Determining Discount Rates Required to Fund Defined Benefit Plans

John A. Turner
Pension Policy Center

Humberto Godinez-Olivares
University of Liverpool

David D. McCarthy
Pension Policy Center

Carmen Boado Penas
University of Liverpool

March 2015

Contact Author:
John A. Turner
3713 Chesapeake St. NW
Washington, DC 20016
United States
Jaturner49@aol.com

This paper analyzes the discount rate required for funding defined benefit pension plans. Current approaches used in the United States either base discount rates for determining funding solely on bond rates of return or solely on the expected rate of return on the pension plan's portfolio. Previous studies by U.S. economists have focused on the related problem of valuing defined benefit liabilities. We present a model that focuses on the probability that for a liability accruing today the assets based on current contributions will be less than the value of that liability at some point in the future when benefits must be paid, requiring further contributions. This probability parameter is implicit in all approaches to determining the adequacy of defined benefit plan funding, but generally has not been made explicit. We argue that from a policy perspective it is a key parameter. The stochastic funding parameter approach generally results in discount rates that are greater than the risk-free rate advocated in much of the economics literature on the topic, and are less than the market-based rates used by state and local government pension plans in the United States. We argue that approaches that focus only on the risk to liabilities or only on the risk to assets do not consider the risk that future contributions will be needed, which involves an analysis of the bivariate distribution of risks of assets and liabilities. With the stochastic funding parameter approach, while the exact discount rate (hurdle rate) used to determine adequate funding depends on the risk to the assets, the risk to the liabilities, and the duration of the liabilities, a simple rule of thumb can be stated. That rule would be to select a discount rate that is less than the expected rate of return on assets but greater than the risk free rate, with the discount being greater the higher the percentage of the portfolio invested in equity and the longer the duration of the liabilities.

1. Introduction

Globally, defined benefit plans have liabilities valued at $23 trillion (The Economist 2014). Actuaries, economists and other financial analysts, however, do not agree as to how those liabilities should be valued and the required level of funding determined. This paper analyzes the question of the required discount rate for determining adequate funding for defined benefit pension plans.

The answer to a problem depends on the question being asked. Financial analysts have put forward several approaches for determining interest rates for calculating required contributions for defined benefit plans. The primary two approaches can for simplicity be called the "economists' approach" and the "actuaries' approach" (Minahan 2010). The disagreement between the economists' approach and the actuaries' approach to determining pension discount rates suggests that the problem may be defined differently by the two groups—they may be answering different questions.

In the United States, private sector and public sector defined benefit plans use different approaches for determining the discount rate used in calculating funding ratios. The Treasury Department currently requires use of a 25-year average of high grade corporate bonds. FASB for accounting purposes requires use of current high grade corporate bonds matching the pension benefit cash flows. The PBGC requires using a 24-month average of high-grade corporate bonds for determining PBGC required contributions. Thus, none of the approaches take into account the portfolio of the plan. State and local government plans, by contrast, use the expected rate of return on their portfolios, as required by GASB.

In this paper, we argue that none of these approaches are appropriate for determining required funding in that none of them explicitly take into account the probability that funding will be inadequate and that future contributions will be required. None of them take into account both the plan's assets and liabilities.

This paper first discusses the previous literature. Because of the controversial nature of the choice of discount rates (at least in the United States), this paper presents the intuition of its approach by starting with simple models to demonstrate the basic points. It then moves to more realistic models that are more complex but also permit a demonstration that the points established in the simple models are robust to more complex analyses. The development of models thus starts with a simple two-period model where either assets or liabilities are risk free, and moves to a more complex multi-period model where both assets and liabilities are risky. The paper concludes with a section on policy implications and concluding remarks.

To anticipate the conclusions, we find that approaches that focus only on assets or only on liabilities do not take into consideration the probability that additional contributions will be required and tend to produce discount rates that are either too low (liabilities approach) or too high (assets approach).

2. The Previous Literature

The economists' approach to determining discount rates for defined benefit plan valuation defines the problem as determining the present value of future pension liabilities. The riskiness of the liabilities determines the discount rate that applies for determining their present value. For example, risk-free liabilities are discounted using a risk-free discount rate. Risk to plan sponsors arises due to mortality risk and future wage rate risk. Risk to participants arises due to default risk. The higher the risk of default by the plan sponsor, the greater the risk to the liability from the participant's perspective, and thus the higher discount rate (Novy-Marx and Rauh 2009, 2013). The choice of discount rates for valuing a liability is independent of the investment portfolio, according to this approach. Novy-Marx and Rauh (2009, 2013), however, note that the question of determining the value of the liabilities is different from determining desired funding.

The actuaries' approach defines the problem as determining the value of assets needed today to fund future liabilities. The expected rate of return on the portfolio determines the discount rate to be used in order to determine how much assets are needed today to pay for future pension liabilities. This approach is used by U.S. state and local government defined benefit pension plans.

The two approaches are answering different questions. The economists' approach is answering the question, what is the appropriate discount rate for valuing future liabilities? The actuaries' approach is answering the question, what is the appropriate discount rate for assuring that assets will be sufficient to fund future liabilities? In this paper, we develop a generalization of the actuaries' approach for when it is required that current contributions be sufficient to fund future liabilities more than 50 percent of the time.

The Actuarial Standards Board (2013) of the American Academy of Actuaries identifies a third possible approach for selecting discount rates. In addition to the economists' and actuaries' approaches, it argues that discount rates can be based on the rate of return implicit in annuity prices. That approach can be viewed as a variant of the economists' approach in that it does not consider the riskiness of the pension portfolio.

A fourth approach has been presented by Day (2004), which can be categorized as a variation of the economists' approach. When assets do not exist to match the plan's liabilities, which is the general situation, he argues that in comparison to a risk-free liability, "In very general terms, the uncertainty of the cash flows makes for a worse liability than before. Thus the liability increases in value rather than decreases as it does for the asset side." Because the liability would increase in value relative to a risk free liability, that implies that a discount rate lower than the risk-free interest rate would be used for valuing the liability which is the opposite of the economists' approach.

Haberman et al. (2003) present the case that pension plans should be evaluated using stochastic rather than deterministic models. They further argue that the measure of risk should take into account the expected size of a shortfall, as well as the probability that a shortfall occurs. Their approach, which is similar in many ways to the approach taken here, focuses on contribution rates rather than on the choice of discount rates.

Because one approach is that defined benefit liabilities should be valued consistently with the valuation of group annuities, the literature on valuation of annuities is relevant to this discussion. Cannon and Tonks (2013), in their analysis of annuities, note that the greater the risk of the liability, the greater the reserves needed by the life insurance company to ensure that the liability can be met. Because long duration liabilities have greater risk due to the possibility of improvements in life expectancy, they require lower discount rates. Thus, their argument is similar to that of Day (2004).

The Day approach is consistent with labor market analysis as to the value of the asset to the asset holders, who are the pension participants. In the labor market, employers do not provide pensions for free. In determining competitive wages, the labor market requires a tradeoff between higher wages versus higher pension benefits for a particular worker. According to the traditional view of the labor market, workers value the insurance aspect of an annuity provided by a defined benefit plan, so that they are willing to forgo more in wages than the expected present value of the payment. Thus, a lower discount rate than the risk free rate would be required. This view of the labor market side of the wage-pension tradeoff is consistent with the capital market side, as analyzed by Day (2004).[i] Because the pension insures against risk for workers, it is worth more to workers and costs more as a liability to employers and annuity providers.

A related literature examines the effect of an increase in risk on precautionary savings by households. That literature demonstrates that risk averse individuals or households increase precautionary savings when a mean preserving increase in risk of future liabilities occurs (for example, Apps et al. 2014). This result is consistent with the pricing of annuities by life insurance companies and with the argument that the discount rate for valuing a risky liability should be lower than the risk free rate.

Jong (2008) presents an approach that takes into account the riskiness of liabilities and the availability of assets to hedge those risks. He considers issues relating to the determination of the expected present value of pension liabilities when those liabilities are based on unknown future labor earnings. For the approach of valuing pension liabilities at market prices (the economists' approach), he notes the problem that typically pension liabilities are not marketed assets. The long maturity of the claims and their indexation to wages for workers accruing benefits make it impossible to find market instruments with similar characteristics. Future wages cannot be hedged perfectly with existing financial market instruments. In the literature on financial asset pricing, the situation where the payoff pattern of an asset cannot be perfectly replicated is referred to as an incomplete market. The holder of the pension liability assumes an unhedgeable risk, which affects the probability distribution of his consumption and final wealth, and thus his utility. The certainty equivalent wealth of the expected utility is the amount the pension plan is willing to pay for the claim. Finding a value for the pension liability amounts to finding a value for the unhedgeable risk. To achieve the same certainty equivalent wealth in the incomplete market, the investor needs to invest more than in the complete markets case with perfect hedging. Jong advises that in comparison to a risk free discount rate, the discount rate should be adjusted downward for the extent of unhedgeable risk. Thus, he has a similar conclusion to the Day approach.

Nijman et al. (2013) analyze the valuation of pension liabilities under the system of risk sharing in the Dutch pension system. Their paper and the following paper both assume the only risk is financial market risk. Bovenberg et al. (2014) argue that in a complete market where all pension cash flows can be replicated, the discount rate is the rate of return on an investment portfolio that completely replicates (or hedges) the cash flows of the liabilities. The market consistent valuation of the liability can be determined using asset pricing theory.

3. Analysis of Three Approaches to Choice of Discount Rates

This section contains further analysis of the three approaches that we call the economists' approach, the actuaries' approach and the Day approach. We start by using a simple two-period model. The following section considers simple scenarios where \$1 is to be either received or paid one year from now, and the value of the payment is known for certain or has a normal distribution.

3.1 The Economists' Approach. This scenario is an exercise in determining the expected present value of a risky future payment to be received. The expected value of the payment valued at the future date of the payment (the mean of the distribution of payments) is 1. Assume the person is risk averse. Because the person is risk averse, the person values the risky future payment to be worth less than a certain payment of \$1.

In determining the amount the person is willing to pay for the future asset, the person uses a discount rate r_p that incorporates a risk premium p ($p > 0$), and is thus higher than the risk-free interest rate ($r_p = r_f + p > r_f$), The amount the person is willing to pay to receive the risky payment, which can be denoted as $PV(1, r_f + p)$, is thus less than the amount he would be willing to invest to receive the risk free payment with the same expected future value.

$$PV = PV(1, r_f + p) \qquad (3.1)$$

This is essentially the economists' approach, which uses the analysis of valuing future income receivable for valuing a future liability. This is the amount someone would pay to receive a risky payment in the future. This approach does not recognize the argument of Day that risky assets and risky liabilities would be valued differently.

3.2 The Actuaries' Approach. Now consider the same scenario except the person must make a payment in one year rather than receive a payment. It is assumed that no asset is available for hedging the liability. The expected rate of return on the person's portfolio, which incorporates the risk of the portfolio is r_e. In this case, the present value of the future payment is determined by discounting by the expected rate of return on the portfolio.

$$PV = PV(1, r_e) \qquad (3.2)$$

Thus, the two approaches use different methodologies for determining the discount rate.

3.3 The Day Approach. Because the plan sponsor is risk averse, and perhaps also because of regulatory reasons, the plan sponsor is more concerned about the outcome of having saved too little than about having saved too much. For that reason, the plan sponsor engages in precautionary saving to try to assure having adequate resources set aside to make the payment. Thus, the plan sponsor needs to save more money in anticipation of the risky payment than the plan sponsor does for a risk-free payment. In practice, because of the tax on reversions, plan sponsors who might terminate their plans do not want to accumulate a surplus.

Because the plan sponsor needs precautionary savings beyond what the plan sponsor would save for a risk free liability, the plan sponsor uses a lower discount rate than the risk free discount rate. The plan sponsor subtracts a factor d ($d > 0$) from the risk free rate, so that the rate used for discounting risky liabilities is $r_d = r_f - d$. The magnitude of the discount d is determined by the amount of precautionary savings the plan sponsor needs, given the person's degree of risk aversion, and would be greater the greater the degree of risk aversion. The degree of risk aversion affects the person's target probability c of not having saved sufficient assets to meet the risky liability. Thus, the amount the person would set aside to pay for the future risky liability would be

$$PV = PV(1, r_f - d). \qquad (3,3)$$

Thus, this result yields a lower discount rate than the other two approaches.

3.4 Critique. All three approaches are flawed in that they do not consider in sufficient detail the actual problem of funding defined benefit plans with confidence. In particular, they do not consider the acceptable probability of success, defined as the outcome that plan sponsors will not need to make further contributions in the future to pay for liabilities already accrued. All three approaches fail to take into account the risk of the pension portfolio in determining the amount of assets needed, assuming that the probability needs to be greater than 50 percent that further contributions will not be needed.

We thus argue that none of the approaches is asking the right question concerning required contributions for funding defined benefit plan liabilities. The right question needs to take into account the probability of success (or the risk as to whether future contributions will be needed, and perhaps also the size of those future contributions).

Thus, we start our analysis by asking this question: What is the discount rate needed for determining contributions to assure that current contributions will be sufficient c percent of the time? More complex versions of this question can take into account the magnitude of the required future contributions, for example that current contributions not exceeding 10 percent of the current contribution in present value will be sufficient c percent of the time?

For the three approaches, the actuaries' approach provides the highest discount rate, the economists' approach provides an intermediate discount rate, and the Day (2004) approach provides the lowest discount rate. All three approaches implicitly have a different acceptable probability of success, and thus, it could be said that all three approaches fit within the framework of this paper. However, none of them explicitly take into account the probability of success in selecting the rate to be used to determine required contributions.

The following section provides a simple model presenting the stochastic funding parameter approach, based on the concept that both the risk to assets and the risk to liabilities affect the probability that the plan sponsor will need to make additional contributions in the future.

4. A Model Based on the Probability of Needing to Make Future Contributions for Currently Accruing Liabilities—The Stochastic Funding Parameter Approach

In the analysis of the choice of discount rates that follows, we make numerous simplifying assumptions. We do so for the purpose of presenting in a simplified framework the key insights of the approach. We first analyze the scenario where assets are risky but liabilities are risk-free. We then reverse the assumptions and analyze the scenario where liabilities are risky but assets are risk-free.

In analyzing the variability in underfunding, the following equation for the variance in the difference between assets A and liabilities L is useful.

$$\mathrm{Var}(A\text{-}L) = \mathrm{Var}(A) + \mathrm{Var}(L) - 2\mathrm{Cov}(A,L) \qquad (4.1)$$

We first consider the situations where only assets are variable or only liabilities are variable, so that in those two situations Var(A-L) equals either Var(A) or Var(L). If perfect hedging were available, the covariance term would equal the sum of the variance terms, and Var(A-L) would equal zero.

For clarity, we distinguish between the discount rate used to value the present value of the expected distribution of future liabilities, versus the interest rate used to determine required contributions. We call the latter interest rate the "hurdle" rate. The hurdle rate is the rate of return used for determining the value of the liability that the portfolio must exceed in order to achieve full funding a given percentage of the time.

4.1 Two-Period Model: Risky Assets, Risk-Free Liabilities. Following the principle of working from simple to more complex analyses, we start with a two-period model. In the first period, the plan determines the amount of contributions it needs to make in order to pay for its second period liabilities. We distinguish between the mean or expected rate of return and the discount rate needed to assure adequate funding (the hurdle rate).

In all the scenarios, we assume that the plan is required to make contributions sufficient so that at least c percent of the time no further contributions are needed in the second period. This approach is a budgeting approach, relating to the sponsor's need to budget for future contributions. A different approach would be a solvency approach, which would focus on the risk to the participant of not receiving the promised benefits. For that approach, participants would want a very high probability of success. Yet another approach would be to focus on the expected present value of required future contributions.

The parameter c defines the success rate. Monte Carlo simulations generally provide results in terms of probability of success (Pfau 2014), and that concept applies in the case of assuring adequate pension funding. This assumption is key in determining how much is needed to be contributed, and is the element that is missing in the simplified versions of the economists' approach and the actuaries' approach. The value of the parameter would presumably depend on the risk that the plan sponsor would be unable (or unwilling) to make the required future contributions. Default risk thus can be taken into account through this parameter. The parameter in principle would be set by pension regulators, and could vary between state government and private sector plans. We discuss in a later section how this parameter might be implemented in policy.

Assume that liabilities are riskless but that assets are risky with normal distribution and known mean and variance. This approach is similar to the actuaries' approach in that it treats the risk of liabilities as irrelevant. Given this distribution, the goal is to determine what level of contribution in the first period needed to assure that assets in the second period will exceed liabilities in the second period at least c percent of the time, i.e., what level of assets in the first period is needed to assure that prob($A_2 \geq L_2$) = c.

To analyze the effect of risk in asset returns, we need to make some assumptions about the probability distribution of rates of return for given mean rates of return. To do so, we need information about the mean and standard deviation of rates of return for portfolios on the efficient frontier. Table 1 provides an example of the relationship between mean and standard deviation of return. It is derived from the efficient frontier for target date funds that are part of the Thrift Saving Plan, which is the 401(k)-type plan for U.S. federal government workers (Thrift Savings Plan 2013). It shows the relationship between the mean rate of return, standard deviation of returns, and the rate of return at different points on the distribution of rates of return corresponding to those parameters for a set of portfolios. These historical data are used for providing an example of the approach being analyzed. Historical data may not be representative of future experience.

The L Funds are the target date funds for different target dates. The Income Fund is the final fund for workers who have reached their target date. For these funds, as expected, as the mean rate of return increases, the standard deviation of returns also increases.

Table 1. L Funds and the Efficient Frontier

https://www.tsp.gov/PDF/formspubs/LFunds.pdf

TSP	Return	Risk	Annual Return for Various Return Percentiles							
Fund	**Mean**	**Std. Dev.**	**50**	**45**	**42**	**40**	**30**	**20**	**10**	**5**
Income	5.8	4.3	5.8	5.3	4.9	4.7	3.5	2.2	0.3	-1.3
L-2020	7.2	11.0	7.2	5.8	5.0	4.4	1.4	-2.1	-6.9	-10.9
2030	7.6	13.0	7.6	6.0	5.0	4.3	0.8	-3.3	-9.1	-13.8
2040	8.0	15.5	8.0	6.1	4.9	4.1	-0.1	-5.0	-11.9	-17.5
2050	8.3	17.5	8.3	6.1	4.8	3.9	-0.9	-6.4	-14.1	-20.5

Table 1 is calculated for a highly unrealistic situation of a two-period model where the first period lasts one year. Later scenarios will investigate whether the results hold under more realistic assumptions.

If the acceptable level of certainty c that future contributions will not be needed is 50 percent, the table shows that the mean rate of return can be used for discounting future liabilities when liabilities are riskless, confirming the actuaries' approach, but under limited circumstances. However, for any higher level of likelihood c that future contributions will not be needed, corresponding to a lower point on the probability distribution of rates of return, the rate of return required for the discount rate assumption is lower than the mean rate of return. For example, if the mean expected return on the portfolio is 5.8 percent and the acceptable level of certainty c that the plan will have sufficient assets for the target benefit level is 0.6 (corresponding to 0.4 in the table), the amount of contributions should be based on an assumed discount rate (hurdle rate) of 4.7 percent, which is higher than the risk-free interest rate, which in 2014 was approximately 3.5 percent. In other words, for a 60 percent success rate we need to set aside assets based on the assumption that the rate of return will be at least 4.7 percent. If the mean expected return is 7.2 percent with the same acceptable level of certainty (60 percent) as to having sufficient assets for the target benefit level, the amount of contributions should be based on an assumed rate of return (hurdle rate) of 4.4 percent.

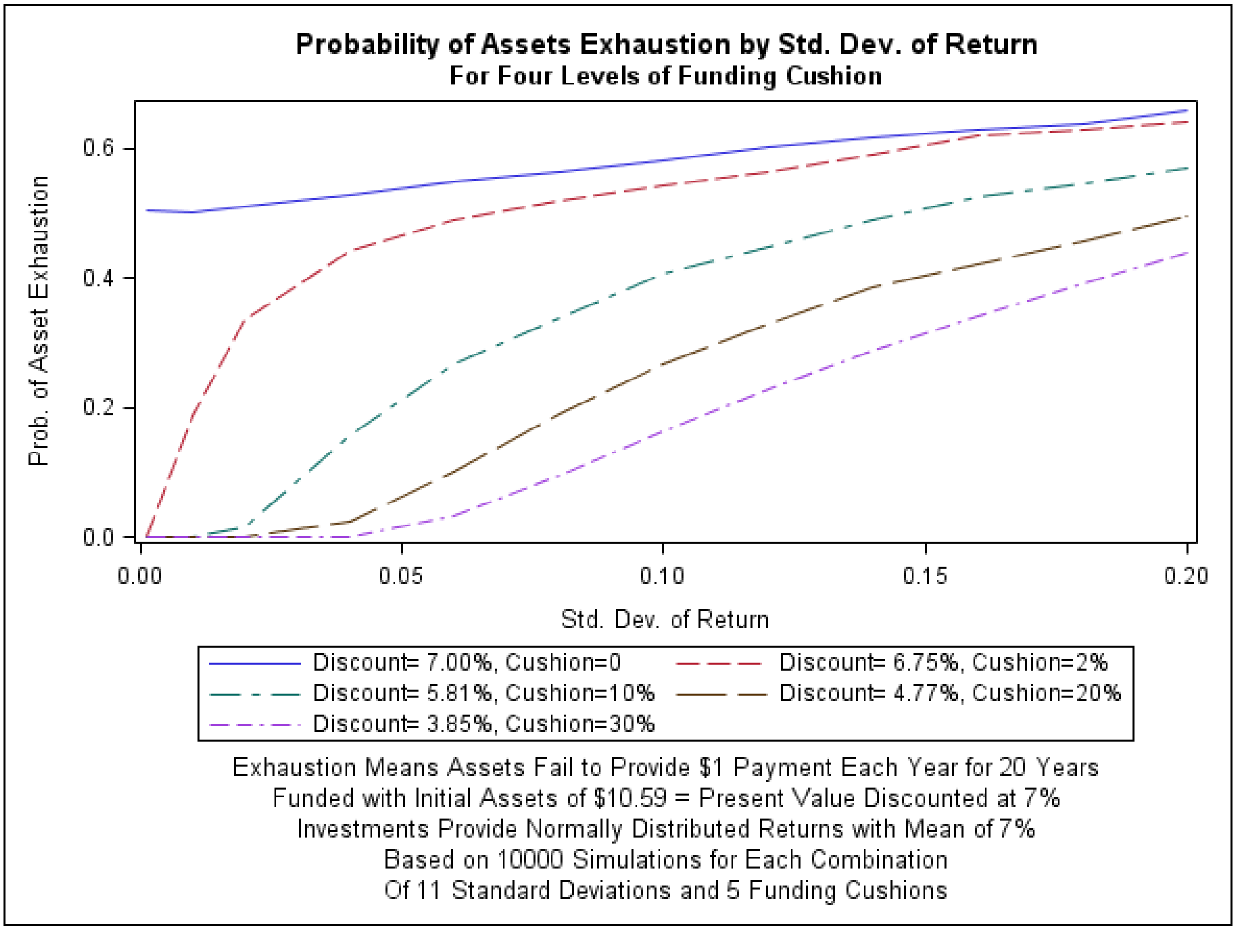
Thus, for a higher mean rate of return, the required discount rate would be lower to compensate for the higher risk. This relationship is exactly opposite from the relationship assumed in funding state and local government pension plans in the United States. Table 1 demonstrates a key point. When the acceptable level of success c is 60 percent or higher, the higher is the expected return for the investment portfolio, and thus the risk, the lower the allowable assumed rate of return for calculating required contributions.

4.2 Multi-Period Model: Risky Assets, Risk-Free Liabilities. We next consider a multi-period model where benefits are paid over a 20-year period. Again, we assume certainty in the liability, but risk as to the rate of return.

Figure 1 shows the probability of exhaustion of assets. Assets are initially set equal to the present value of an annuity paying $1 per year for 20 years, discounted at 7 percent, which is the mean of the assumed distribution of rates of return for this example. Given this assumption and a constant (certain) 7 percent rate of return, the final balance after 20 years would always be zero, with no risk of insufficiency.

Variable returns increase the probability of asset exhaustion, which can be alleviated with funding cushions. Funding cushions require higher contributions so that initial assets are set at higher levels. In Figure 1, different funding cushion levels are selected and then the hurdle rate associated with them is determined. The labels for the chart indicate the size of the cushion provided by reducing the hurdle rate below the discount rate of 7 percent. In this figure, the five lines show the probability of failure (exhaustion of assets before the end of the payout period measured on the y axis) as a function of the standard deviation of returns around an assumed mean of 7 percent for five different levels of asset cushion.

Figure 1.



Source: authors' calculations

As indicated in the table at the bottom of figure 1, the discount rates considered for determining required contributions range from 5.61 percent to 7.00 percent. In all cases, the mean rate of return is 7.00 percent, with the lower hurdle rates providing a cushion of assets compared to the mean rate.

With a 7.00 percent discount rate, there is no asset cushion. The probability of asset exhaustion rises from 50 percent with the assumption of a low level of risk to over 60 percent with a standard deviation of 0.20. As in the two-period model, an asset cushion, which is provided by assuming a hurdle interest rate lower than the discount rate, is clearly needed if the risk of needing to make future contributions is set by regulation to not be allowed to exceed 50 percent. If a hurdle rate of return of 5.81 percent is assumed (compared to the mean of 7 percent), the probability of failure stays under 40 percent for most of the range of standard deviations considered, which gives a probability of success of 60 percent.

Thus, the model of 20 periods shows that if a parameter of success of greater than 50 percent is required, with risky assets, a hurdle rate of return lower than the expected rate of return is needed for determining required contributions. This result retains the same qualitative result of the two-period model, which is that a hurdle rate greater than the risk-free rate but lower than the expected rate of return on the portfolio is needed when the stochastic parameter of success is greater than 50 percent.

4.3 Two-Period Model, Risky Liabilities, Risk-Free Assets. We now switch the focus and investigate the situation of risky liabilities and risk-free assets. Thus, we are now focusing on how risky liabilities affect the determination of the discount rate for defined benefit pension funding. This approach is similar to the economists' approach in that it treats the risk of assets as irrelevant.

In the short run, mortality is unpredictable for individuals, but mortality rates are highly predictable for large groups. In the long run, even the rates are uncertain and everything about the uncertainty is uncertain – its growth rate, its variance, its distribution, and its impact on liabilities. To gain quantitative traction, we assume that possible magnitudes of future liabilities are normally distributed with known mean and variance. Although a major factor affecting uncertainty in liabilities is uncertainty in mortality, which spreads liabilities over many years, we separate the issues of uncertain duration of liabilities from uncertain magnitude of liabilities by presenting a simple two-period model – the present funding period and the future payment period D years from present. Early retirement also affects both the duration and magnitude of liabilities, but is not considered here. Subscripts p and f distinguish present from future, and we denote the mean and standard deviation[1] of future liabilities by μ_f and σ_f. We assume that rates of return on assets are constant at a known rate r.

The parameters L_f and A_f, represent liabilities and assets D years into the future. The stochastic funding requirement is that future liabilities L_f be less than future assets A_f, with probability c:

$$P(L_f \leq A_f) = c, \text{ where } 0 < c < 1 \qquad (4.2)$$

Rescaling liabilities to a standard normal distribution (the normal distribution with mean zero and standard deviation of 1), that requirement can be rewritten as:

$$P\left(\frac{L_f - \mu_f}{\sigma_f} \leq \frac{A_f - \mu_f}{\sigma_f}\right) = c \qquad (4.3)$$

[1] Although this assumption has implications regarding the distribution of uncertainty in future mortality rates, these need not be explored here. A right-skewed distribution of future mortality may more accurately reflect expert demographic opinion, but we choose to assume a normal distribution because in this paper conceptual clarity is more important than quantitative accuracy and a normal distribution permits us to more easily build upon the familiar reasoning of confidence intervals. The fact that our normal distribution is an assumption rather than a consequence of the central limit theorem does not compromise our ability to draw upon the logic of confidence intervals when deriving required funding cushions.

Because $(L_f - \mu_f) / \sigma_f$ is standard normal, we can also write:

$$P\left(\frac{L_f - \mu_f}{\sigma_f} \leq \text{probit(c)}\right) = \text{c} \qquad (4.4)$$

where $\text{probit}(c)$ is the inverse of the cumulative distribution of the standard normal function. It returns the positive or negative number below which a standard normal variable falls with probability c.[2]

The right hand sides of the two previous inequalities are therefore equal:

$$\frac{A_f - \mu_f}{\sigma_f} = \text{probit(c)} \qquad (4.5)$$

So

$$A_f = \mu_f + \text{probit(c)}\, \sigma_f \equiv \text{CFL} \qquad (4.6)$$

Asset level A_f will be at least equal to liabilities with probability c. We call that asset level the confident funding level CFL. Note that if c=.5 (corresponding to the median (and mean) of the standard normal distribution of liabilities), probit(c) = 0. This result shows that if a 50 percent probability of success is sufficient, having future assets equal to the mean of the liability distribution, with no need for an asset cushion, is sufficient. If a higher probability of success is required, then a higher level of assets must be targeted. Thus, again it is seen that a required standard of success greater than 50 percent implies using a lower discount rate (hurdle rate) than the expected rate of return. An alternative approach used in practice is to build in margins in mortality and other assumptions to provide a cushion.

In the previous equation, the target level of assets A_f is expressed in absolute terms. It is useful to translate this equation into relative terms by subtracting μ_f from both sides of the CFL definitional equation and then dividing by μ_f, which yields

$$\frac{CFL_f - \mu_f}{\mu_f} = \text{probit(c)} \left(\frac{\sigma_f}{\mu_f}\right) \qquad (4.7)$$

The left-hand side is the fraction by which CFL exceeds the expected value of future liabilities, and we call it the funding cushion FC_f. It is the funding cushion in the second period. A consequence of this definition, which we use below, is that $1+FC_f = CFL_f / \mu_f$. Note that σ_f / μ_f

[2] Statistics students may recall using probit(.975)=1.96 when constructing two-tailed 95% confidence intervals. In words, a standard normal variable falls below 1.96 97.5 percent of the time.

is the coefficient of variation $\mathbf{CV_f}$ for liability uncertainty. Using this terminology, confident funding is achieved when the future funding cushion satisfies:

$$FC_f = \text{probit}(c)\ CV_f \qquad (4.8)$$

Thus, the required funding cushion is greater, and the hurdle rate for determining funding lower, the greater the required probability of success c and the greater is the variance of liabilities relative to its mean. And again, no funding cushion is required if the required probability of success is 50 percent or lower.

Figure 2 explores this relationship.

Figure 2.

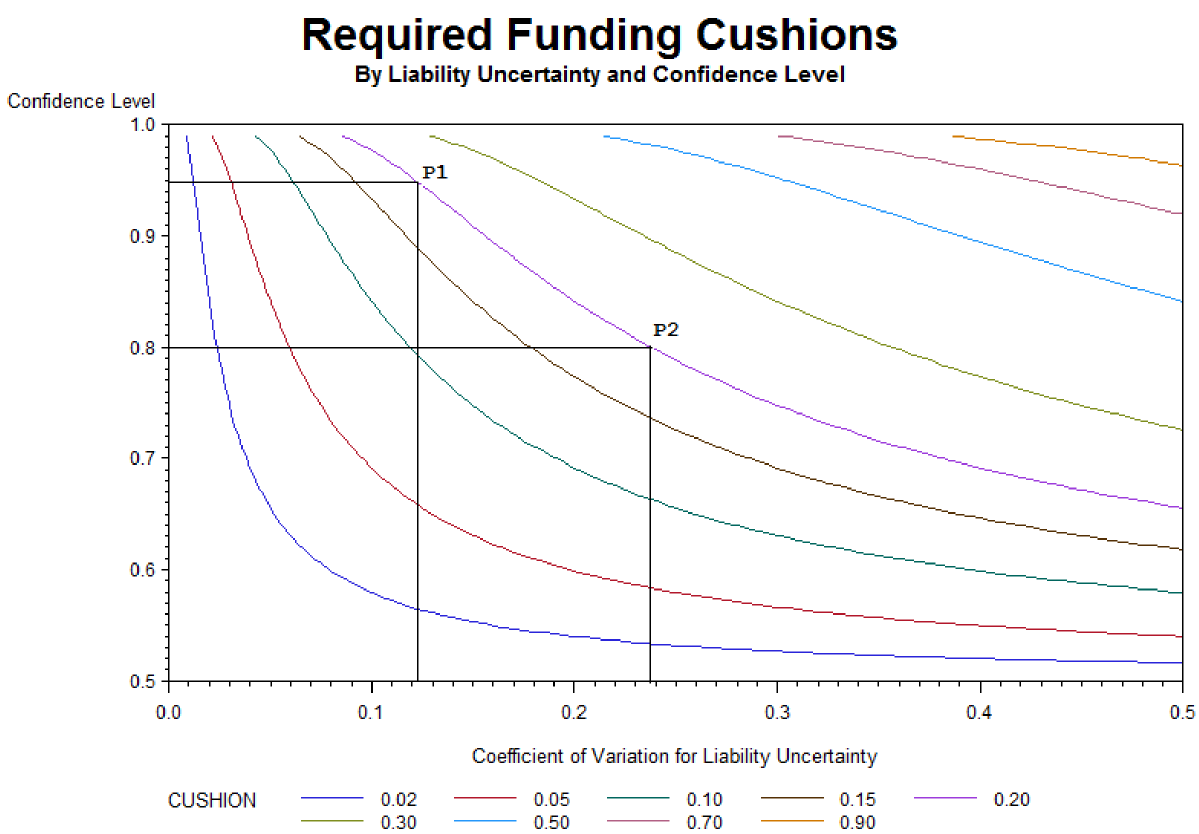


Figure 2 makes the point for risky liabilities that has been made for risky assets. The required funding cushion increases with both the level of confidence required and the level of risk, here measured by the coefficient of variation for liability uncertainty. Points P1 and P2 illustrate the tradeoff between the liability uncertainty (measured by the coefficient of variation) and the level of confidence required that future contributions will not be needed. Points P1 and P2 show in this example that a 20 percent funding cushion can provide 95 percent confidence that liabilities will not exceed assets if the CV of liability does not exceed 12 percent, and 80 percent confidence if the CV of liabilities does not exceed 24 percent. For a confidence level of 0.6 (60 percent) and a coefficient of variation of about 0.24, a funding cushion of 5 percent relative to full funding valued at the mean rate of return is needed.

Because our focus is on the required downward adjustment to the mean rate of return on the asset portfolio to assure adequate funding in the face of risk, we now explore the relationship between the magnitude of the funding cushion and magnitude of the downward adjustment needed to determine the hurdle rate.

Let D represent the duration[3] of the liabilities, while μ_P and CFL_P represent the present values of μ_f and CFL; that is

$$\mu_f = \mu_P \times (1 + r)^D \qquad (4.9)$$

and

$$CFL_f = CFL_P \times (1 + r)^D \qquad (4/10)$$

where r is the mean of the distribution of rates of return. Assume also that we wish to adjust the discount rate (hurdle rate) r downward to a level r’ that will yield the higher present value CFL:

$$\mu_f = CFL_p \times (1 + r')^D \qquad (4.11)$$

Equating these two expressions for μ_f:

$$\mu_P \times (1 + r)^D = CFL_p \times (1 + r')^D \qquad (4.12)$$

[3] Readers may be more familiar with the concept of duration in the context of bonds where duration is the approximate change in price resulting from a 100 basis point change in interest rate. Like a bond, an annuity is a series of cash flows having a present value that is sensitive to the discount rate in the same way that a bond price is sensitive to interest rates, and, just as for bonds, that sensitivity can be gauged using the concept of duration. Of the three common forms of duration, “modified duration” is the precise concept we use here because this concept is applicable to bonds having expected cash flows that do not change when the yield changes and payments from the annuity we are discussing do not vary with the discount rate.

So

$$\left(\frac{1+r}{1+r\prime}\right)^{D} = \left(\frac{CFL_p}{\mu_\mathrm{p}}\right) \qquad (4.13)$$

Solving for r′ yields:

$$r' = \frac{1+r}{\left(\frac{CFL_p}{\mu_\mathrm{p}}\right)^{(1/D)}} - 1 \qquad (4.14)$$

As noted, $1+FC_f = CFL_f / \mu_f$, and similarly $1+FC_p = CFL_p / \mu_p$, so we can rewrite this equation as:

$$r' = \frac{1+r}{\left(1+\mathrm{FC_p}\right)^{(1/D)}} - 1 \qquad (4.15)$$

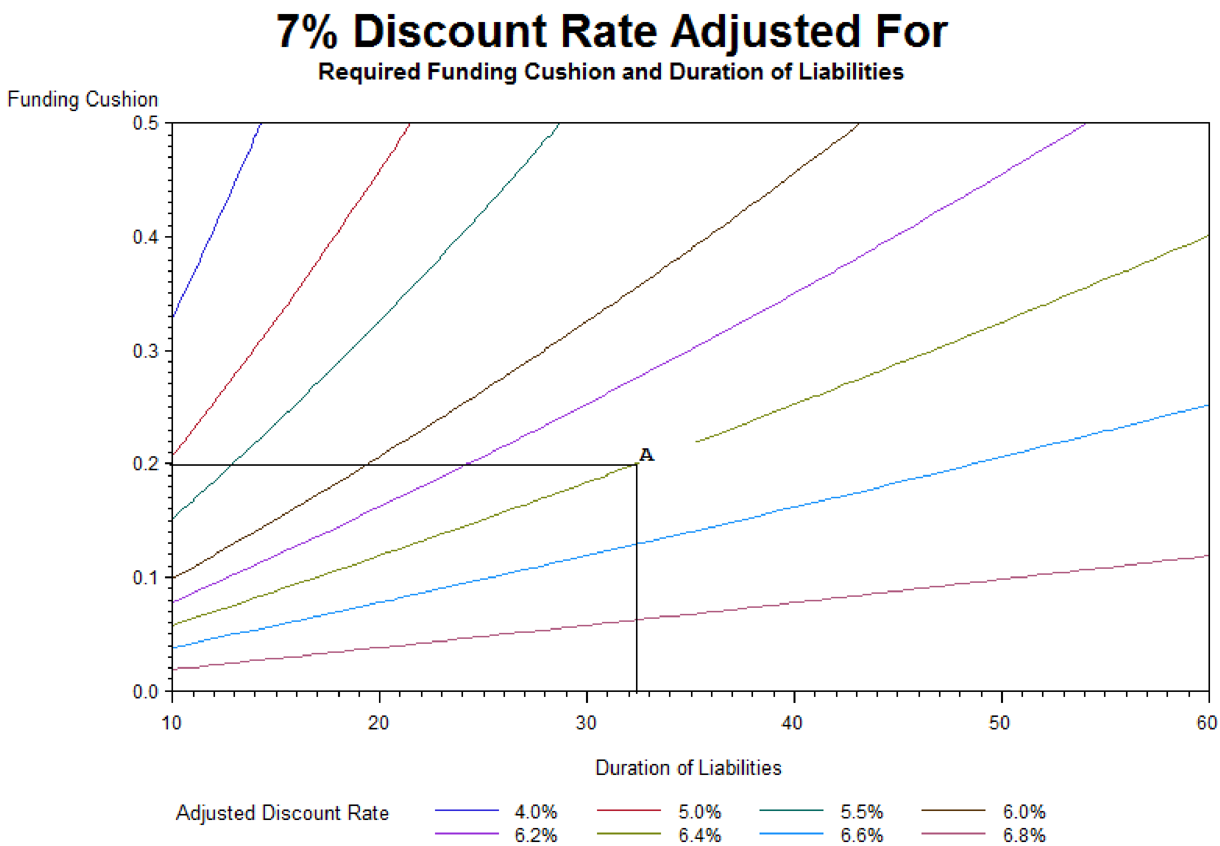
Thus, the greater the value of the funding cushion, the greater the reduction in the discount rate.

We can use equation (4.8) to express this adjustment as a function of the coefficient of variation for future labilities CV_f and the required probability c of funding sufficiency.

$$r' = \frac{1+r}{(1+\mathrm{probit(c)\ CV_f})^{(1/D)}} - 1 \qquad (4.16)$$

Figure 3 provides a contour plot for equation (4.15).

Figure 3.



In Figure 3, the adjusted discount rates at the bottom of the figure from left to right correspond to the lines going from top to bottom in the figure. In Figure 3, point A, for example, shows that a new plan where the duration of liabilities is 32.4 years can build in a funding cushion of 20 percent by lowering the discount rate from 7 percent to 6.4 percent, while a plan with a duration of liabilities of 20 years would have a funding cushion of 10 percent. A target funding cushion of 5 percent requires that discount rate be adjusted below the expected rate of return to 6.6 percent.

The chart shows the combinations of liability uncertainty and confidence level that this cushion could provide. As expected, controlling for the duration of liabilities, a higher discount rate provides a lower funding cushion. Also as expected, the longer the duration of liabilities, the higher the discount rate that can achieve a given funding cushion.

4.4 Risky Assets, Risky Liabilities. The previous analyses either ignore the risk to assets by assuming they are not risky, or ignore the risk to liabilities by assuming the liabilities are not risky. Thus, those approaches are consistent with the assumptions of the economists' approach and the actuaries' approach. However, taking into account the risk of needing to make future contributions, the implied discount rates (hurdle rates) are different from those determined by the two approaches. We now investigate the more complex and more realistic scenario where both assets and liabilities are risky.

Following the approach developed when the assets are risky and the liabilities risk-free, we consider a multi-period model where benefits are paid over a 20-year period. Under this scenario, we assume uncertainty in the liability, so the benefits paid over the 20-year period are unknown. Thus, the length of the payout period is known, but the amount paid each period is unknown. We do not specify the cause of the liability risk. We also assume riskiness of asset rates of return.

The variables A_f and L_f, represent assets and liabilities D years into the future. We assume a bivariate normal distribution with mean μ_{Af} and μ_{Lf} and variance σ_{Af} and σ_{Lf} respectively.

such that

$$(A, L) \sim N(mu, \Sigma) \qquad (4.17)$$

where $mu = (\mu_A, \mu_L)$ is the vector with the mean of the assets and liabilities. $\Sigma = \begin{pmatrix} \sigma_A^2 & \rho\sigma_A\sigma_L \\ \rho\sigma_A\sigma_L & \sigma_L^2 \end{pmatrix}$ is the variance-covariance matrix with $\rho = corr(A, L)$ The correlation $corr(A, L)$ between assets and liabilities depends in part on the investment portfolio. If assets exist that perfectly hedge the liabilities, the correlation would be 1. For the standard deviations, if the assets are primarily stocks, we would assume that the assets would be considerably more risky than the liabilities. On the other hand, if there were a perfect hedge, we would assume that the standard deviations would be the same.

Again, the stochastic funding requirement is that future assets A_f will exceed the future liabilities L_f, with probability c. $P(A_f \geq L_f)$ could be rewritten as

$$P(A_f - L_f \geq 0) = c \qquad (4.18)$$

The correlation between assets and liabilities depends in part on the assets in the portfolio, so, if we set

$$\rho = \frac{\sigma_{AfLf}}{\sigma_{Af}\sigma_{Lf}} \text{ where } \rho = corr(A_f, L_f) \text{ and } \sigma_{AfLf} = cov(A_f, L_f) \qquad (4.19)$$

it follows that

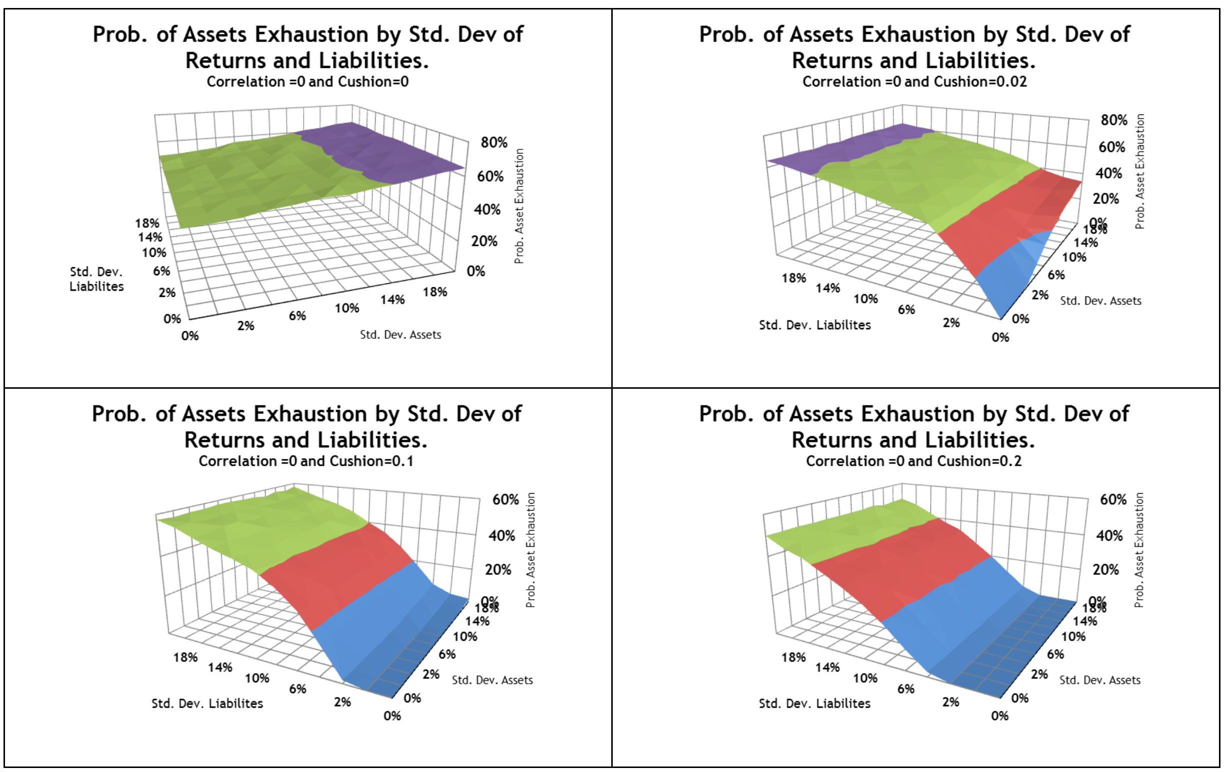
$$A_f - L_f \sim N\left(\mu_{Af} - \mu_{Lf}, \sigma_{Af}^2 + \sigma_{Lf}^2 - 2\rho\,\sigma_{Af}\sigma_{Lf}\right) \qquad (4.20)$$

If $\mu_{Af} = \mu_{Lf}$, the difference of the means is zero. Therefore, 50 percent of the time assets will equal or exceed liabilities or, equivalently, if a 50 percent probability of success is sufficient, there is no need for an asset cushion. That result is due to the bivariate normal distribution being symmetric around its mean.

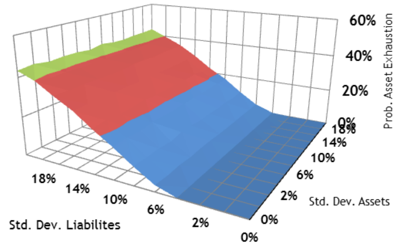
As in Figure 1, we calculate the probability of exhaustion of assets under uncertain liabilities, but now with assets also being subject to risk. For comparison purposes, we use the same assumptions as before, assets are initially set equal to the present value of an annuity paying $1 per year for 20 years, discounted at 7 percent, which is the mean of the assumed distribution of rates of return for this example. However, with risky liabilities, the payment each year is simulated under different levels of standard deviation. We assume a mean of $1 each period for the liabilities and different levels of standard deviation.

We first set the levels of funding cushions, then determine the probability that future contributions will not be needed and the hurdle rate associated with that cushion.

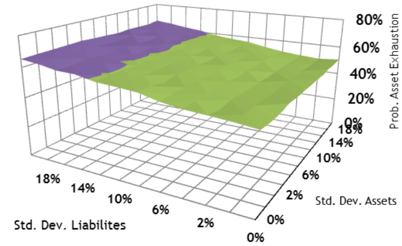
Figure 4 shows the results under three different values for the correlation between assets and liabilities (0, 0.5 and 1), five levels of funding cushion (0, 0.01, 0.1, 0.2, 0.3), and varying levels of risk as to assets and liabilities. When there is a zero cushion, the probability of asset exhaustion is about 50 percent in all cells (except the no risk cell), which is what would be expected. When there is a cushion, the probability of asset exhaustion falls, as expected. It is roughly symmetric comparing risks in assets and liabilities. It increases at increasing standard deviations of assets and liabilities, as expected. With a relatively small cushion, a small reduction in the hurdle rate is possible.



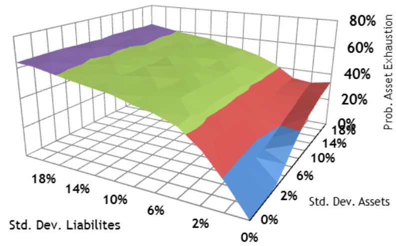
Prob. of Assets Exhaustion by Std. Dev of Returns and Liabilities.
Correlation =0 and Cushion=0.3



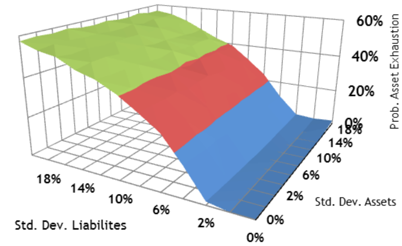
Prob. of Assets Exhaustion by Std. Dev of Returns and Liabilities.
Correlation =0.5 and Cushion=0



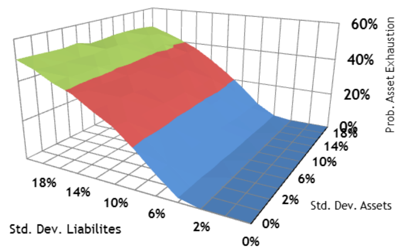
Prob. of Assets Exhaustion by Std. Dev of Returns and Liabilities.
Correlation =0.5 and Cushion=0.01



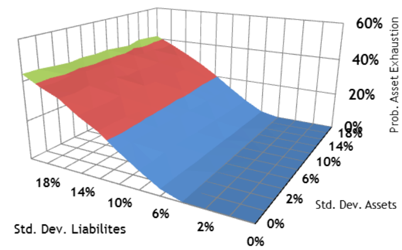
Prob. of Assets Exhaustion by Std. Dev of Returns and Liabilities.
Correlation =0.5 and Cushion=0.1



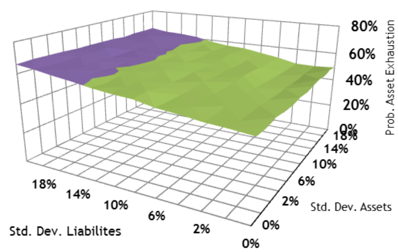
Prob. of Assets Exhaustion by Std. Dev of Returns and Liabilities.
Correlation =0.5 and Cushion=0.2



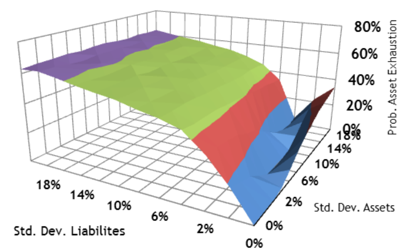
Prob. of Assets Exhaustion by Std. Dev of Returns and Liabilities.
Correlation =0.5 and Cushion=0.3



Prob. of Assets Exhaustion by Std. Dev of Returns and Liabilities.
Correlation =1 and Cushion=0



Prob. of Assets Exhaustion by Std. Dev of Returns and Liabilities.
Correlation =1 and Cushion=0.01



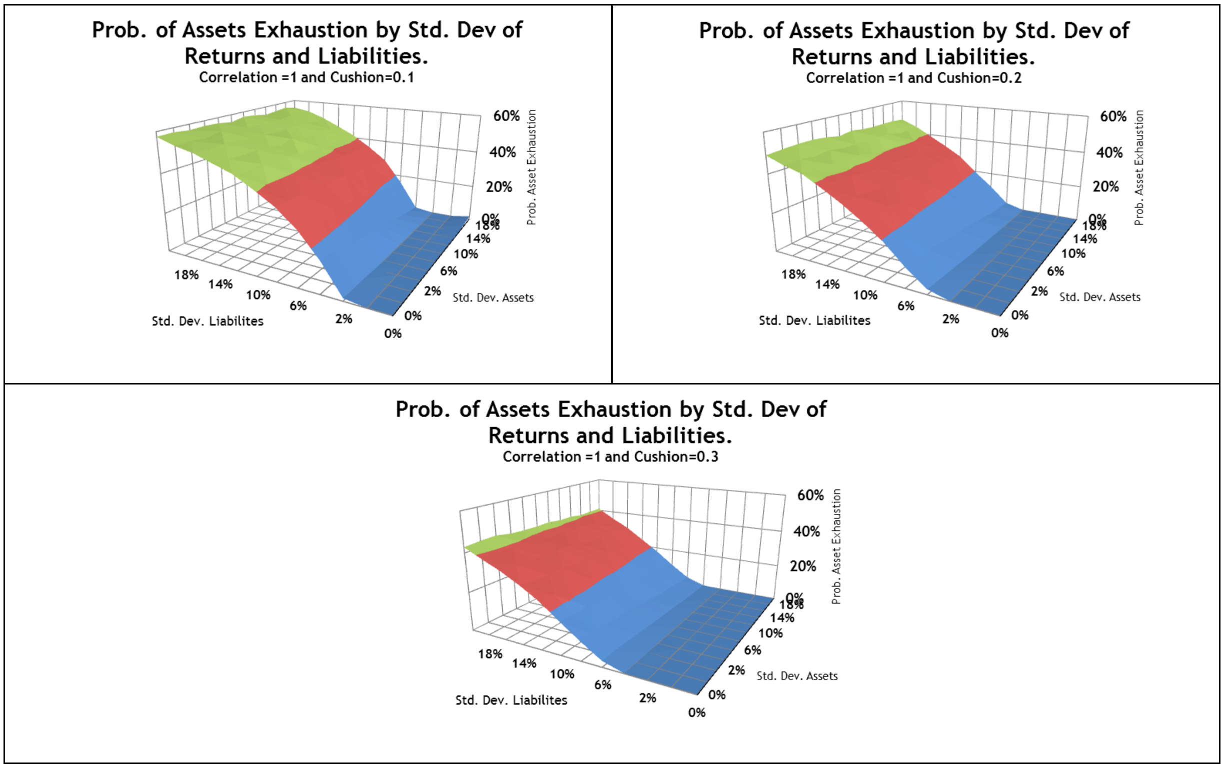


Figure 1 earlier in the paper has 0 standard deviation of liabilities, which yields equivalent results to the bivariate simulations when it is assumed that liabilities are constant. Comparing the top row, where there is no risk to liabilities but risk to assets versus lower rows where both liabilities and assets are risky has very little effect on the probability of exhaustion or the hurdle rate. That result is explained by the assumption of a binomial normal distribution.

Table 2. Probability of asset exhaustion by standard deviation of returns and liabilities, correlation equals 0, cushion equals 10 percent. (note: Std. assets are indicated by the row across the top of the chart)

[[3]] **Prob. of Assets Exhaustion by Std. Dev of Returns and Liabilities. Corr**

	Std. Liabilities	0%	1%	2%	4%	6%	8%	10%
Std. Assets	0%	0.00%	0.00%	1.37%	15.53%	26.47%	33.63%	40.70%
	1%	0.00%	0.00%	1.71%	15.20%	26.03%	35.11%	41.41%
	2%	0.00%	0.01%	1.66%	14.97%	27.18%	34.90%	40.57%
	4%	0.00%	0.00%	1.64%	15.17%	26.34%	35.52%	40.61%
	6%	0.00%	0.01%	2.31%	14.96%	27.14%	35.28%	39.58%
	8%	0.00%	0.03%	2.42%	15.83%	26.90%	34.80%	40.64%
	10%	0.00%	0.13%	2.98%	16.76%	27.90%	34.59%	40.47%
	12%	0.01%	0.29%	3.54%	16.30%	27.85%	35.27%	39.85%
	14%	0.14%	0.82%	4.15%	17.31%	27.32%	34.21%	39.80%

16%	0.49%	1.25%	5.11%	17.11%	27.23%	35.37%	41.51%
18%	1.08%	2.11%	5.94%	18.01%	27.73%	34.42%	40.44%
20%	1.70%	3.03%	6.86%	18.30%	28.49%	35.14%	40.49%

Authors' calculations

Examining the top row, which is for 0 standard deviation of liabilities and varying standard deviations of assets, it is seen that the results are similar to those obtained in Figure 1, using a different simulation program. For example, with a 10 percent standard deviation of assets, the probability of failure is 40.7 percent, which is close to the roughly 35 percent value from Figure 1.

In Table 2, looking across the rows, an increase in the standard deviation of assets raises the probability of asset exhaustion, while looking down the columns an increase in the standard deviation of liabilities has little effect on the probability of asset exhaustion. The explanation for this lack of symmetry is because the two measures of risk are not directly comparable. In our simulations, the standard deviation of liabilities is for a mean of $1 in every period. However, for the assets, the standard deviation in assets is for a mean that starts out at PV_L(1+cushion) in the first period and changes each period with investment earnings and benefit payments, eventually declining the neighborhood of zero. Thus, the standard deviation of assets and of liabilities are not comparable in magnitude.

Table 3. Hurdle rates corresponding to Table 2

	[[3]]	**IRR (Hurdle Rate) of Assets Exhaustion by Std. Dev of Returns and Lia**					
	Std. Liabilities	**0%**	**1%**	**2%**	**4%**	**6%**	**8%**	**10%**
Std. Assets	**0%**	5.805%	5.805%	5.805%	5.805%	5.805%	5.805%	5.805
	1%	5.805%	5.806%	5.805%	5.806%	5.805%	5.805%	5.805
	2%	5.804%	5.804%	5.806%	5.805%	5.805%	5.806%	5.806
	4%	5.805%	5.804%	5.804%	5.805%	5.807%	5.806%	5.804
	6%	5.802%	5.808%	5.807%	5.804%	5.802%	5.806%	5.806
	8%	5.805%	5.807%	5.803%	5.799%	5.800%	5.804%	5.809
	10%	5.809%	5.802%	5.802%	5.806%	5.807%	5.804%	5.811
	12%	5.801%	5.804%	5.810%	5.807%	5.804%	5.804%	5.804
	14%	5.804%	5.807%	5.805%	5.807%	5.805%	5.805%	5.801
	16%	5.808%	5.801%	5.809%	5.799%	5.799%	5.808%	5.809
	18%	5.804%	5.809%	5.815%	5.806%	5.803%	5.802%	5.797
	20%	5.798%	5.805%	5.804%	5.789%	5.815%	5.799%	5.803

Authors' calculations

Table 3 demonstrates the point that has been made in simpler models earlier in the paper, that the hurdle rate needs to be lower than the expected rate of return on the portfolio, but is higher than the rate of return on bonds.

5. Conclusions

Around the world, defined benefit plans have liabilities of more than $23 trillion, but actuaries, financial analysts, and economists differ as to how these liabilities should be measured. The analysis in this paper shows that for risky assets and risky liabilities, a discount rate lower than the expected or mean rate of return on the defined benefit plan portfolio is needed to provide a target funding cushion whenever the target probability of success exceeds 50 percent. In addition, the higher the duration of liabilities, the less the discount rate needs to be reduced below the expected rate of return to provide a target funding cushion. Thus, these analyses show that for the question we ask, the "economists' approach" of using the risk-free interest rate for discounting risk-free liabilities is not generally correct. Generally, a higher discount rate is appropriate.

Also, these analyses show that for the question we ask, the "actuaries' approach" of using the expected rate of return for discounting liabilities is not generally correct, and that generally a lower rate is appropriate.

With the stochastic funding parameter approach, the risk to both the assets and liabilities affect the hurdle rate (discount rate) used for determining required funding. Generally, that rate is less than the expected rate of return on the portfolio, but higher than a rate or rates derived from current bond yields.

The policy implications of this analysis are clear. Current approaches used in the United States and elsewhere that determine interest rates for funding defined benefit plan liabilities solely by valuing the liabilities or solely based on the expected rate of return on the assets are both flawed. With the stochastic funding parameter approach, while the exact discount rate (hurdle rate) used to determine adequate funding depends on the risk to the assets, the risk to the liabilities, and the duration of the liabilities, a simple rule of thumb can be stated. That rule would be to select a discount rate that is less than the expected rate of return on assets but greater than the risk free rate, with the discount being greater the higher the percentage of the portfolio invested in equity and the longer the duration of the liabilities.

Further work needs to be done with modeling more realistic scenarios on both the assets side and the liabilities side. It could also explore measures of success that incorporate the magnitude of a shortfall as well as its probability.

Acknowledgments. We have received funding for this project from the Society of Actuaries. We have received valuable comments from Lans Bovengerg, and members of the Project Oversight Group of the Society of Actuaries, including Edward Bartholomew, David Cantor, Victor Modugno, and Lawrence Pollack.

References

Actuarial Standards Board (2013): 'Selection of Economic Assumptions for Measuring Pension Obligations." American Academy of Actuaries. September. http://www.actuarialstandardsboard.org/pdf/asops/asop027_172.pdf

Apps, P.; Andrienko, Y. and Rees, R. (2014): "Risk and Precautionary Saving in Two-Person Households." *American Economic Review* **104(3)**: 1040-1046.

Bovenberg, L.; Mehlkopf, R. and Bilsen, S. v. (2014): "Variable Annuities in Pension Schemes with Risk Sharing: Valuation, Investment and Communication." Unpublished paper.

Day, T. (2004): "Financial Economics and Actuarial Practice." *North American Actuarial Journal* **8(3)**. http://dev.soa.org/Library/Journals/NAAJ/2004/july/naaj0403-6.aspx

The Economist (2014): "My Money or Your Life." August 23. http://www.economist.com/news/finance-and-economics/21613335-new-financial-instruments-may-help-make-pension-schemes-safer-my-money-or-your

General Accountability Office (2014): "Pension Plan Valuation: Views on Using Multiple Measures to Offer a More Complete Picture." September, GAO-14-264. http://www.gao.gov/assets/670/666287.pdf

Haberman, S.; Day, C.; Fogarty, D.; Khorasanee, M.Z.; McWhirter, M.; Nash, N.: Ngwira, B.; Wright, I.D.; and Yakoubov, Y. (2003): "A Stochastic Approach to Risk Management and Decision Making in Defined Benefit Schemes." *British Actuarial Journal* **9(3)**: 493-618.

Jong, F. de (2008) "Valuation of Pension Liabilities in Incomplete Markets." *Journal of Pension Economics and Finance* **7(3)**: 277-294. http://arno.uvt.nl/show.cgi?fid=93100

Minahan, J.R. (2010): "Valuing and Funding Public Pension Liabilities." February 26.

Nijman, T.E.; Stalborch, S. M. van; Toor, Johannes A.C. van and Werker, B. J.M. (2013): "Formalizing the New Dutch Pension Contract." Netspar Occasional Paper. http://arno.uvt.nl/show.cgi?fid=128354

Novy-Marx, R. and Rauh, J.D. (2009): "The Liabilities and Risks of State-Sponsored Pension Plans." *Journal of Economic* Perspectives **23(4)**: 191–210.

_____ (2013): "The Revenue Demands of Public Employee Pension Promises." *American Economic Journal: Economic Policy* **6(1)**: 193-229. http://rnm.simon.rochester.edu/research/RDPEPP.pdf

Thrift Savings Plan (2013): "L Funds." https://www.tsp.gov/PDF/formspubs/LFunds.pdf

Pfau, W. (2014): “New Research on How to Choose Portfolio Return Assumptions.” *Advisor Perspectives*, July 1.

[i] Just as workers do not seem to value annuities, they also may place a relatively low value of the annuity benefits provided by defined benefit plans, which may explain in part the decline of those plans.