Longevity assets and pre-retirement consumption/portfolio decisions

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AFFI Paris, 2015

Outline

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Co-author

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The Longevity Risk

- *Definition*: risk of unexpected changes in the mortality of a group of individuals
- Still illiquid market: lack of standardization, information asymmetries, basis risk
- More and more transfer of longevity risk from pension funds to re-insurers (OTC)

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- Closed form solution to the consumption/portfolio problem (finite horizon – before retirement)
- HARA preferences (with both consumption and final wealth subsistence levels)
- Any number of risky assets and state variables
- Limits: market completeness and no basis risk

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State Variables

- s state variables $z(t) \in \mathbb{R}^s$ ($z(t_0)$ deterministic)

$$\underset{s\times 1}{dz(t)} = \underset{s\times 1}{\mu_z(t,z)}dt + \underset{s\times n}{\Omega(t,z)'}\underset{n\times 1}{dW(t)}$$

- force of mortality $\lambda(t) \in z(t)$
- survival probability between t and T: $\mathbb{E}_t^{\mathbb{P}}\left[e^{-\int_t^T \lambda(u)du}\right]$

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Completeness

- The market is complete:

$$\exists! \xi(t,z): \quad \Sigma(t,z)' \xi(t,z) = \mu(t,z) - r(t,z)\mathbf{1}$$

- (under suitable condition on ξ) there exists $\mathbb{Q}$:

$$dW^{\mathbb{Q}}(t) = \xi(t,z)\,dt + dW(t)$$

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Longevity Bonds (two examples)

- An asset which pays $\Xi(t)$ if an agent is still alive in t:

$$\mathbb{E}_{t_0}^{\mathbb{Q}}\left[\Xi(t)\, e^{-\int_{t_0}^{t} r(u,z)+\lambda(u,z)du}\right]$$

- An asset which pays $\Xi(\tau)$ at the death time τ:

$$\mathbb{E}_{t_0}^{\mathbb{Q}}\left[\int_{t_0}^{\infty} \lambda(s)\,\Xi(s)\, e^{-\int_{t_0}^{s} r(u,z)+\lambda(u,z)du}\, ds\right]$$

- Any other combination is possible
- The longevity asset $\Lambda(t) \in S(t)$

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Wealth, Consumption, Income

- Investor's wealth is

$$R(t) = \theta_S(t)' S(t) + \theta_G(t) G(t)$$

- Accumulated labor income (state variable)

$$dL(t) = w(t,z)\,dt + \sigma_L(t,z)' dW(t)$$

- Instantaneous consumption is (control variable)

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Dynamic Budget Constraint

- Wealth differential:

$$dR(t) = \theta_S(t)' dS(t) + \theta_G(t) dG(t) + \underbrace{d\theta_S(t)'(S(t) + dS(t)) + d\theta_G(t) G(t)}_{dR_a(t)}$$

- (non-self) Financing condition:

$$dR_a(t) = -c(t) dt + dL(t) + \lambda(t,z) R(t) dt$$

- Dynamic budget constraint

$$dR(t) = \left(R(t)(r(t,z) + \lambda(t,z)) + \theta_S(t)' I_S(\mu(t,z) - r(t,z)1) + w(t,z) - c(t)\right) dt + \left(\theta_S(t)' I_S \Sigma(t,z)' + \sigma_L(t,z)'\right) dW(t)$$

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Maximization

- The agent solves:

$$\max_{\theta_S(t),c(t)} \mathbb{E}_{t_0}\left[\int_{t_0}^{T} \frac{(c(t)-c_m)^{1-\delta}}{1-\delta} e^{-\int_{t_0}^{t}\rho(u,z)+\lambda(u,z)du} dt\right] + \mathbb{E}_{t_0}\left[\frac{(R(T)-R_m)^{1-\delta}}{1-\delta} e^{-\int_{t_0}^{T}\rho(u,z)+\lambda(u,z)du}\right]$$

- under the constraint

$$R(t_0) = \mathbb{E}_{t_0}^{\mathbb{Q}}\left[\int_{t_0}^{T} \left(c(t)-w(t,z)+\sigma_L(t,z)'\xi(t,z)\right) e^{-\int_{t_0}^{t} r(u,z)+\lambda(u,z)du} dt\right] + \mathbb{E}_{t_0}^{\mathbb{Q}}\left[R(T) e^{-\int_{t_0}^{T} r(u,z)+\lambda(u,z)du}\right]$$

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Outline

Optimal Consumption

- The optimal consumption at any time t is:

$$c^*(t) = c_m + \frac{R(t) - H(t,z)}{F(t,z)}$$

- Function $H(t,z)$ is a sum of discounted cash flows:

$$H(t,z) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^T \frac{c_m - w(s,z) + \sigma_L(s,z)'\,\xi(s,z)}{e^{\int_{t_0}^s r(u,z)+\lambda(u,z)du}}ds + \frac{R_m}{e^{\int_{t_0}^T r(u,z)+\lambda(u,z)du}}\right]$$

- Function $F(t,z)$ is a kind of discount factor

$$F(t,z) = \mathbb{E}_t^{\mathbb{Q}_\delta}\left[\begin{array}{c}\int_t^T e^{-\int_t^s\left(\frac{\delta-1}{\delta}r(u,z)+\frac{1}{\delta}\rho(u,z)+\lambda(u,z)+\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\xi(u,z)'\xi(u,z)\right)du}ds \\ +e^{-\int_t^T\left(\frac{\delta-1}{\delta}r(u,z)+\frac{1}{\delta}\rho(u,z)+\lambda(u,z)+\frac{1}{2}\frac{1}{\delta}\frac{\delta-1}{\delta}\xi(u,z)'\xi(u,z)\right)du}\end{array}\right]$$

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A New Probability

- The new subjective probability $\mathbb{Q}_\delta$ is such that

$$dW(t)^{\mathbb{Q}_\delta} = \frac{\delta - 1}{\delta}\xi(t,z)\,dt + dW(t)$$

- which is a weighted mean

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Optimal Portfolio

- The optimal portfolio at any time t is:

$$
\begin{aligned}
I_S \theta_S^*(t) = & \frac{R(t) - H(t,z)}{\delta} \Sigma(t,z)^{-1} \xi(t,z) \\
& - \Sigma(t,z)^{-1} \sigma_L(t,z) \\
& + \frac{R(t) - H(t,z)}{F(t,z)} \Sigma(t,z)^{-1} \Omega(t,z) \frac{\partial F(t,z)}{\partial z} \\
& + \Sigma(t,z)^{-1} \Omega(t,z) \frac{\partial H(t,z)}{\partial z}
\end{aligned}
$$

Outline

Interest Rate & Force of Mortality

- Interest rate is mean reverting ($\xi_r = \phi_r \sqrt{r(t)}$):

$$dr(t) = \alpha_r (\beta_r - r(t))\, dt + \sigma_r \sqrt{r(t)} dW_r(t)$$

- Force of mortality is mean reverting too ($\xi_\lambda = \phi_\lambda \sqrt{\lambda(t)}$):

$$d\lambda(t) = \alpha_\lambda \left(\underbrace{\frac{1}{\alpha_\lambda} \frac{\partial \gamma(t)}{\partial t} + \gamma(t)}_{\beta_\lambda(t)} - \lambda(t) \right) dt + \sigma_\lambda \sqrt{\lambda(t)} dW_\lambda(t)$$

- but towards a divergent mean (Gompertz-Makeham):

$$\gamma(t) = \phi_0 + \frac{1}{b} e^{\frac{t-m}{b}}$$

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Assets

- Stock:

$$A(t)^{-1}\,dA(t) = \mu\,dt + \sigma_A dW_A(t) + \sigma_{Ar} dW_r(t)$$

- rolling Bond:

$$B(t) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{t+T_B} r(u)du}\right]$$

- rolling Longevity Bond:

$$\Lambda(t) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{t+T_\Lambda} r(u)+\lambda(u)du}\right]$$

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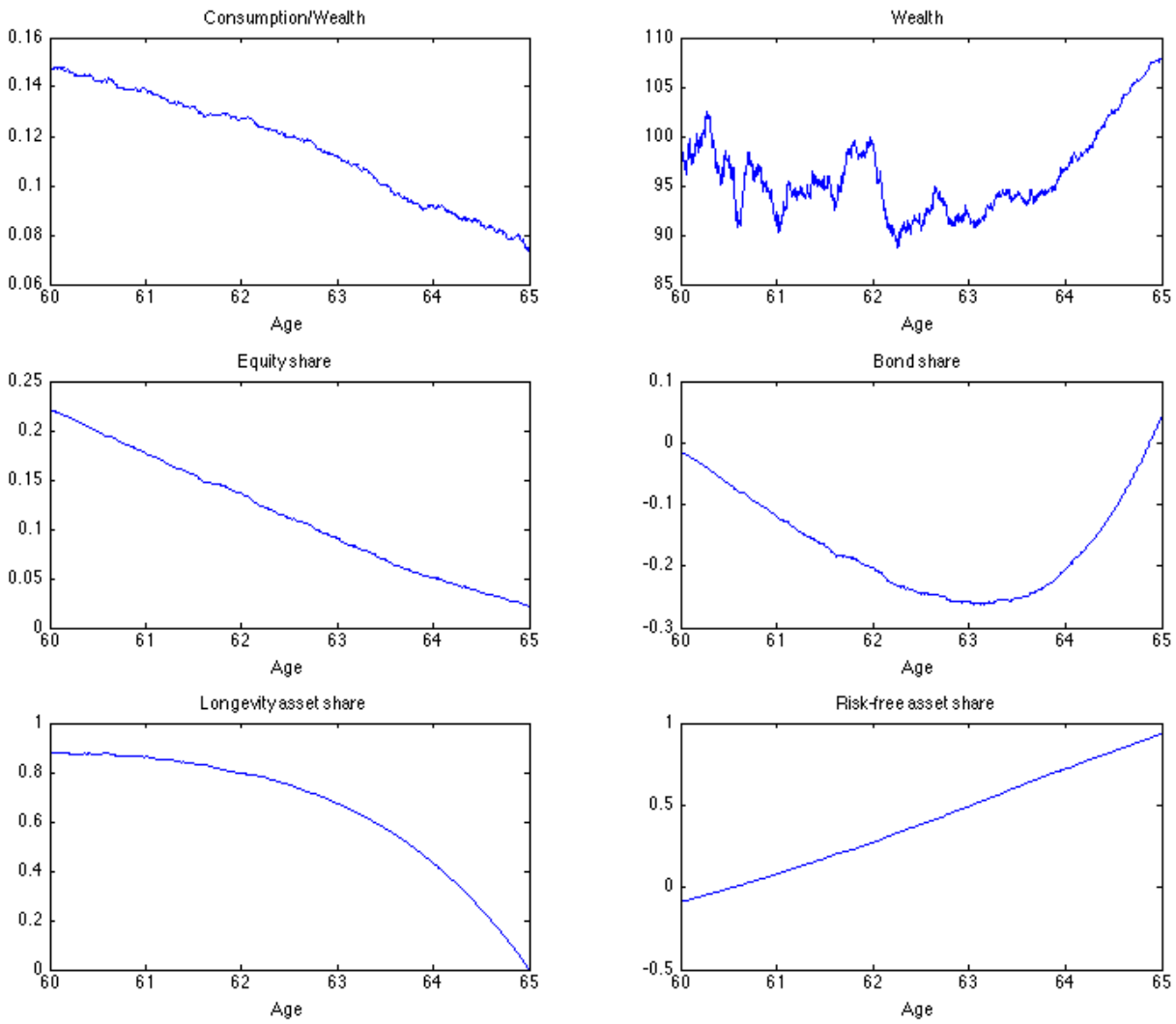
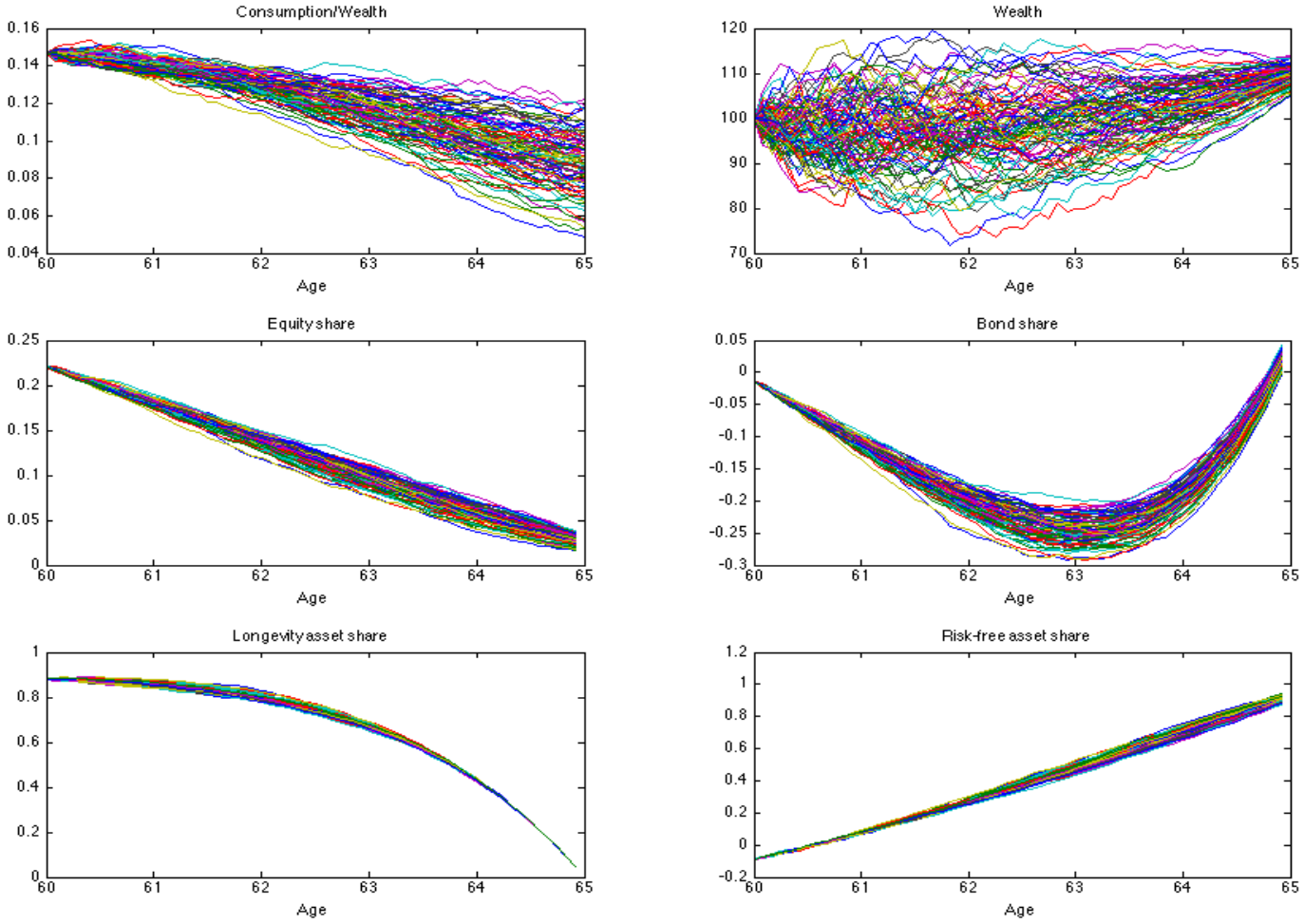
Calibration

Rate/Bond	Stock	Wealth/Pref.	Mort./Long.
$\alpha_r = 0.0904668$	$\sigma_A = 0.14926$	$R_0 = 100$	$\alpha_\lambda = 0.561$
$\beta_r = 0.0621328 = r_0$	$\sigma_{Ar} = 0.0046306$	$w = 10$	$\sigma_\lambda = 0.0352$
$\sigma_r = 0.0543625$	$\xi_A = 0.1108301$	$T = 65$	$\phi_0 = 0.0009944$
$\phi_r = -0.5590635$		$\rho = 0.01$	$b = 12.9374$
$T_B = 10$		$R_m = 100$	$m = 86.4515$
		$c_m = 0$	$t_0 = 60$
		$\delta = 2.5$	$T_L = 10$

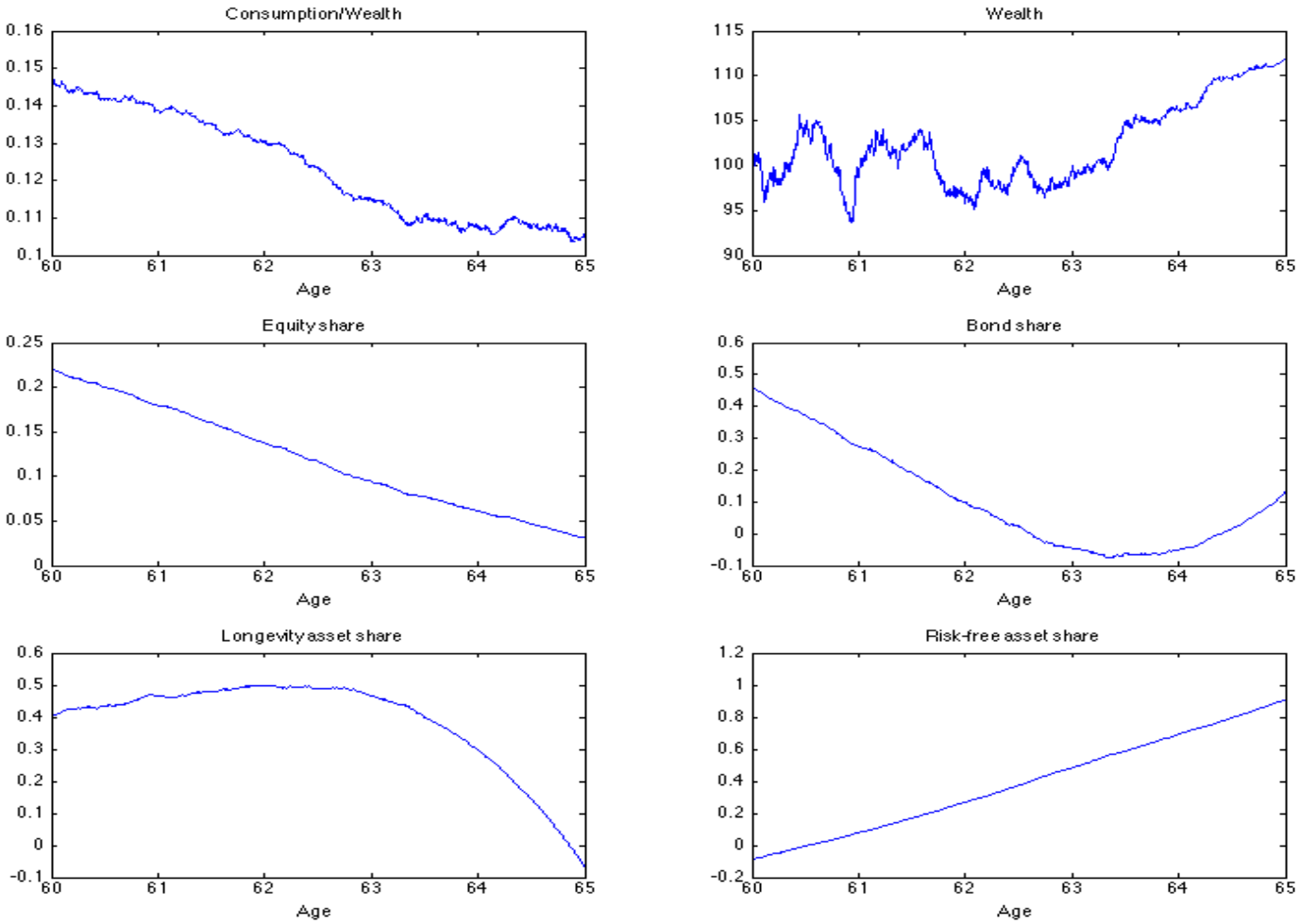
Numerical Results



100 Paths


Consumption/Wealth
Wealth
Equity share
Bond share
Longevity asset share
Risk-free asset share
Age

$\phi_\lambda = 0.1$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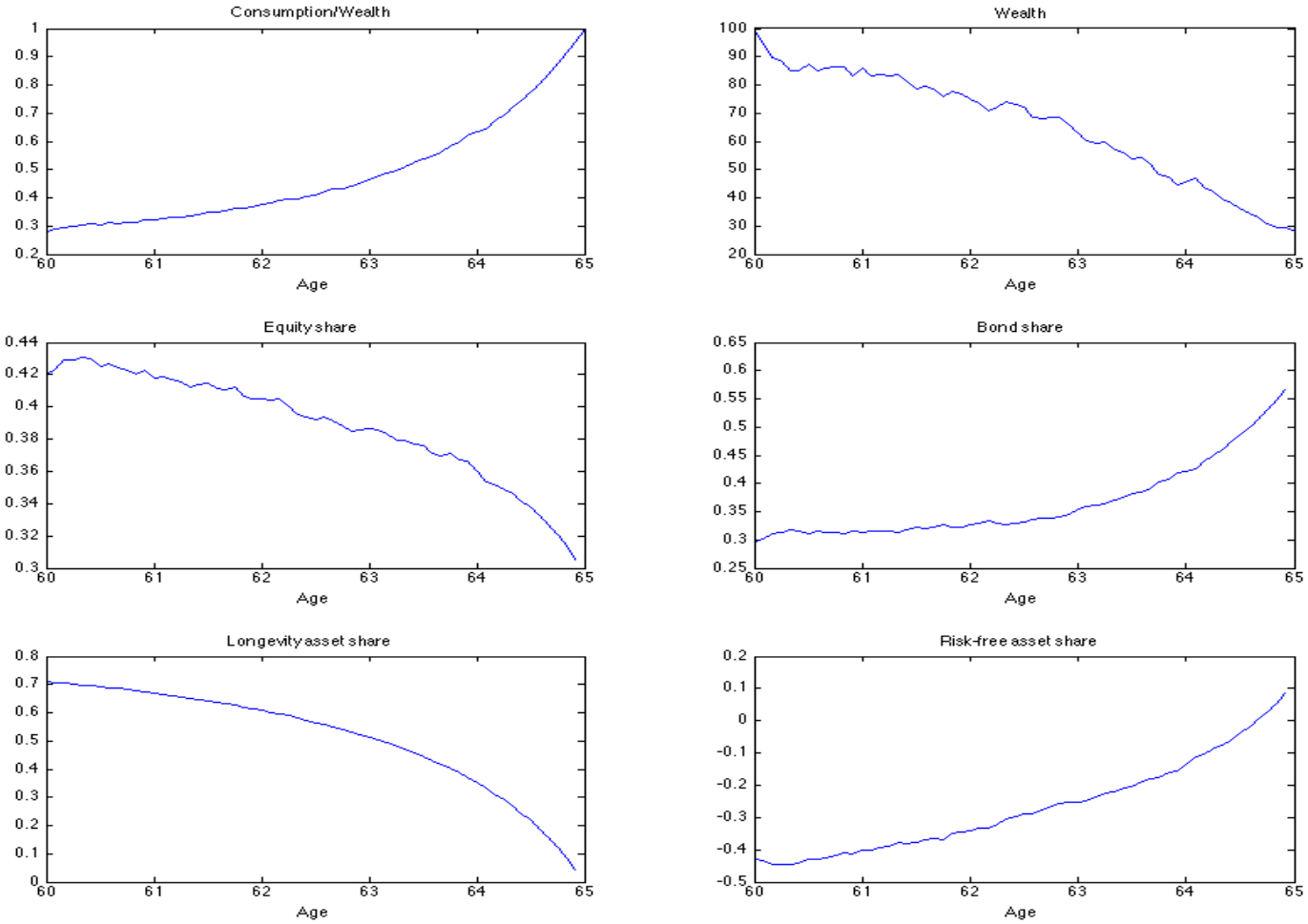
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Main Results

- If there is (no) need for a minimum final wealth then optimal consumption % is decreasing (increasing) over time
- Portfolio volatility is first increasing and then decreasing over time
- Equity share is always decreasing over time
- Bond share has a parabolic convex behaviour over time
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Sensitivity Analysis

- With increasing ϕ_λ there is a strong re-allocation from the longevity asset to the bond
- Sex: women should invest more in longevity (about 100% at first, decreasing to 37%)
- Time horizon ($t_0 = 55$): first 3-year period consumption/wealth is almost constant (and then start decreasing)
- Risk aversion: stock is the most reactive, then longevity (the lower δ the higher the shares)
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