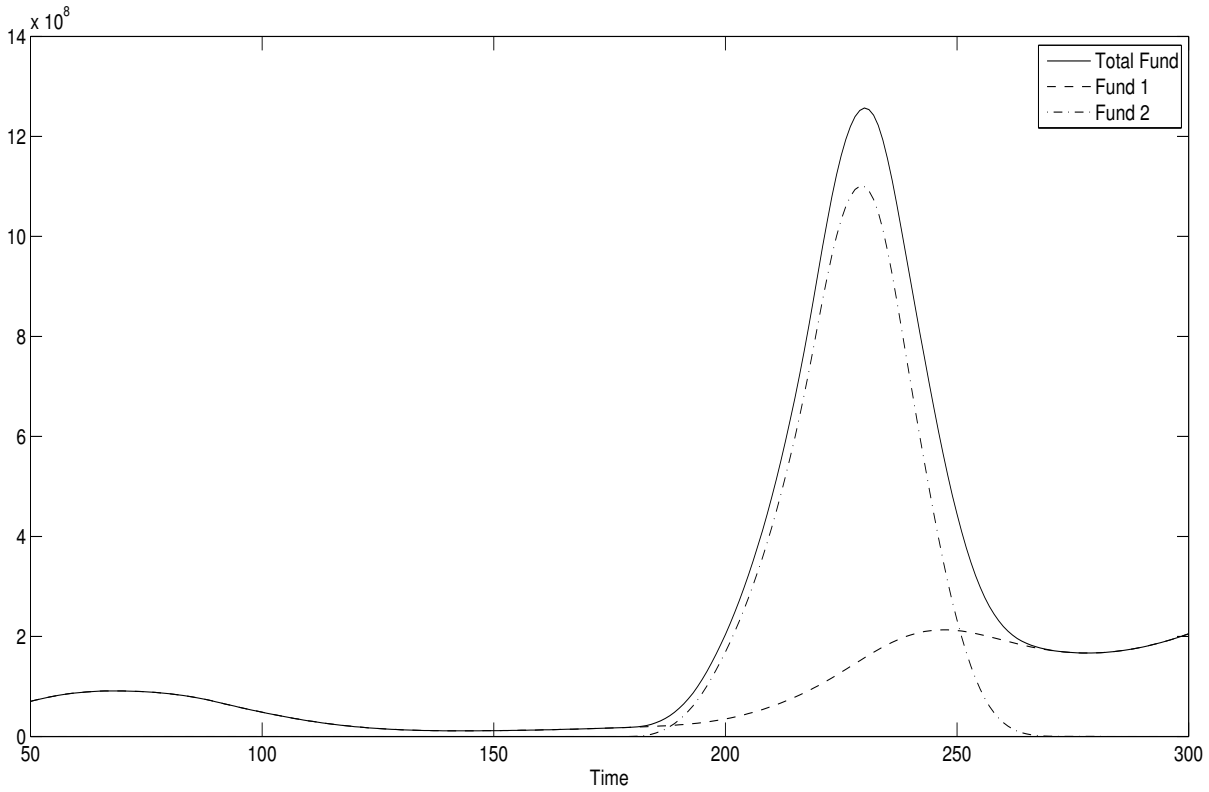
The application of the Separation Theorem

Being that all assumptions of the separation theorem are satisfied, we verify that:
$\beta_1(t)$ is constant for each $t \geq t_i$ and it is equal to $\beta_1(t_i) = 4.468$



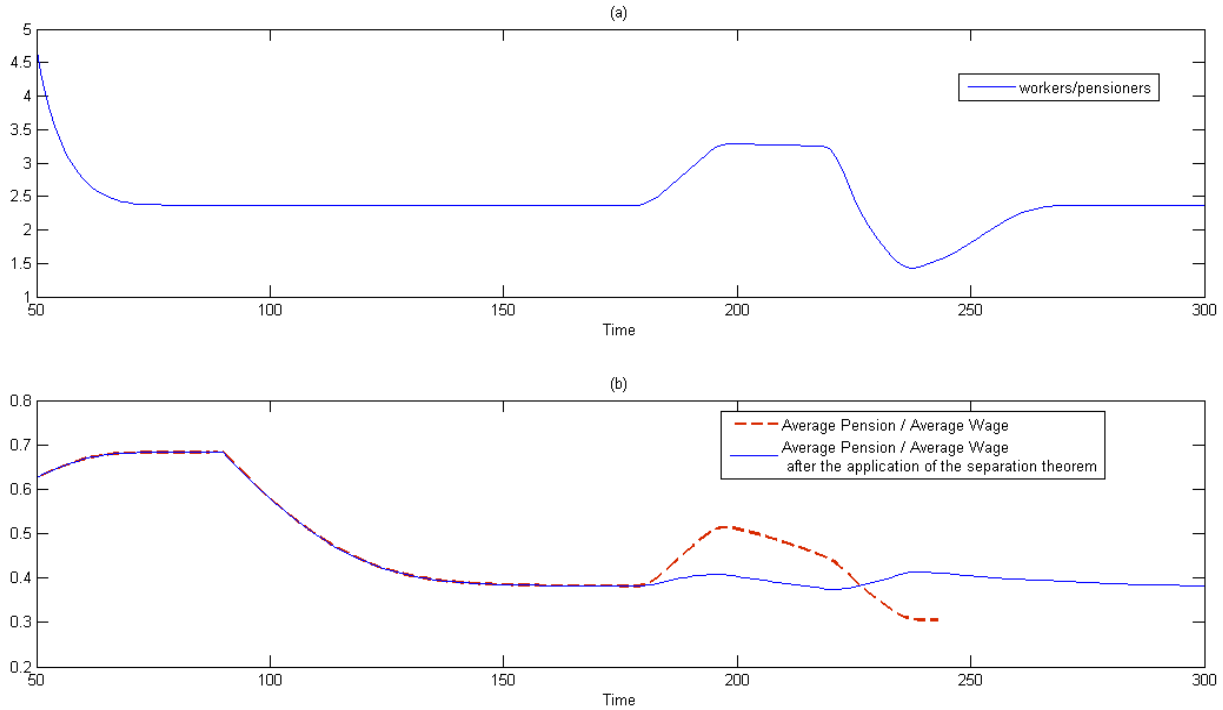
The application of the Separation Theorem (cont.)

The fund of the Pivot Pension System is greater than 0.



Consequence of the application of the Separation Theorem

The application of the Separation Theorem permits and maintains the intergenerational equity



Conclusions and further developments

The main contribution of this work consists in providing an operating procedure to control these issues in an efficient manner. We proved that it is not possible to exclusively follow the PAYG scheme but, at the same time, the procedure proposed does not involve the very burdensome shift of the pension system to the fully-funded scheme.

Further developments of this study are going to provide results in the case that the financial rate and the growth rate of productivity are both modelled by stochastic processes and with the management of the longevity risk.

References

1. M. Angrisani, Funded and unfunded systems: two ends of the same stick, Proceedings of the 28th International Congress of Actuaries (ICA 2006), Paris, 2006.
2. M. Angrisani, The logical sustainability of the pension system, Pure Math. Appl. (PU.M.A.) 19 (2008), pp. 67-81.
3. M. Angrisani, A. Attias, S. Bianchi, and Z. Varga, Sustainability of a pay-as-you-go pension system by dynamic immigration control, Appl. Math. Comput. 219 (2012), pp. 2442-2452.
4. M. Angrisani and C. Di Palo, An extension of Aaron's sustainable rate of return to partially funded pension systems, International Journal of Sustainable Economy 4 (2012), pp. 213-233.
5. N.S. Blomquist and H. Wijkander, Fertility waves, aggregate savings and the rate of interest, Journal of Population Economics 7 (1994), pp. 27-48.
6. G. Bonoli, G. and T. Shinkawa, Population ageing and the logics of pension reform in Western Europe, East Asia and North America, in Ageing and pension reform around the world: evidence from eleven countries, G. Bonoli and T. Shinkawa, eds., Edward Elgar Publishing, Northampton Massachusetts, 2005, pp. 1-23.

References (cont.)

7 M. Feldstein and E. Ranguelova, Accumulated pension collars: A market approach to reducing the risk of investment-based social security reform, No. w7861, National Bureau of Economic Research, 2000.

8 R. Holzmann, E. Palmer, and D. Robalino (eds.), Nonfinancial Defined Contribution pension schemes in a changing pension world. Volume 1: Progress, issues, and implementation, Vol. 1 World Bank Publications, Washington, DC, 2012.

9 R. Holzmann, E. Palmer, and D. Robalino (eds.), Nonfinancial Defined Contribution pension schemes in a changing pension world. Volume 2: Progress, issues, and implementation, Vol. 2 World Bank Publications, Washington, DC, 2012.

10 Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at: www.mortality.org or www.humanmortality.de (data downloaded on March 2011).

11 M. Knell, Pay-As-You-Go?A Relict from the Past or a Promise for the Future, Winning contribution of the Hannes Androsch Prize, 2011.

12 R. Lee, The demographic transition: three centuries of fundamental change, The Journal of Economic Perspectives, 17 (2003), pp. 167-190.

References (cont.)

13 F. Modigliani, M.E.A. Ceprini, and A.S. Muralidhar. An MIT solution to the social security crisis, Sloan School of Management, Massachusetts Institute of Technology, 1999.

14 National Social Insurance Board (RFV), The Swedish Pension System Annual Report 2001, Stockholm, 2002.

Thank you for your attention

massimo.angrisani@uniroma1.it

c.dipalo@unicas.it