Longevity assets and pre-retirement consumption/portfolio decisions

F. Menoncin\textsuperscript{1} L. Regis\textsuperscript{2}

\textsuperscript{1}Department of Economics and Management
University of Brescia (IT)

\textsuperscript{2}IMT Institute for Advanced Studies (IT)

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Outline

1 First Part
   - Co-Author
   - Motivation
   - The Framework
   - Investor’s Problem

2 Main Results
   - Optimal Consumption/Portfolio

3 Numerical Simulation
   - State Variables
   - Assets and Calibration
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The Longevity Risk

- **Definition**: risk of unexpected changes in the mortality of a group of individuals
- Still illiquid market: lack of standardization, information asymmetries, basis risk
- More and more transfer of longevity risk from pension funds to re-insurers (OTC)
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Our Contributions

- Closed form solution to the consumption/portfolio problem (finite horizon – before retirement)
- HARA preferences (with both consumption and final wealth subsistence levels)
- Any number of risky assets and state variables
- Limits: market completeness and no basis risk
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State Variables

- $s$ state variables $z(t) \in \mathbb{R}^s$ ($z(t_0)$ deterministic)

\[
dz(t) = \mu_z(t, z) dt + \Omega(t, z)' dW(t)
\]

- force of mortality $\lambda(t) \in z(t)$

- survival probability between $t$ and $T$: $\mathbb{E}_t^P \left[ e^{-\int_t^T \lambda(u)du} \right]$
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Menoncin-Regis Portfolio/Longevity assets
Continuously open and friction-less financial market over the time set \([t_0, +\infty]\)

- \(n\) risky assets \(S(t) \in \mathbb{R}_+^n\) (\(S(t_0)\) deterministic):

\[
I_S^{-1}dS(t) = \mu(t, z)dt + \Sigma(t, z)'dW(t)
\]

- one risk-less asset \(G(t) \in \mathbb{R}_+\):

\[
G(t)^{-1}dG(t) = r(t, z)dt
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Financial Market

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G(t)^{-1}dG(t) = r(t,z)dt
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Completeness

- The market is complete:

\[ \exists ! \xi(t,z) : \Sigma(t,z)' \xi(t,z) = \mu(t,z) - r(t,z) 1 \]

- (under suitable condition on \( \xi \)) there exists \( Q \):

\[ dW^Q(t) = \xi(t,z) \, dt + dW(t) \]
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Longevity Bonds (two examples)

- An asset which pays $\Xi(t)$ if an agent is still alive in $t$:
  $$
  \mathbb{E}_t^Q \left[ \Xi(t) e^{-\int_{t_0}^t r(u,z)+\lambda(u,z)du} \right]
  $$

- An asset which pays $\Xi(\tau)$ at the death time $\tau$:
  $$
  \mathbb{E}_t^Q \left[ \int_{t_0}^{\infty} \lambda(s) \Xi(s) e^{-\int_{t_0}^{s} r(u,z)+\lambda(u,z)du} ds \right]
  $$

- Any other combination is possible
- The longevity asset $\Lambda(t) \in S(t)$
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Wealth, Consumption, Income

- Investor’s wealth is
  \[
  R(t) = \theta_S(t)' S(t) + \theta_G(t) G(t)
  \]

- Accumulated labor income (state variable)
  \[
  dL(t) = w(t, z) dt + \sigma_L(t, z)' dW(t)
  \]

- Instantaneous consumption is (control variable)
  \[
  c(t) dt
  \]
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- Instantaneous consumption is (control variable)

\[ c(t) \, dt \]
Dynamic Budget Constraint

- Wealth differential:

\[
dR(t) = \theta_S(t)' dS(t) + \theta_G(t) dG(t) \\
+ d\theta_S(t)' (S(t) + dS(t)) + d\theta_G(t) G(t) \\
\]

\[
dR_a(t)
\]

- (non-self) Financing condition:

\[
dR_a(t) = -c(t) dt + dL(t) + \lambda(t,z) R(t) dt
\]

- Dynamic budget constraint

\[
dR(t) = (R(t) (r(t,z) + \lambda(t,z)) + \theta_S(t)' I_S (\mu(t,z) - r(t,z) 1) + w(t,z) - c(t)) dt \\
+ (\theta_S(t)' I_S \Sigma(t,z)' + \sigma_L(t,z)') dW(t)
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Dynamic Budget Constraint

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\[ dR(t) = (R(t) (r(t, z) + \lambda(t, z)) + \theta_S(t) I_S (\mu(t, z) - r(t, z)) 1) + w(t, z) - c(t) dt + (\theta_S(t) I_S \Sigma(t, z) + \sigma_L(t, z) ) dW(t) \]
Maximization

- The agent solves:

\[
\max_{\theta S(t), c(t)} \mathbb{E}_{t_0} \left[ \int_{t_0}^{T} \left( \frac{(c(t) - c_m)^{1-\delta}}{1-\delta} e^{-\int_{t_0}^{t} \rho(u,z) + \lambda(u,z) du} \right) dt + \mathbb{E}_{t_0} \left[ \frac{(R(T) - R_m)^{1-\delta}}{1-\delta} e^{-\int_{t_0}^{T} \rho(u,z) + \lambda(u,z) du} \right] \right]
\]

- under the constraint

\[
R(t_0) = \mathbb{E}_{t_0}^Q \left[ \int_{t_0}^{T} (c(t) - w(t,z) + \sigma_L(t,z)' \xi(t,z)) e^{-\int_{t_0}^{t} r(u,z) + \lambda(u,z) du} dt \right] + \mathbb{E}_{t_0}^{Q} \left[ R(T) e^{-\int_{t_0}^{T} r(u,z) + \lambda(u,z) du} \right]
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\[
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Optimal Consumption

The optimal consumption at any time \( t \) is:

\[
c^*(t) = c_m + \frac{R(t) - H(t, z)}{F(t, z)}
\]

Function \( H(t, z) \) is a sum of discounted cash flows:

\[
H(t, z) = \mathbb{E}_t^Q \left[ \int_t^T \frac{c_m - w(s, z) + \sigma_L(s, z)' \xi(s, z)}{e^{\int_t^s r(u, z) + \lambda(u, z)du}} ds + \frac{R_m}{e^{\int_t^T r(u, z) + \lambda(u, z)du}} \right]
\]

Function \( F(t, z) \) is a kind of discount factor

\[
F(t, z) = \mathbb{E}_t^{Q_\delta} \left[ \int_t^T e^{-\int_t^s \left( \frac{\delta-1}{\delta} r(u, z) + \frac{1}{\delta} \rho(u, z) + \lambda(u, z) + \frac{1}{2} \frac{\delta-1}{\delta} \xi(u, z)' \xi(u, z) \right) du} ds + e^{-\int_t^T \left( \frac{\delta-1}{\delta} r(u, z) + \frac{1}{\delta} \rho(u, z) + \lambda(u, z) + \frac{1}{2} \frac{\delta-1}{\delta} \xi(u, z)' \xi(u, z) \right) du} \right]
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$$F(t,z) = \mathbb{E}^{Q_\delta}_t \left[ \int_t^T e^{-\int_t^s \left( \delta - 1 \delta r(u,z) + \frac{1}{\delta} \rho(u,z) + \lambda(u,z) + \frac{1}{2 \delta} \frac{\delta - 1}{\delta} \xi(u,z)' \xi(u,z) \right) du} ds \
+ e^{-\int_t^T \left( \delta - 1 \delta r(u,z) + \frac{1}{\delta} \rho(u,z) + \lambda(u,z) + \frac{1}{2 \delta} \frac{\delta - 1}{\delta} \xi(u,z)' \xi(u,z) \right) du} ds \right]$$
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A New Probability

- The new subjective probability $Q_\delta$ is such that

$$dW(t)^{Q_\delta} = \frac{\delta - 1}{\delta} \xi(t, z) dt + dW(t)$$

- which is a weighted mean

$$dW(t)^{Q_\delta} = \left(1 - \frac{1}{\delta}\right)dW(t)^{Q} + \frac{1}{\delta}dW(t)$$
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The optimal portfolio at any time \( t \) is:

\[
I_{S} \theta_{S}^{*}(t) = \frac{R(t) - H(t, z)}{\delta} \Sigma(t, z)^{-1} \xi(t, z) \\
- \Sigma(t, z)^{-1} \sigma_L(t, z) \\
+ \frac{R(t) - H(t, z)}{F(t, z)} \Sigma(t, z)^{-1} \Omega(t, z) \frac{\partial F(t, z)}{\partial z} \\
+ \Sigma(t, z)^{-1} \Omega(t, z) \frac{\partial H(t, z)}{\partial z}
\]
Interest Rate & Force of Mortality

- Interest rate is mean reverting ($\xi_r = \phi_r \sqrt{r(t)}$):

  $$dr(t) = \alpha_r (\beta_r - r(t)) dt + \sigma_r \sqrt{r(t)} dW_r(t)$$

- Force of mortality is mean reverting too ($\xi_{\lambda} = \phi_\lambda \sqrt{\lambda(t)}$):

  $$d\lambda(t) = \alpha_\lambda \left( \frac{1}{\alpha_\lambda} \frac{\partial \gamma(t)}{\partial t} + \gamma(t) - \lambda(t) \frac{\beta_\lambda(t)}{\beta_\lambda(t)} \right) dt + \sigma_\lambda \sqrt{\lambda(t)} dW_\lambda(t)$$

- but towards a divergent mean (Gompertz-Makeham):

  $$\gamma(t) = \phi_0 + \frac{1}{b} e^{\frac{t-m}{b}}$$
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Assets

- Stock:

\[ A(t)^{-1} dA(t) = \mu dt + \sigma_A dW_A(t) + \sigma_{Ar} dW_r(t) \]

- rolling Bond:

\[ B(t) = \mathbb{E}_t^Q \left[ e^{-\int_t^{T_B} r(u) du} \right] \]

- rolling Longevity Bond:

\[ \Lambda(t) = \mathbb{E}_t^Q \left[ e^{-\int_t^{T_\Lambda} r(u) + \lambda(u) du} \right] \]
Assets

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- rolling Bond:
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## Calibration

<table>
<thead>
<tr>
<th>Rate/Bond</th>
<th>Stock</th>
<th>Wealth/Pref.</th>
<th>Mort./Long.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_r = 0.0904668$</td>
<td>$\sigma_A = 0.14926$</td>
<td>$R_0 = 100$</td>
<td>$\alpha_\lambda = 0.561$</td>
</tr>
<tr>
<td>$\beta_r = 0.0621328 = r_0$</td>
<td>$\sigma_{Ar} = 0.0046306$</td>
<td>$w = 10$</td>
<td>$\sigma_\lambda = 0.0352$</td>
</tr>
<tr>
<td>$\sigma_r = 0.0543625$</td>
<td>$\xi_A = 0.1108301$</td>
<td>$T = 65$</td>
<td>$\phi_0 = 0.0009944$</td>
</tr>
<tr>
<td>$\phi_r = -0.5590635$</td>
<td></td>
<td>$\rho = 0.01$</td>
<td>$b = 12.9374$</td>
</tr>
<tr>
<td>$T_B = 10$</td>
<td></td>
<td>$R_m = 100$</td>
<td>$m = 86.4515$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c_m = 0$</td>
<td>$t_0 = 60$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\delta = 2.5$</td>
<td>$T_L = 10$</td>
</tr>
</tbody>
</table>
Numerical Results

- Consumption/Wealth
- Wealth
- Equity share
- Bond share
- Longevity asset share
- Risk-free asset share
100 Paths

- Consumption/Wealth vs. Age
- Wealth vs. Age
- Equity share vs. Age
- Bond share vs. Age
- Longevity asset share vs. Age
- Risk-free asset share vs. Age

Menoncin-Regis
Portfolio/Longevity assets
First Part
Main Results
Numerical Simulation
Summary

State Variables
Assets and Calibration

Portfolio/Longevity assets
$R_m = 0$
First Part
Main Results
Numerical Simulation
Summary

Menoncin-Regis
Portfolio/Longevity assets
Main Results

- If there is (no) need for a minimum final wealth then optimal consumption % is decreasing (increasing) over time.

- Portfolio volatility is first increasing and then decreasing over time.

- Equity share is always decreasing over time.

- Bond share has a parabolic convex behaviour over time.

- Risk free asset share is increasing over time.

- Longevity asset share is decreasing over time, its level heavily depends on $\phi_\lambda$ and has a very low volatility.
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Sensitivity Analysis

- With increasing $\phi_\lambda$ there is a strong re-allocation from the longevity asset to the bond
- Sex: women should invest more in longevity (about 100% at first, decreasing to 37%)
- Time horizon ($t_0 = 55$): first 3-year period consumption/wealth is almost constant (and then start decreasing)
- Risk aversion: stock is the most reactive, then longevity (the lower $\delta$ the higher the shares)
- Subsistence consumption: a more conservative strategy is adopted (less stock and bond, more riskless), while longevity is almost unaffected
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