A two-account life insurance model for scenario-based valuation including event risk

Ninna Reitzel Jensen  
University of Copenhagen  
Universitetsparken 5  
DK-2100 København Ø  
Denmark  
ninna@math.ku.dk

Kristian Juul Schomacker  
Edlund A/S  
Bjerregårds Sidevej 4  
DK-2500 Valby  
Denmark  
kristian.schomacker@edlund.dk

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Main contributions

- Two interacting accounts: a technical account $Y$ and one describing the assets $X$.
- Focus on valuation of non-guaranteed payments via introduction of economic scenarios.
- Common framework for valuation of participating life (PL) and unit-linked (UL) insurance policies.
## Valuation in life insurance/pensions

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### Payment streams

\[
\begin{align*}
\text{dB}^{PL} (t) &= k(\varepsilon(t)) \, dB^u (t) + dB^f (t) - dC (t), \\
\text{dB}^{UL} (t) &= X(t-) \, dB^p (t) + dB^f (t) - dC (t).
\end{align*}
\]
Account projections – Participating life

\[ dX(t) = X(t-1) dR_X(t) + d\varsigma(t) - d\beta^f(t) - k^\epsilon(t) d\beta^u(t) \]

\[ + g(t) d\epsilon(t) - \pi_g(t) d\epsilon(t), \]

\[ dY(t) = Y(t) r^\epsilon(t) dt + d\varsigma(t) - d\beta^f(t) - k^\epsilon(t) d\beta^u(t) \]

\[ + d(t) d\epsilon(t) + \alpha \left( t, r^\epsilon(t), k^\epsilon(t) \right) dt. \]
Additional benefits

- Upscaling factor \( k^{(\varepsilon(t))} \) is determined by

\[
d(t) = \left( k^{(\varepsilon(t))} - k^{(\varepsilon(t-))} \right) V^{u,*,m,\uparrow}(t-),
\]

where

\[
V^{u,*,m,\uparrow}(t) = \sum_{j \in \mathcal{J}} p_{0j}^m (0, t) V^{u,*,\uparrow}_{j}(t, r^*)
\]

= market expected technical reserve

with state-wise technical reserves

\[
V^{u,*,\uparrow}_{j}(t, r^*) = \mathbb{E}^{*} \left[ \int_{t}^{T} e^{-r^*(s-t)} dB^{u}(s) \bigg| Z(t) = j \right].
\]
Account projections – Unit-linked

\[ dX(t) = \underbrace{X(t-) dR_X(t)}_{\text{investment returns}} + \underbrace{\varsigma(t) - d\beta^f(t) - X(t-) d\beta^p(t)}_{\text{expected premiums and benefits}} + \underbrace{(Y(R-) - X(R-))^+ d\varepsilon_R(t) - \pi_g(t) d\varepsilon(t)}_{\text{final guarantee and guarantee fee}} , \]

\[ dY(t) = Y(t) \underbrace{r^* dt}_{\text{technical interest rate}} + \varsigma(t) - d\beta^f(t) - X(t-) d\beta^p(t) + \underbrace{u(t) d\varepsilon(t)}_{\text{guarantee upgrade}} , \quad t \leq R , \]

\[ Y(t) = 0 , \quad t > R . \]
Overlapping generations example – Participating life

- Two policy holders aged 25 enter 20 years apart.

- **Product:**
  - Term insurance of 1 upon death before $T$.
  - Pure endowment of 3 upon survival until $T$.
  - Continuous premium payment of 0.04614 while active.

- **Guarantee fee is a constant fraction of the yield:**

$\pi_g = \theta_3[R_X(t)X(t - 1)]^+.$
Overlapping generations example – continued

- Market value of portfolio is $W(0) = \sum_{i=1,2} W_i(0)$ where
  
  $$W_i(0) = V^i(0) + \mathbb{E}^Q \left[ \int_0^T e^{-\int_0^s r(v)dv} \left( k_i^{(e(s))} - 1 \right) d\beta^u(s) \right].$$

- Fairness on portfolio level since $W_1(0) + W_2(0) = 0$...
- ... but unfair since $W_1(0) < 0$ and $W_2(0) > 0$.

Figure: $k_2$ ends higher than $k_1$. 
Single-policy example – Unit-linked

▶ Same death sum and premium as PL, but different guarantee.
▶ The size of endowment is

\[
\text{"Asset value at time } R\text{"} \\
\text{"Probability of surviving to time } R\text{"} \\
\cdot
\]

▶ The guarantee upgrade is

\[
u(t) = \theta_1 [X(t-) - \pi_g(t) - Y(t-)]^+.
\]

▶ At expiration \([Y(R-) - X(R-)]^+\) is added to the assets.

\[\begin{align*}
0 & \quad 5 & \quad 10 & \quad 15 & \quad 20 & \quad 25 & \quad 30 & \quad 35 & \quad 40 \\
0 & \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad 5 \\
\end{align*}\]

Figure: Sample paths for assets and guarantee.
Participating life vs. Unit-linked

- By construction the unit-linked and participating life product have the same average cash flow.
- However, the products differ in riskiness.

**Figure:** Unit-linked has bigger up- and downside.
Summing up

- Two-account model with event risk.
- Focus on valuation of non-guaranteed payments.
- Common valuation framework for PL and UL.

- Questions?


References IV


