Utility functions for financial calculators

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Introduction

People need ***help to plan for retirement***. Some of the help must come in the form of a ***calculator*** to decide how much to ***save*** and to spend, and where to ***invest***. These are the decisions that people can ***control*** (in the cab).

Current advice and calculators are embarrassingly primitive and much ***panders to ignorance and instinct*** (some fortunately is not wrong!)

Users do not need to know the details of the workings of the calculators, but those who design them need to use ***plausible and coherent utility functions*** that represent the users' preferences and relevant demographic and economic assumptions. These functions can be as complex as necessary (because they belong under the hood).

Users ultimately have to express preferences between alternative stochastic distributions of ***random vectors*** of consumption, leisure and bequests. This is not easy but can perhaps be elicited through offering them ***graphic representations***.

Current advice is primitive

Many are **financially illiterate** and take no interest in retirement savings. Advice can be **inconsistent, biased and counter-productive** (Gokhale and Kotlikoff; Inderst and Ottaviani; Hackethal et al). Merton (2014) suggests that we face a crisis in “financial well-being”.

- Inappropriate inflation and investment assumptions – equity risk premium too high; no momentum and mean reversion.
- No allowance for consistency between current and future consumption – by using arbitrary targets or “needs”.
- Misleading risk appetite assessments using short term portfolio volatility and life expectancy.
- Incoherent ruin probabilities (VaR), which can encourage risk taking by ignoring the size of some downside losses.

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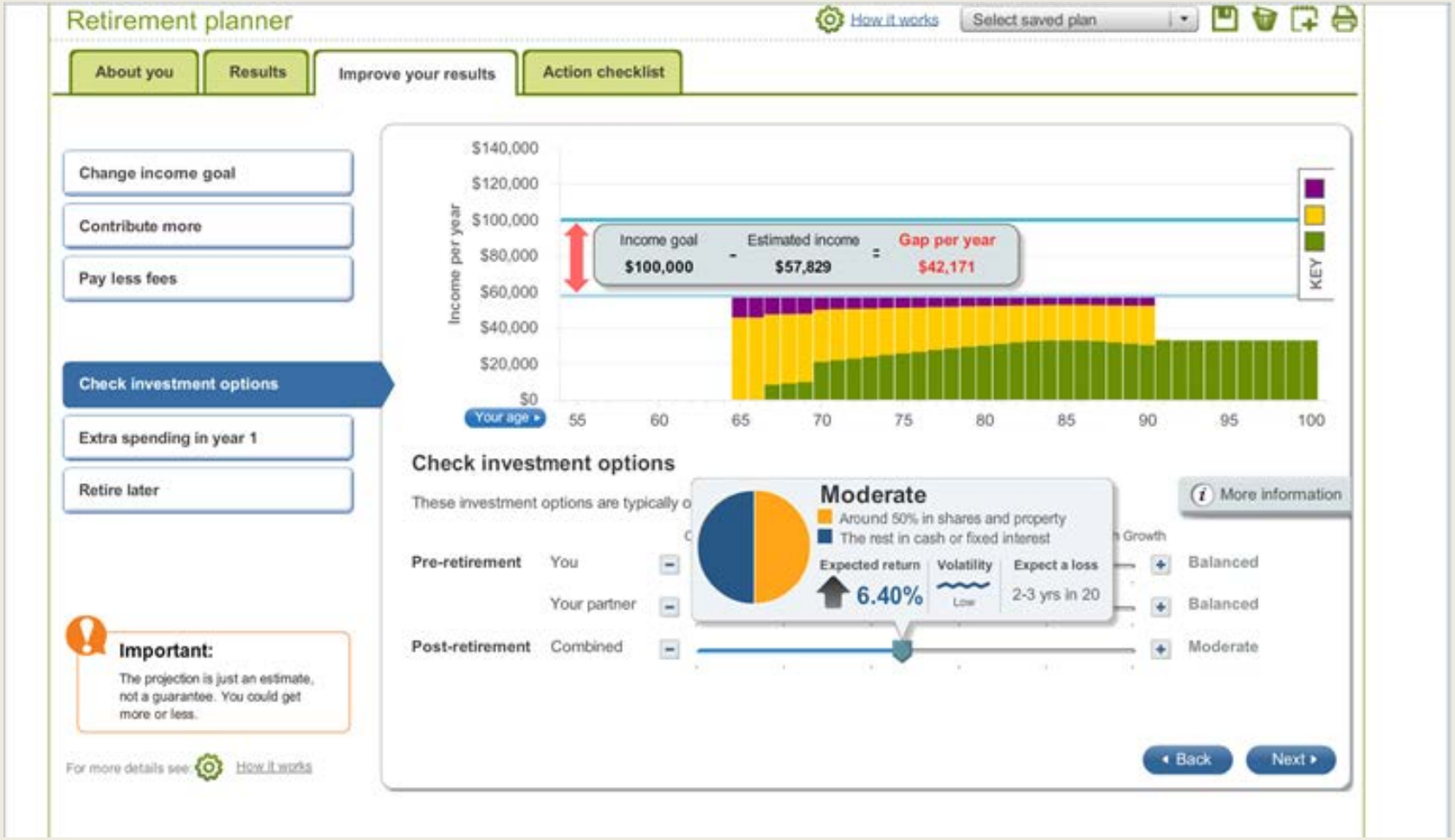

https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/retirement-planner

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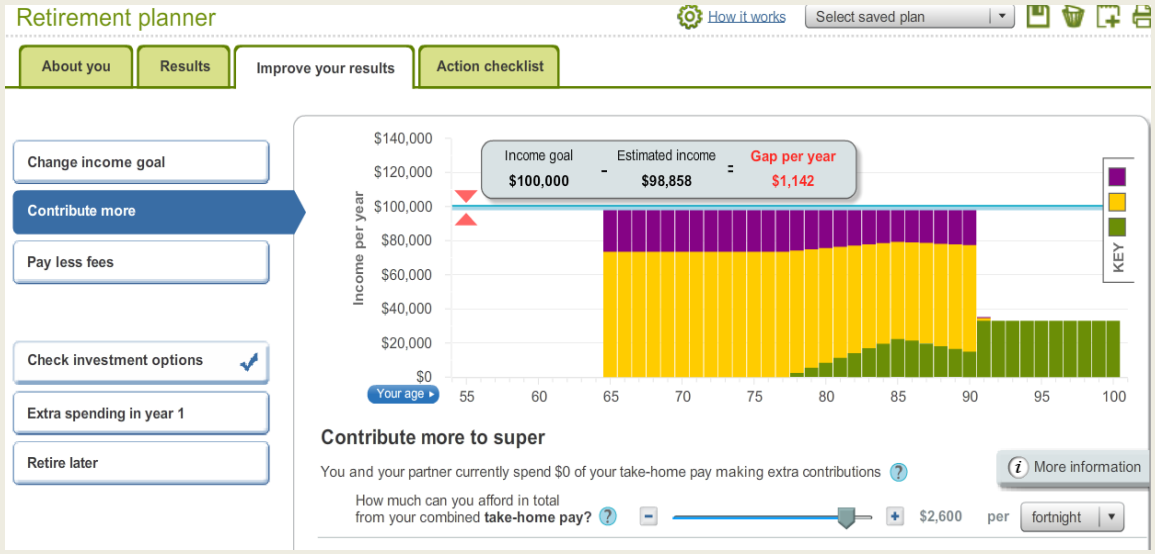
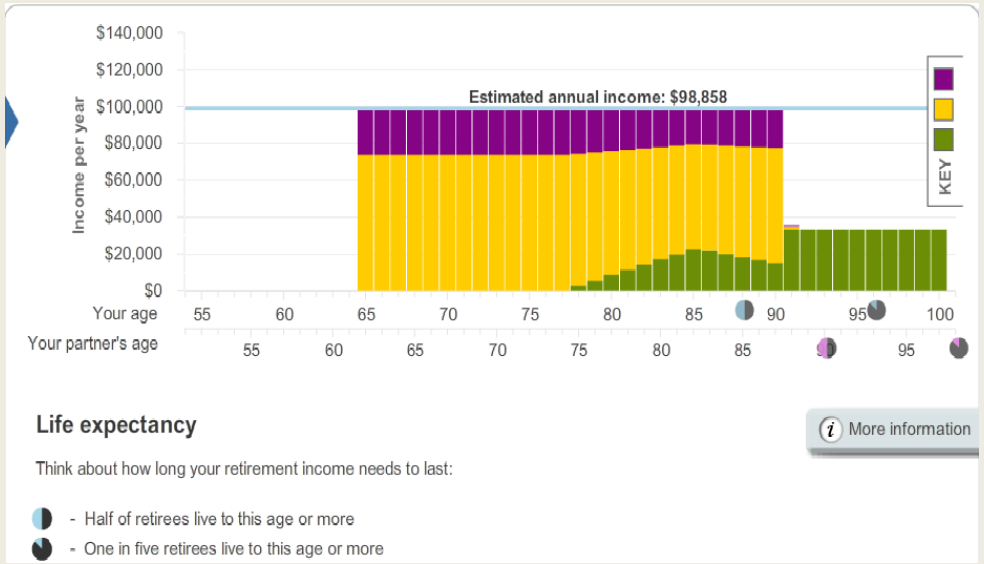


As my wife and I are earning $154,000 pa we are assumed to have an income goal of $100,000 pa and are “advised” to contribute $67,000 pa to make up the shortfall.

She has a 50:50 chance of living past ninety, but the money runs out before then - pity!



Identifying and addressing ignorance and instinct

“Bounded rationality, bounded willpower”

The difficulties with traditional logic can be explained by two systems :

- *System 1* is instinctive, responds rapidly, is low effort, often quick to make emotional judgments and works off rules of thumb
- *System 2* is slower, reasoned and requires higher effort to engage, although not always right.

The difficulty of engaging system 2 means that we all have a tendency to make snap judgements that we can regret, because we did not take the time and effort to engage our deeper thinking processes.

But the lessons of modern psychology (Mischel, 2014) and the ancient world religions tell us we can – with some effort – decide differently. “So then with the mind I myself serve the law of God; but with the flesh I serve the law of sin.” (Romans 7:25 King James Version)

It is surely possible to provide training so that people can plan better – more consistently – and in a way where they implement their decisions.

Explaining apparent irrationality

System 1 heuristics explain some apparent irrationality:

- Hyperbolic discounting, with high short term rates of discount for small amounts of money.
- Discount rates are lower when people consider probabilities (too complex for system 1). Similarly, when time preferences are framed as choosing lifetime patterns of consumption, people prefer level ones.
- People choose prior commitments (such as forced savings and “bucket” allocations) to take decisions away from system 1.

Other apparent irrationality can be explained by frictional costs:

- The Prospect Theory “kink” in the utility function can be explained by consequential financial losses (eg cannot use the car and so sell it).
- Information costs of making equity investments mean greater risk aversion.
- If my consumption falls below those of my reference group, I cannot participate as much in their activities.

(See Frederick et al (2002) and Clark et al (2000) for explorations of some of these biases but without these explanations.)

Also need to take into account – in reserve perhaps – hedonic adjustment means that failing to maintain consumption is not a disaster, particularly when a basic state support is provided.

The standard utility function

$$U_0 = \sum_1^T \beta k^{t-1} \left(\prod_{t=0}^{k-2} p_j\right)\{p_{t-1} \frac{1}{1-\gamma}\left(\frac{C_t}{\emptyset_t} l^{\alpha}\right)^{1-\gamma} + b(1-p_{t-1})\frac{1}{1-\gamma}(D_t)^{1-\gamma}\}$$

Where β represents the net time preference rate;

C_t consumption in period t;

φ_t represents an allowance for the family state (which in Hubener et al (2013) is contingently modelled using empirical transition rates for divorce and widowhood;

l^{α} represents the utility of the hours of leisure;

p_t the probability of survival in the year t to t+1;

γ>0 is the coefficient of risk aversion;

b the intensity of the bequest motive, and

D_t the bequest.

Possible adaptations

$$U_0 = \sum_1^T \beta^{t-1} \left(\prod_{j=0}^{t-2} p_j\right)\{p_{t-1} \frac{1}{1-\gamma}\left(\frac{c_t}{\emptyset_t} l^\alpha\right)^{1-\gamma} + b(1-p_{t-1})\frac{1}{1-\gamma}(D_t)^{1-\gamma}\}$$

β – should perhaps be contingent on actual investment returns because of reference group – to represent level consumption throughout life.

Φ_t can be partly controlled; should one include probabilities of divorce?

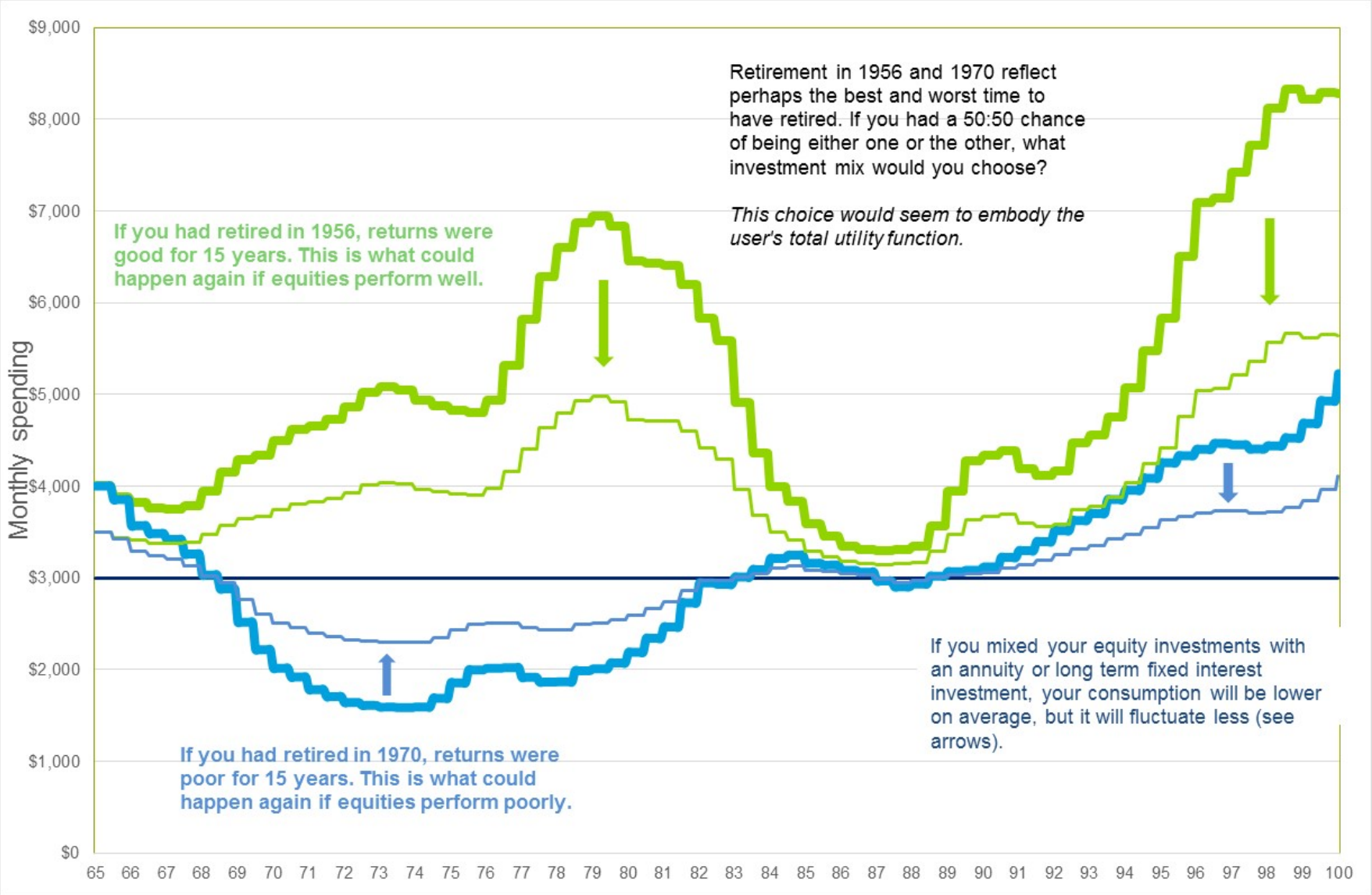
l^α could include a constraint related to age and health in order to generate a retirement date, or retirement date could be an input variable.

p_t could also be set to 1 as any coherent plan that has zero utility for the state of death would use a life annuity

γ>0 can perhaps be enhanced/replaced by a penalty whenever consumption drops, which would also produce smoothing. Alternatively consider inter-temporal elasticity; constant IEC implies (essentially): $U_0 = (c_0^\rho + c_1^\rho)^{1/\rho}$

b and D_t can be removed, and an amount set aside separately for precautionary and bequest purposes.

Eliciting the utility function



An Example: Utility Specification

The modified version of the CRRA utility function described earlier is used.

$$U_0 = \sum_{t=1}^{T} \left(\prod_{j=0}^{t-1} p_j \right) \left[\frac{C_t^{1-\gamma}}{1-\gamma} + \xi \min\left(0, \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{C_{t-1}^{1-\gamma}}{1-\gamma} \right) \right] (p_t)$$

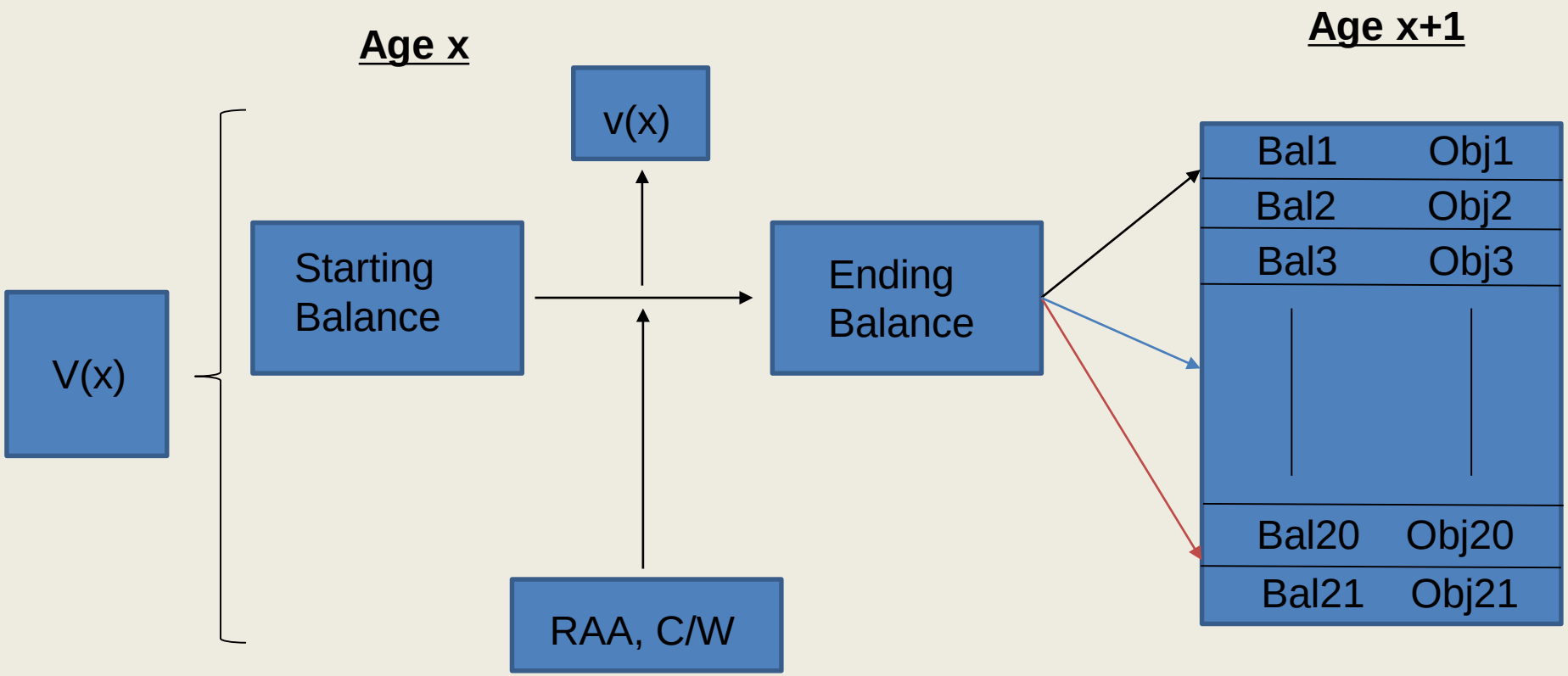
In this formulation:

- A penalty ($\xi = 0.1$) applies where the current period's consumption is lower than the past period's consumption
- Bequest motives are not allowed for
- Leisure or family states are not considered
- Appropriate timing adjustments are not shown

The optimisation procedure involves a stochastic dynamic programming process

An Example: Illustration of a simple process (no penalty)

For each simulation at age x: $V(X) = v(x) + V(x+1) \times (1 - q_x)$



An Example: Input and Output

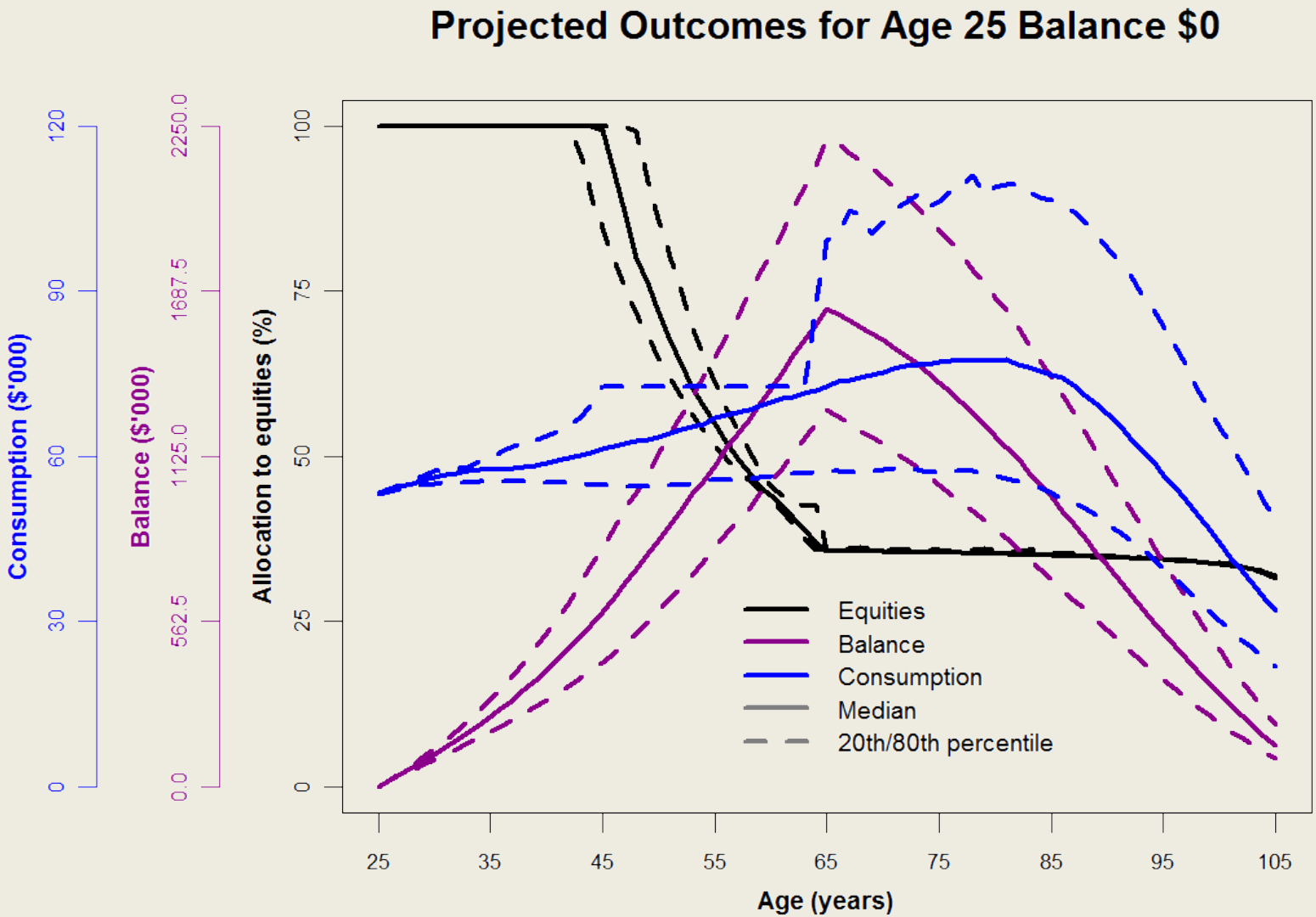
Inputs and Assumptions

- Economic data for simulation bootstrapped from relevant indices over the period 1993-2012 – results expressed on a real (AWE) basis
- Mortality based on male ALT 2005-07 rates – no mortality improvement allowed for
- Ages considered are 25 through to 109 with retirement at 65.
- Labour is the only source of income pre-retirement and superannuation is the only source of income post-retirement

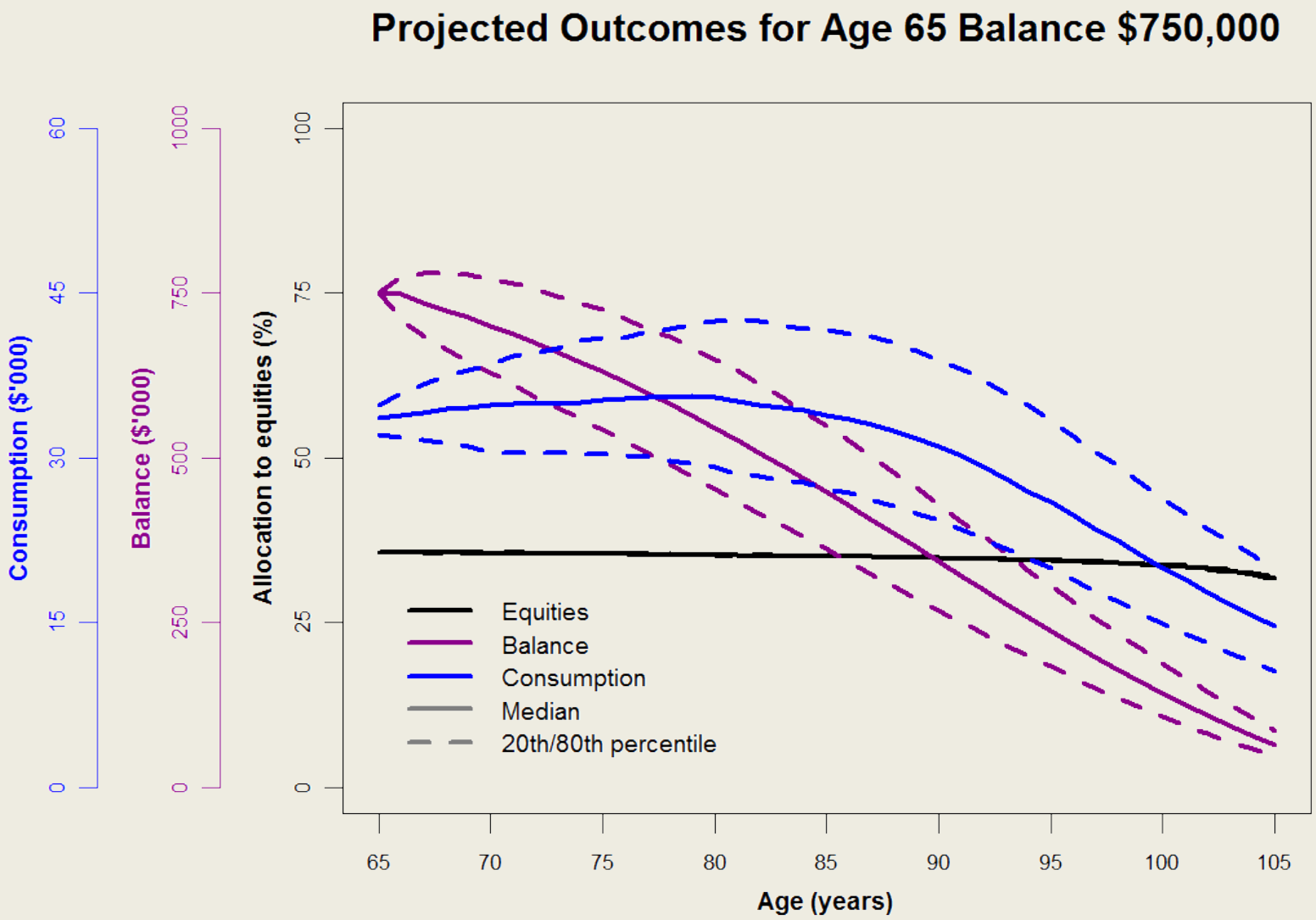
Outputs

Optimum results (not shown here) are generated for a combination of balances and prior consumption values (state variables). The resulting output provides optimum risky asset allocation and consumption amounts.

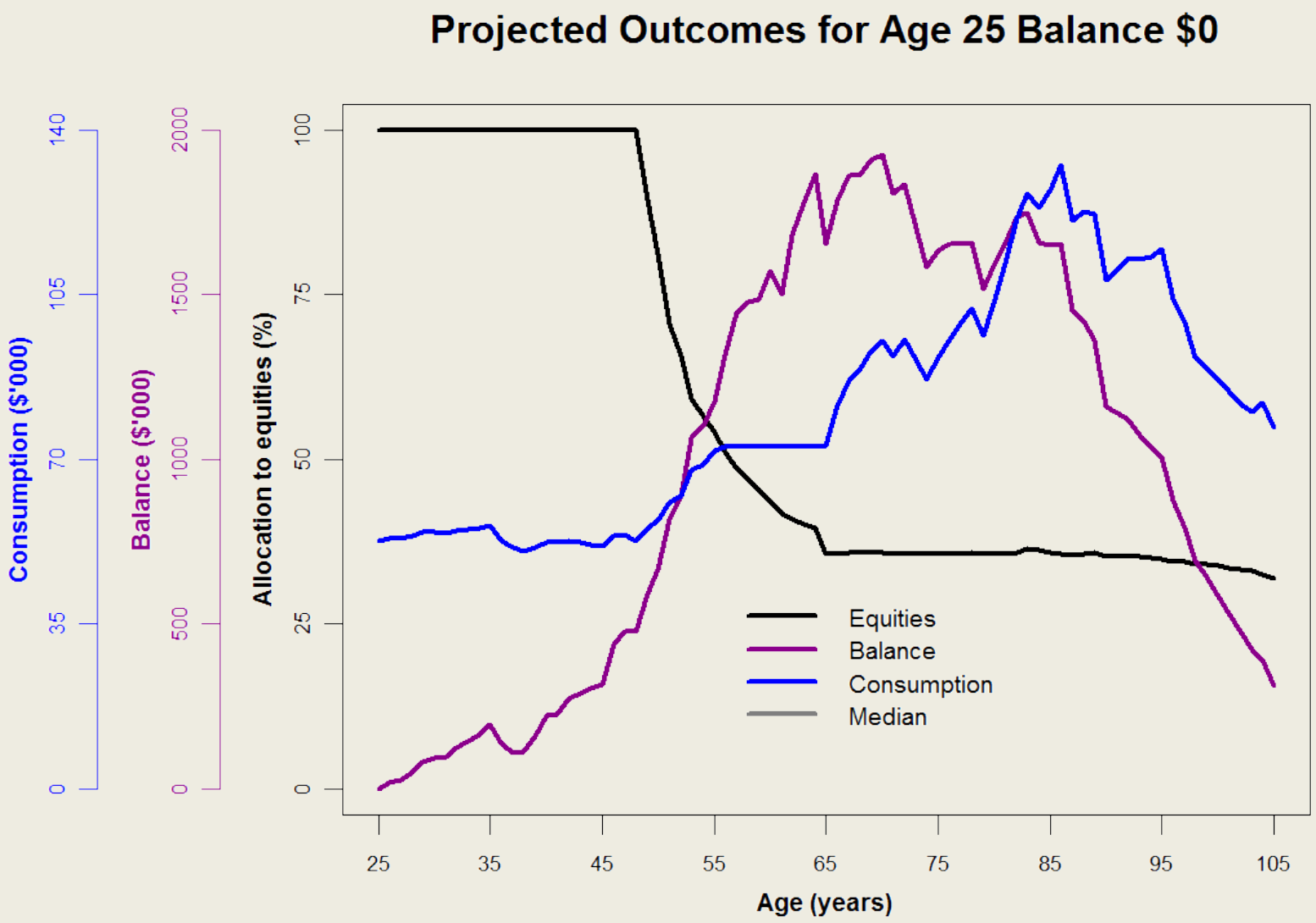
An Example: Median Outcomes for age 25



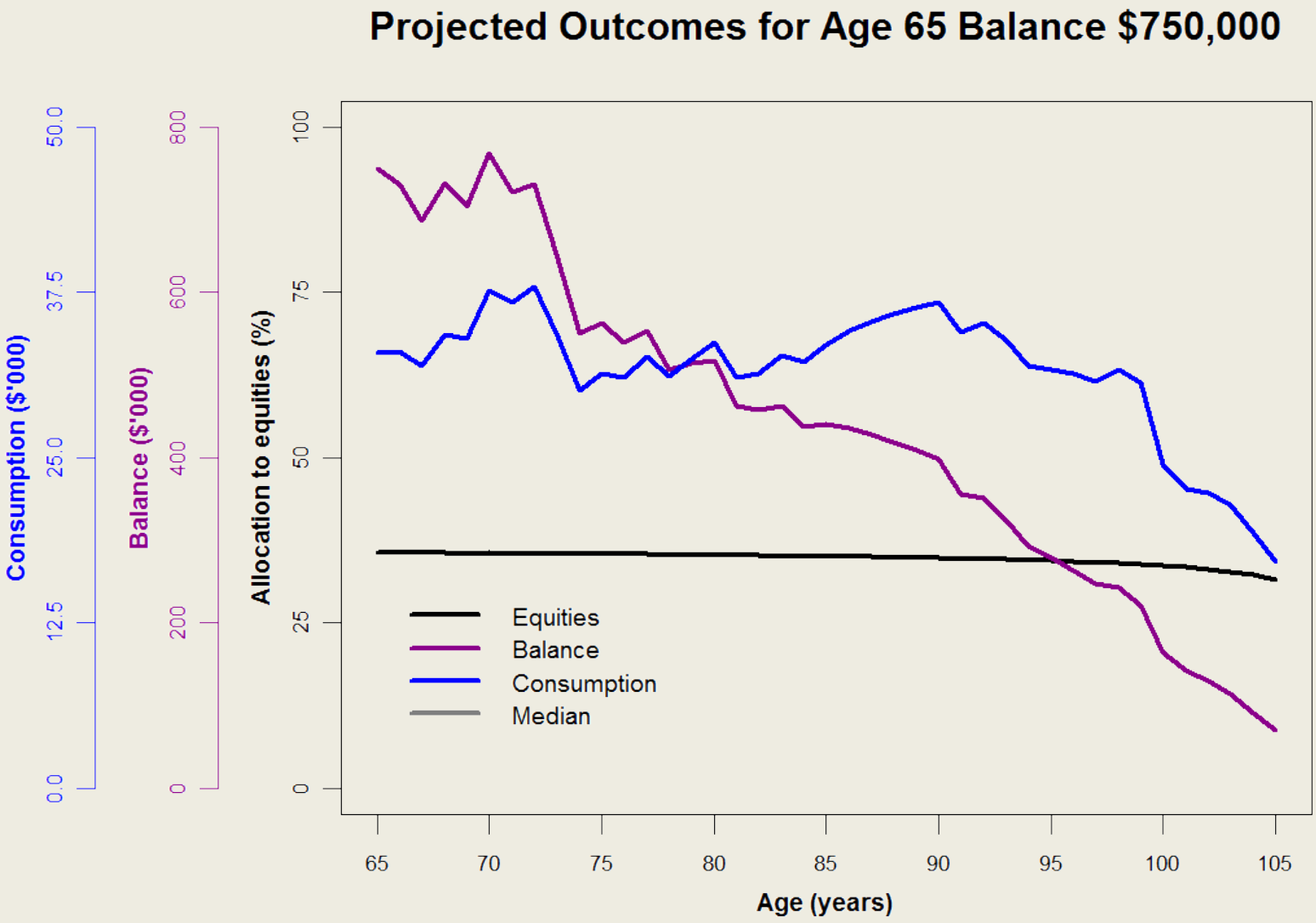
An Example: Median Outcomes for age 65



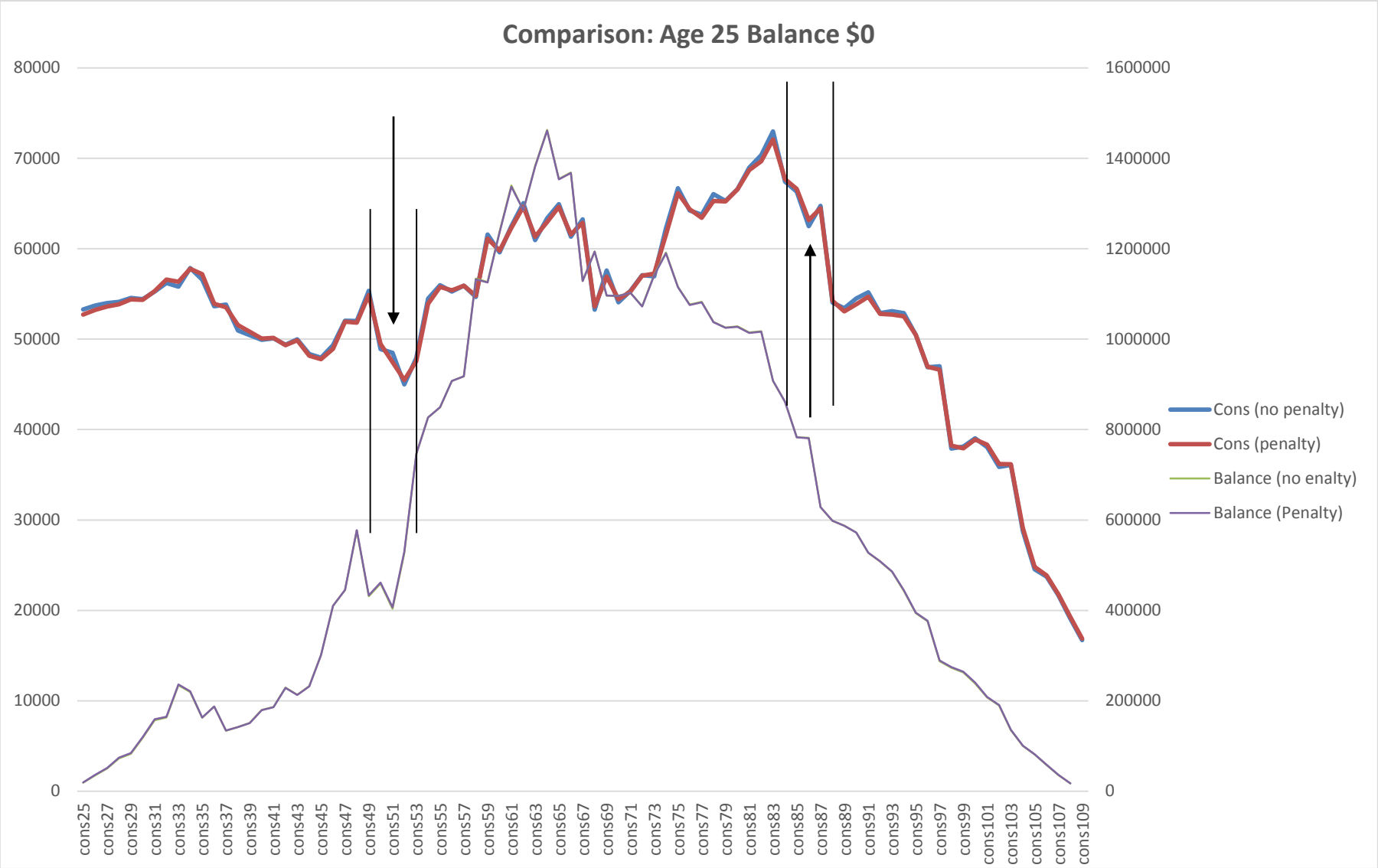
An Example: Median Path for age 25



An Example: Median Path for age 65



An Example: Comparison



Future Steps

- Experiment with alternative ways of eliciting utility functions – or is coherence an illusion and different preferences incommensurable?
- Develop realistic calculator
  - Australian social security and taxation
  - Marital status, children, home ownership etc …
  - Add alternative annuities
- Develop alternative presentations to increase engagement
- Add further asset classes and refine optimisation procedures