Guarantee valuation in Notional Defined Contribution pension systems

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The aim of this presentation is to:

- describe the notional defined contribution (NDC) pension scheme with its advantages and shortcomings;
- develop a pricing framework for guarantees in NDC first pillar pension systems;
- and compare it to the insurance and financial markets framework.
Overview of pension systems

- Basic financing techniques
  - Pay as you go (PAYG): current contributors pay current pensioners (Unfunded schemes)
  - Funding: contributions are accumulated in a fund which earns a market return (Funded schemes)

- Benefit formulae
  - Defined Benefit: Pension is calculated according to a fixed formula which usually depends on the members salary and the number of contribution years.
  - Defined Contribution: Pension is dependent on the amount of money contributed each year and their return.

⇒ Notional Accounts: Mix of PAYG and Defined Contribution!
Notional Defined Contribution

What’s NDC?

- A pension scheme managed by the State,
- with Pay-as-you-go (PAYG) as financing technique: 'current contributors pay current pensioners'.
- The pension formula depends on the amount contributed and the return which equal to the notional rate.
- At retirement age: Accumulated capital $\Rightarrow$ Annuity which takes into account:
  - Life expectancy of the individual
  - The indexation of pensions and technical interest rate
- We assume that the notional rate is equal to the rate of increase of the total contributions.
Mathematically, the pension for someone retiring at age $x_r$ at time $t$ can be stated as:

$$P(x_r, t) = \frac{NDC(x_r, t)}{a(x_r, t)}$$

where:

$$NDC(x_r, t) = \sum_{x=x_0}^{x_r-1} C(x, t - x_r + x) \prod_{i=t-x_r+x}^{t} (1 + nr_i)$$

and $a(x_r, t)$ is the whole time annuity at time $t$ for age $x_r$.

⇒ It is crucial to have an **appropriate capital** in order to have an adequate pension!
Advantages of NDC

- It's *more or less* actuarially fair (takes into account life expectancy and contributions)
- Portability of pension rights between jobs, occupations and sectors is permitted.
- It promises to deal with the effects of population ageing more or less automatically.
- Arbitrariness in benefit indexation rules and adjustment factors is avoided.
Challenge faced

Disadvantages of a pure NDC system (w/out guarantees):

- The return risk is fully borne by the participants.
- The amount of pension at retirement is unknown!
- The notional rate depends on demography’s and salary’s rate of increase: it can be close to 0% or negative!

Why should we consider guarantees?

- We offer a minimum value for each contribution at retirement.
- **However**, which is the price for the guarantee provider?
- ...and how can we price the option on an asset which is not traded in the market?

**Answer:** pricing in incomplete markets techniques!
Utility indifferent pricing

**Definition**

The indifference price on an option is defined as the function price which makes these two situations equivalent:

- **hold a financial portfolio** (which maximizes capital and hedges the option) and **hold (resp. sell)** the written option at time $t$ and receive (resp. pay) the payoff at maturity;
- **only** hold the financial portfolio.

We are in an incomplete market, so the price will depend on
- the utility function used
- and the risk aversion $\gamma$.

In our case, we use the exponential utility:

$$U(x) = -e^{-\gamma x}, \gamma > 0$$ (2)
The pension system

- The working population pays a contribution rate $\pi \in (0, 1)$ of their income to the pension system;
- each contribution earns a rate of return (notional rate) based on the total contribution base at time $t$.

The total contribution base is then: $Y(t) = \pi P(t)L(t)$, where:

- $P(t)$ represents the working population
- and $L(t)$ is the mean salary earned by the workers.
- Their evolution is driven by a Geometric Brownian Motion ($\to$ lognormally distributed)
The pension system (C’td)

For the **pension system provider** or state, the guarantee scheme entails:

- holding the contribution base which represents the returns of contribution,
- a loss of \( K - \frac{Y(T)}{Y(t)} \) if the contribution base performed badly

Mathematically, it becomes the put option:

\[
g(Y(T)) = \left( K - \frac{Y(T)}{Y(t)} \right)^+ \tag{3}
\]

with

\[
dY(t) = d(\pi P(t)L(t)) = (g + \alpha_P + \rho_{L,P}\sigma_L\sigma_P) Y(t) \, dt + \sigma_L Y(t) dW_L(t) + \sigma_r Y(t) dW_P(t) \tag{4}
\]

\( d \) is the drift, \( \sigma \) the salary risk, and \( \rho \) the demographic risk.
Reminder: The salary and population risks are not traded in the market → the market is incomplete!

- We can’t use the classical models! (e.g. Black & Scholes)
- However, the risks linked to the contribution are correlated to those which are traded!
- We can then price the underlying non-hedgeable risks, by using the traded risks as a proxy.
Financial portfolio

We create a self-financing portfolio with the assets available in the market:

\[
\frac{dX(s)}{X(s)} = \theta_0(s) \frac{dS_0(s)}{S_0(s)} + \theta_P(s) \frac{dP(s, T)}{P(s, T)} + \theta_S(s) \frac{dS(s)}{S(s)}
\]

\[
= (r(s) + \theta_S(s) \lambda_S \sigma_S - \theta_P(s) q \sigma(s, T)) dt \\
+ \theta_S(s) \lambda_S dW_S(s) - \theta_P(s) q \sigma(s, T) dW_r(s)
\]

(5)

\[
X_t = x \quad 0 \leq t \leq s \leq T
\]

with:

- \( \theta_0(s) \) proportion invested in the bank account \( S_0(s) \);
- \( \theta_S(s) \) proportion invested in the risky asset \( S(s) \);
- \( \theta_P(s) \) proportion invested in the zero-coupon bond \( P(s, T) \).
- **Constraint**: \( \theta_0(s) + \theta_S(s) + \theta_P(s) = 1 \)
The price

**Proposition**

The price of a European option \( G = g \left( \frac{Y(T)}{Y(t)} \right) \) under the exponential utility in a market like the one presented is given by

\[
p(x, y, r, t) = P_Q(t, T) \frac{\delta}{\gamma} \log \left( E_{Q^T} \left[ e^{\frac{\gamma}{\delta} g \left( \frac{Y(T)}{Y(t)} \right)} \big| Y(t) = y \right] \right)
\]

(6)

where:
- \( \delta \) constant which depends on the level of correlation between the pension risks and the traded risks;
- \( \gamma \) is the risk aversion;
- \( \frac{dQ^T}{dP} \) is the forward measure;
Particular cases: Independent & Complete markets

- If the notional index is totally independent we find a zero-utility insurance premium:

\[ p(x, y, r, t) = P_Q(t, T) \frac{1}{\gamma} \log \left( E_P \left[ e^{\gamma g\left( \frac{Y(T)}{Y(t)} \right)} \middle| Y(t) = y \right] \right) \]  \hspace{1cm} (7)

- If the notional index is traded we find a complete markets price:

\[ p(x, y, r, t) = P_Q(t, T) E_Q \left[ g \left( \frac{Y(T)}{Y(t)} \right) \middle| Y(t) = y \right] \]  \hspace{1cm} (8)

\[ = \text{Black & Scholes Formula!} \]
Comparison between the intermediate price, the insurance and B&S Price

**Figure**: The value of a put option guaranteeing $i_G = 4\%$ (first row), $i_G = 6\%$ (second row): un-correlated exponential price (continuous line), imperfect correlation exponential price (discontinuous line) and perfect correlation B&S case (pointed line)

**Source**: the authors.
Link between pure insurance, intermediate price and Black & Scholes:

- For some risk aversions and guarantees we can have lower prices than Black & Scholes.
  \( \Rightarrow \) I’m ready to pay ’more’ for a better hedge.

For the other graphics:

- Price \( \uparrow \) when risk aversion \( \uparrow \).
  \( \Rightarrow \) I’m ready to pay a higher price to insure me against losses when I’m more ’afraid’ of them.

- Price \( \uparrow \) with guaranteed rate \( \uparrow \).
  \( \Rightarrow \) I’m ready to pay a higher price linked to higher guarantees.
Assume the following economy:

<table>
<thead>
<tr>
<th>Salary (Gross - Annual)</th>
<th>39.532,00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salary rate of increase</td>
<td>1%</td>
</tr>
<tr>
<td>Contribution rate</td>
<td>16.86%</td>
</tr>
<tr>
<td>Life expectancy at 65</td>
<td>20.02</td>
</tr>
</tbody>
</table>

Then the Monthly Minimum Pensions (MMP) and the Total Guarantee Cost (TGC) for different minimum returns are:

<table>
<thead>
<tr>
<th></th>
<th>0%</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
</tr>
</thead>
<tbody>
<tr>
<td>TGC (% of capital)</td>
<td>2.26%</td>
<td>4.14%</td>
<td>6.69%</td>
<td>9.27%</td>
<td>11.53%</td>
</tr>
<tr>
<td>MMP - State</td>
<td>1.505,80</td>
<td>1.822,50</td>
<td>2.235,42</td>
<td>2.764,12</td>
<td>3.427,86</td>
</tr>
<tr>
<td>MMP - Contributors</td>
<td>1.471,70</td>
<td>1.732,05</td>
<td>2.002,60</td>
<td>2.252,07</td>
<td>2.420,45</td>
</tr>
</tbody>
</table>
We obtained a closed-form formula which prices different options written on the notional index;

- The notional index is the rate of return on the contributions and is stochastic;
- This price doesn’t depend on the initial capital!
- This tool allows to put a price on promises to the participants for a better risk management.

**Issue:** How can we finance this guarantees? Who should pay? Contributors? The State?
References


References (C’td)


Thank you for your attention!