PENSION REFORM IN BELGIUM: A NEW POINTS SYSTEM BETWEEN DB and DC

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Abstract

More than in other countries, the Belgian first pillar of public pension needs urgent and structural reforms in order to restore financial sustainability and intergenerational equity. In the last decades only small parametric changes have been made and time has come to think about the future. Last year, a commission of academic experts has been appointed by the Belgian government in order to propose a new pension architecture.

This commission has proposed to implement a pay as you go system based on a "points" mechanism with a risk sharing logic between active people and retirees. This system can be considered as an intermediate solution between DB (Defined Benefit) and DC (Defined Contribution). The purpose of this paper is to present the underlying principles of this system, to discuss its advantages and disadvantages and to address various actuarial challenges generated by the new proposed formula.

Keywords

Pension reform, Social security, Pay as you go, Musgrave rule

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1. Introduction

As in many countries, the Belgian first pillar of pension, based on a pay as you go mechanism and a Defined Benefit architecture, is under pressure and needs fundamental reforms to guarantee simultaneously long term sustainability and social fairness. Even if some parametric changes have been decided these last years, no fundamental decisions have been taken to restore the long-term viability and the global coherence of the system (see for instance Devolder, 2010).

The absence of global reforms contrasts a lot with the situation in other countries where fundamental reforms emerged these last years, such as the Notional Defined Contribution (NDC) in Sweden (Holzman et al., 2012; Palmer, 2000; Settergren, 2001), the introduction of a sustainability factor in Germany (Borsch-Supan et al., 2003) of other techniques of automatic adjustment (Vidal-Melia et al, 2006; Knell, 2010). In Belgium, however, there have been no real reforms, despite its negative financial trend. The evolution of the expenditure is negatively affected not only by ageing and "pappy boom" (as in many western countries), but also by the bad level of activity (see for instance The Ageing report, 2015). Nevertheless, after years of silence, an increasing political awareness seems finally to appear.

In this context, in 2013, the Belgian government decided to ask a commission of academic experts to propose a new structure for the Belgian public pensions. The results of this commission have been published in a report (Commission 2020-2040, 2014). The aim of this paper is to present some of the ideas proposed by this commission. In particular, we would like to address the problem of the risk sharing between contributors and retirees. In a classical pension architecture, based on a DB philosophy (resp. a DC philosophy) all the risks are borne by the contributors (resp. the retirees). The idea presented by the commission is to create an automatic adjustment of the replacement rate and the contribution rate based on a sharing of the risks between contributors and retirees. One possible solution in this context is to use the so called "Musgrave rule" based on a modified replacement rate. This rule is note related to the gross salary, but to the salary, net of pension contributions. However, other sharing rules between the two generations are possible. One of the aims of this paper is to present various sharing rules and to measure the level of solidarity generated by these adjustment techniques. We use for that a simple deterministic model, where the main driver is the dependence ratio (ratio between the number of retirees and the number of workers). In particular, we introduce two different ways to model the impact of a change of the dependence ratio on the contribution and the replacement rates. The first one is based on a ratio between the change in the contribution rate and the change in the replacement rate. The second one

introduces a convex invariant (convex combination between the contribution rate and the replacement rate). In both cases, DB and DC schemes appear to be extreme solutions for the parameters and the "Musgrave rule" is one of the possible intermediate systems. Other examples of mix between DB and DC are proposed.

The paper is organized as follows. Section 2 briefly summarizes the pension formula for the existing Belgian first pillar for employees. Section 3 explains the new points system proposed by the commission of experts. In section 4 we illustrate the link between this system and a NDC scheme.

In section 5, we develop the "Musgrave rule" generating an intermediate system between DB and DC (system called DM – Defined Musgrave). This approach is generalized in sections 6 and 7. In section 6, we define a risk sharing coefficient, comparing in case of change of the dependence ratio, the impact in terms of contribution rate and replacement rate. In this context we introduce a Defined Equal sharing plan where the change of the dependence ratio has a same proportional effect on the contribution and the replacement rates. We compute also this risk sharing coefficient for the DM and show that this coefficient is not constant. We prove also that in general this system is closer to DB than DC. In section 7, we propose another approach of mixing by considering a convex combination between the contribution rate and the replacement rate. The "Musgrave rule" is presented as a natural example. Section 8 concludes the paper.

2. The existing Defined Benefit system in the Belgian first pillar

The existing first pillar of pension in Belgium is a classical combination of Pay-as-you-Go (PAYG) and Defined Benefit (DB). For instance, for employees, the pension formula is based on the average of indexed salaries; more precisely, the normal retirement age is 65 and the amount of pension is given by:

$$
\begin{aligned}
&P = 0,60.\left(\frac{1}{45}\sum_{t=T-45}^{T} S_t.\ h(t,T)\right) \\
&\text{where:} \quad P = \text{pension} \\
&\qquad\qquad T = \text{retirement year} \\
&\qquad\qquad S_t = \text{salary of year } t \text{ (with application of a wage ceiling)} \\
&\qquad\qquad h(t,T) = \text{indexation coefiicient between year } t \text{ and year } T
\end{aligned}
\tag{2.1}
$$

This system presents many drawbacks:

- financial sustainability is under pressure;
- no transparency;
- too many incentives for early retirements;
- no actuarial fairness.

3. A new points system

The pension reform's commission has proposed to move from this DB framework to a new system having three main characteristics:

-a) PAYG financing

b) benefit computation based on a points system

c) risk sharing mechanism between DB and DC.

In such a system, the pension formula is described as follows:

a) during the active career: each year, every contributor receives a number of points given by the ratio between his own salary and a reference salary, fixed each year. The reference salary could be for instance the mean salary of the economy:

$$n_t = \frac{S_t}{S_t^r}$$

$$\text{where: } n_t = \text{number of points for year t} \qquad (3.1)$$

$$S_t = \text{individual salary of year t}$$

$$S_t^r = \text{reference salary of year t}$$

b) the total number of points accumulated at retirement age by a pension participant, for a career of M years, is given by:

$$N_T = \sum_{t=T-M}^{T} n_t \qquad (3.2)$$

For example, when M=45, somebody earning each year exactly the reference salary, will obtain exactly 45 points at retirement. The total number of points is therefore a sort of metric of the length of the career, corrected of course by the level of salary.

c) the pension at retirement age is given by the following formula:

$$\begin{aligned} &P_T = N_T . V_T . \rho_T \\ &\text{where}: \quad N_T = \text{total number of points} \\ &\qquad\qquad V_T = \text{value of the point (in €)} \\ &\qquad\qquad \rho_T = \text{actuarial coefficient} \end{aligned} \tag{3.3}$$

The value of the point V is fixed every year taking into account simultaneously different goals to combine. It should ensure a sustainable individual pension based on an adequate replacement rate, while providing a relative stability to the contribution rate. Different political choices can be made concerning the risk sharing between contributors and retirees. This point will be addressed in detail in section 5.

The actuarial correction must take into account the retirement age and the length of the career (total or partial actuarial neutrality) and is based on life expectancies.

d) after retirement, the pension is indexed, taking into account the evolution of the salaries and the sustainability of the regime:

$$\begin{aligned} &P_{T+1} = P_T . h_T . \eta_T \\ &\text{where}: \quad h_T = \text{indexation coefficient} \\ &\qquad\qquad \eta_T = \text{sustainability factor} \end{aligned} \tag{3.4}$$

As a first simple example of determination of the value of the point, consider a target in terms of replacement rate for a representative agent ("DB" philosophy). Let us assume an affiliate with a salary equal each year to the reference salary ; in order to obtain a full pension we ask this affiliate to work during a reference period denoted by N*. We don't apply in this case any actuarial correction ($\rho = 1$) and the pension (3.3) becomes:

$$P_T = N^* . V_T \tag{3.5}$$

The value of the point can be fixed by reference to a replacement rate :

$$P_T = \delta . S_T^r \tag{3.6}$$

The value of the point is then :

$$V_T = \frac{\delta . S_T^r}{N^*} \tag{3.7}$$

For a replacement rate of 60% and a reference period for the career of 45 years, the value of the point is then equal to 1.33 % of the reference salary.

4. Comparison with a NDC scheme

There are clear analogies between this points system and a Notional Defined Contribution scheme (NDC) .

In a NDC scheme (see for instance Holzman et al.,2012), the pension can be written (assuming a retirement age of 65):

$$P = \frac{1}{a_{65}} . \sum_{t=T-45}^{T} \pi . S_t . \, g(t,T)$$

$$\begin{aligned} \text{where}: \quad & P = \text{pension} \\ & T = \text{retirement year} \\ & S_t = \text{salary of year t (with ceiling)} \\ & \pi = \text{contribution rate} \\ & g(t,T) = \text{revalorization based on notional rates} \\ & a_{65} = \text{annuity price at retirement age} \end{aligned} \tag{4.1}$$

It is easy to see that formula (4.1) can be seen as a particular choice of the general point formula (3.3). Introducing in formula (4.1) the reference salary, we obtain:

$$P = \frac{1}{a_{65}} . \sum_{t=T-45}^{T} \pi . \frac{S_t}{S_t^r} . \, (g(t,T).S_t^r)$$

If we assume, as usual in NDC systems, that the revalorization coefficients are in line with the increase of the mean salary, we can write:

$$g(t,T).S_t^r = S_T^r$$

So finally:

$$P = \frac{1}{a_{65}} . \pi . S_T^r . N_T$$

The value of the point is then:

$$V_T = \frac{\pi . S_T^r}{a_{65}}$$

The replacement rate for a reference salary is no constant anymore but given by:

$$\delta_T = \frac{\pi.N_T}{a_{65}}$$

5. Between DB and DC: the Musgrave rule

The point system presented in section 3 is a very flexible architecture and can be modelled using various calibrations. The key question is how to fix the value of the point and how to adapt automatically the system to exogenous shocks. We have already proposed two ways to define the point (formula (3.7) in a DB philosophy and formula (4.1) in a NDC context).

In a DB framework, there is an absolute guarantee for the retirees (fixed replacement rate) and the contributors must support the risks; in a DC framework, there is an absolute stability for the contributors (fixed contribution rate) and the replacement rate is adjusted.

The points system can extend this duality by allowing many other mechanisms of risk sharing between retirees and active workers.

In order to model the demographic risk, let us assume an initial stable situation (denoted by state 1) composed only of reprentative agents (same salary and same career) receiving a pension based on a replacement rate δ_1 (with $0 < \delta_1 < 1$) and a contribution rate π_1. We denote by D_1 the dependence ratio (ratio between the number of retirees and the number of contributors).

The system is then given by the two equations :

- budget equation : $D_1.P_T = \pi_1.S_T$
- pension equation: $P_T = \delta_1.S_T$

The equilibrium between the parameters is obtained when the following classical condition is fulfilled:

$$\pi_1 = D_1.\delta_1 \tag{5.1}$$

Accordingly to formula (3.7) , the value of the point is given by :

$$V_T = \frac{\delta_1.S_T^r}{N*} \tag{5.2}$$

Suppose now that the system moves to another stage characterized by another dependence ratio denoted by D_2. We will often assume that $D_2 > D_1$ (ageing of the population).

We want to define the new parameters δ_2 and π_2, still linked by :

$$\pi_2 = D_2.\delta_2$$

a) in a DB scheme, the replacement rate has to remain constant and the contribution rate must

increase (risk is only borne by the contributors) :

$$\delta_2 = \delta_1 = \delta$$
$$\pi_2 = \pi_1 . \frac{D_2}{D_1} \tag{5.3}$$

b) in a DC structure (for instance NDC) , the contribution rate has to remain fix and the replacement rate will decrease (risk is only borne by the retirees) :

$$\pi_2 = \pi_1 = \pi$$
$$\delta_2 = \delta_1 . \frac{D_1}{D_2} \tag{5.4}$$

c) Musgrave (1981) has proposed another invariant leading to a form of sharing of the risk between the two generations. Let us define the Musgrave ratio as the ratio between the pension and the salary net of pension contributions :

$$M_1 = \frac{P_T}{S_T(1-\pi_1)} = \frac{\delta_1}{(1-\pi_1)} \tag{5.5}$$

In a DB structure, the Musgrave ratio increases when the dependence ratio increases:

$$M_2 = \frac{\delta_2}{(1-\pi_2)} = \frac{\delta_1}{(1-\pi_1 D_2 / D_1)}$$

In a DC structure , the Musgrave ratio decreases :

$$M_2 = \frac{\delta_2}{(1-\pi_2)} = \frac{\delta_1}{(1-\pi_1)} . \frac{D_1}{D_2}$$

In the Musgrave rule, called here DM (Defined Musgrave) ,we want to stabilize this coefficient :

$$M_1 = M_2 = M \tag{5.6}$$

In this philosophy , the new replacement rate can be easily obtained using (5.1) :

$$M = \frac{\delta_2}{(1-\pi_2)} = \frac{\delta_2}{(1-D_2\delta_2)}$$

so :

$$\delta_2 = \frac{M}{1+M.D_2} \tag{5.7}$$

By using (5.1) again , the contribution rate becomes :

$$\pi_2 = \frac{D_2.M}{1+M.D_2} \tag{5.8}$$

We can also compare old and new values of the two parameters :

a) replacement rate:

$$\delta_2 = \frac{M}{1+M.D_2} = \frac{\delta_1/(1-\pi_1)}{1+\frac{\delta_1}{1-\pi_1}.D_2} \tag{5.9}$$

$$= \delta_1 . \frac{1}{1+\delta_1.(D_2-D_1)}$$

In a DC scheme , the new replacement rate was given by :

$$\delta_2 = \delta_1 . \frac{D_1}{D_2} \tag{5.10}$$

In a DB scheme , the replacement rate remains constant :

$$\delta_2 = \delta_1 \tag{5.11}$$

We can obtain the following rule, showing that Defined Musgrave can be seen as an intermediary between Defined Benefit (no influence of the dependence ratio on the replacement rate) and Defined Contribution (full influence of the dependence ratio) :

<u>Property 5.1. :</u>

If the dependence ratio increases ($D_2 > D_1$) and if the initial contribution rate $0 < \pi_1 < 1$, then we have the following inequality:

$$\delta_1 > \delta_1 . \frac{1}{1+\delta_1(D_2-D_1)} > \delta_1 . \frac{D_1}{D_2} \tag{5.12}$$

Or: $\delta_2^{DB} > \delta_2^{DM} > \delta_2^{DC}$

<u>Proof:</u>

(i) $\delta_1 > \delta_1 . \frac{1}{1+\delta_1(D_2-D_1)}$ is a direct consequence of:

$D_2 > D_1$

(ii) $\delta_1 . \frac{1}{1+\delta_1(D_2-D_1)} > \delta_1 . \frac{D_1}{D_2}$

or: $D_2 > D_1.(1+\delta_1(D_2-D_1))$

Or: $D_2 - D_1 > D_1.\delta_1(D_2-D_1) = \pi_1.(D_2-D_1)$

Or: $\pi_1 < 1$

b) contribution rate:

$$\pi_2 = \frac{D_2.M}{1+M.D_2} = \frac{D_2.\frac{\pi_1 / D_1}{1-\pi_1}}{1+D_2.\frac{\pi_1 / D_1}{1-\pi_1}}$$

$$= \pi_1.\frac{D_2}{D_1+\pi_1.(D_2-D_1)}$$

In a DB scheme the new contribution rate was given by:

$$\pi_2 = \pi_1.\frac{D_2}{D_1}$$

In a DC scheme the contribution rate remains constant:

$$\pi_2 = \pi_1$$

Then we have the following property similar to property 5.1. :

<u>Property 5.2. :</u>

If the dependence ratio increases ($D_2 > D_1$) and if the initial contribution rate $0 < \pi_1 < 1$, then we have the following inequality:

$$\pi_1 < \pi_1.\frac{D_2}{D_1+\pi_1(D_2-D_1)} < \pi_1.\frac{D_2}{D_1} \qquad (5.13)$$

Or: $\pi_1^{DC} = \pi_2^{DC} < \pi_2^{DM} < \pi_2^{DB}$

<u>Proof:</u>

(i) $\pi_1 < \pi_1.\frac{D_2}{D_1+\pi_1(D_2-D_1)}$

Or: $D_1+\pi_1(D_2-D_1) < D_2$

Or: $\pi_1 < 1$

(ii) $\pi_1.\frac{D_2}{D_1+\pi_1(D_2-D_1)} < \pi_1.\frac{D_2}{D_1}$ is direct consequence of:

$D_1+\pi_1.(D_2-D_1) > D_1$

Remark that, if the population is getting younger rather than ageing (i.e. $D_2 < D_1$), the order relations given in properties 5.1 and 5.2 are reversed.

Formulas (5.12) and (5.13) show that the Defined Musgrave approach can be seen as an intermediary between DB and DC in terms of contribution rates and replacement rates. In a DM scheme, the two rates move together in opposite directions (increase of the contribution rate and

decrease of the replacement rate).Contributors and retirees are affected by the demographic risk. Table 1 summarizes the three choices (DB, DC, and DM).

TABLE 1: Contribution rate and replacement rate formula in DB, DC and DM

	Contribution rate	**Replacement rate**	**Value of the point**
Defined Benefit	$\pi_2 = \pi_1 . \frac{D_2}{D_1}$	$\delta_2 = \delta_1 = \delta$	$V_2 = V_1 = \frac{\delta . S_T^R}{N*}$
Defined Contribution	$\pi_2 = \pi_1 = \pi$	$\delta_2 = \delta_1 . \frac{D_1}{D_2}$	$V_2 = V_1 . \frac{D_1}{D_2}$
Defined Musgrave	$\pi_2 = \pi_1 . \frac{D_2}{D_1 + \pi_1 . (D_2 - D_1)}$	$\delta_2 = \delta_1 . \frac{1}{1 + \delta_1 . (D_2 - D_1)}$	$V_2 = V_1 . \frac{1}{1 + \delta_1 . (D_2 - D_1)}$

Example 5.1:

Let us assume an initial steady state characterized by the following parameters:

- initial dependence ratio: D = 0.40
- initial replacement rate: δ = 0.50
- initial contribution rate : π = 0.40 x 0.50 = 0.20

Then table 2 gives for various values of the new dependence ratio the new contribution rate and replacement rate in the three structures (DB, DC and DM) :

TABLE 2 : numerical comparison between DB, DC and DM

		D_2=0,25	**D_2=0,35**	**D_2=0,45**	**D_2=0,50**	**D_2=0,60**
DB	δ	0.50	0.50	0.50	0.50	0.50
	π	0.13	0.18	0.23	0.25	0.30
DC	δ	0.80	0.57	0.44	0.40	0.33
	π	0.20	0.20	0.20	0.20	0.20
DM	δ	0.54	0.51	0.49	0.48	0.45
	π	0.14	0.18	0.22	0.24	0.27

6. Risk sharing coefficient

The defined Musgrave mechanism can be considered as an intermediate scheme between DB and DC. We can develop a whole family of intermediate solutions between DB and DC where a change of the dependence ratio is not supported only by the retirees (DC) or only by the contributors (DB) but is shared between the two groups.

In order to introduce this family, we start from a steady state given by the equilibrium equation (5.1).

A change in the dependence ratio D generates new values for the contribution rate and the replacement rate, still solutions of the equation:

$$\pi_2 = D_2 . \delta_2 \tag{6.1}$$

We introduce the following notation:

$$\begin{aligned} \pi_2 &= \pi_1 (1+\lambda_\pi) \\ \delta_2 &= \delta_1 (1-\lambda_\delta) \end{aligned} \tag{6.2}$$

The parameters λ represent the relative changes in the two rates.

Equilibrium relation (6.1) gives the following relation between these parameters:

$$(1+\lambda_\pi) = \frac{D_2}{D_1} . (1-\lambda_\delta) \tag{6.3}$$

We can also introduce a risk sharing coefficient, comparing the efforts supported by the contributors and the retirees:

$$\rho = \frac{\lambda_\pi}{\lambda_\delta} \tag{6.4}$$

If this coefficient is equal to 1, the risk is equally shared by the two generations.

We compute first the values of these new parameters in the three systems introduced in section 5.

a) *<u>In a DB scheme</u>*, we have by definition:

$$\lambda_\delta = 0$$

Equation (6.3) gives the other parameter:

$$1+\lambda_\pi = \frac{D_2}{D_1}$$

or:

$$\lambda_\pi = \frac{D_2 - D_1}{D_1} \tag{6.5}$$

The risk sharing coefficient is then: $\rho = +\infty$ (6.6)

b) *In a DC scheme*, we have by definition:

$$\lambda_\pi = 0$$

Equation (6.3) gives the other parameter:

$$1 = \frac{D_2}{D_1}(1-\lambda_\delta)$$

or:

$$\lambda_\delta = \frac{D_2 - D_1}{D_2} \tag{6.7}$$

The risk sharing coefficient is then: $\rho = 0$ (6.8)

c) *In a DM scheme*, it comes, using table 1:

- for the contribution part:

$$\pi_2 = \pi_1 . \frac{D_2}{D_1 + \pi_1.(D_2 - D_1)} = \pi_1.(1+\lambda_\pi)$$

or:

$$\lambda_\pi = \frac{(D_2 - D_1)(1-\pi_1)}{D_1 + \pi_1.(D_2 - D_1)}$$

- for the replacement rate:

$$\delta_2 = \delta_1 . \frac{1}{1+\delta_1.(D_2 - D_1)} = \delta_1.(1-\lambda_\delta)$$

or:

$$\lambda_\delta = \frac{\delta_1.(D_2 - D_1)}{1+\delta_1.(D_2 - D_1)}$$

The risk sharing coefficient in a DM scheme is :

$$\begin{aligned}\rho = \frac{\lambda_\pi}{\lambda_\delta} &= \frac{(D_2 - D_1).(1-\pi_1)}{D_1 + \pi_1(D_2 - D_1)} . \frac{1+\delta_1.(D_2 - D_1)}{\delta_1.(D_2 - D_1)} \\ &= \frac{1-\pi_1}{\pi_1} = \frac{1}{\pi_1} - 1\end{aligned} \tag{6.9}$$

For natural values of the contribution rate ($\pi < 0.5$) , this sharing coefficient is higher than 1, showing that in a Defined Musgrave , contributors made a bigger effort than the retirees (system closer to DB than DC).

In example 5.1 ($\pi_1 = 0.2$), this coefficient is equal to 4.

Let us remark that in a Defined Musgrave, this coefficient depends on the contribution rate. Therefore, successive applications of this rule will change the value of the risk sharing coefficient (along with the change in the contribution rate). So a defined Musgrave cannot be considered as a system with a constant risk sharing coefficient.

Apart from these three systems, we are now able to develop other sharing rules.

d) For instance, a natural candidate is characterized by a risk sharing coefficient equal to 1.

We could call this rule DE (Defined Equal sharing).

In this case, we have: $\lambda_\pi = \lambda_\delta = \lambda$

Then equation (6.3) becomes:

$$(1+\lambda) = \frac{D_2}{D_1}.(1-\lambda)$$

or:

$$\lambda = \frac{D_2 - D_1}{D_2 + D_1} \tag{6.10}$$

In this case, using formula (6.2) the new contribution and replacement rates are respectively given by:

$$\begin{aligned} \pi_2 &= \pi_1.\frac{2D_2}{D_1 + D_2} \\ \delta_2 &= \delta_1.\frac{2D_1}{D_1 + D_2} \end{aligned} \tag{6.11}$$

e) In general, we can characterize a pension scheme by of its level of solidarity between retirees and contributors, summarized by the coefficient ρ. A value of $\rho > 1$ (resp. $\rho < 1$) generates more effort from the contributors (resp. the retirees), the DB and the DC being the two limit techniques. The parameters are then solution of the two equations:

$$\begin{aligned} (1+\lambda_\pi) &= \frac{D_2}{D_1}(1-\lambda_\delta) \\ \lambda_\pi &= \rho.\lambda_\delta \end{aligned}$$

The solution is given by:

$$\begin{aligned} \lambda_\delta(\rho) &= \frac{D_2 - D_1}{D_2 + \rho.D_1} \\ \lambda_\pi(\rho) &= \rho.\frac{D_2 - D_1}{D_2 + \rho.D_1} \end{aligned} \tag{6.12}$$

The contribution and replacement rates become:

$$\begin{aligned} \pi_2 &= \pi_1.\frac{D_2.(1+\rho)}{D_2 + \rho.D_1} \\ \delta_2 &= \delta_1.\frac{D_1.(1+\rho)}{D_2 + \rho.D_1} \end{aligned} \tag{6.13}$$

The DC, DB and DM frameworks seen before are particular cases of these general formulas for risk sharing coefficient ρ respectively equal to $\rho = 0; \rho = +\infty; \rho = (1-\pi_1)/\pi_1$.

Example 6.1

Using the same assumptions as in example 5.1, table 3 compares the effect of the risk sharing coefficient on the contribution rate and the replacement rate

TABLE 3: numerical comparison of the contribution rate and the replacement rate for various values of the risk sharing coefficient

		D_2=0,25	D_2=0,35	D_2=0,45	D_2=0,50	D_2=0,60
ρ=0 (*DC*)	δ	0.8	0.57	0.44	0.40	0.33
	π	0.20	0.20	0.20	0.20	0.20
ρ=0.5	δ	0.67	0.55	0.46	0.43	0.38
	π	0.17	0.19	0.21	0.21	0.23
ρ=1 *(DE)*	δ	0.62	0.53	0.47	0.44	0.40
	π	0.15	0.19	0.21	0.22	0.24
ρ=4 (*DM*)	δ	0.54	0.51	0.49	0.48	0.45
	π	0.14	0.18	0.22	0.24	0.27
ρ=20	δ	0.51	0.50	0.50	0.49	0.49
	π	0.13	0.18	0.22	0.25	0.29
ρ = +∞ **(DB)**	δ	0.50	0.50	0.50	0.50	0.50
	π	0.13	0.18	0.23	0.25	0.30

7. A convex invariant

The Musgrave rule can also be revisited using a convex combination between the replacement rate and the contribution rate and asking this combination to stay constant in case of a demographic shock.

This interpretation will allow us to consider again a whole family of intermediate schemes between

DB and DC. The Defined Musgrave is based on the following invariant (cf. equation (5.5)):

$$M = \frac{\delta}{1-\pi}$$

This relation can be written :

$$M = \delta + M.\pi$$

or :

$$\frac{1}{1+M}.\delta + \frac{M}{1+M}.\pi = \frac{M}{1+M}$$

or:

$$\alpha.\delta + (1-\alpha).\pi = \alpha \qquad (7.1)$$

This last relation shows that in a DM, a convex combination of the contribution rate and the replacement rate has to remain constant.

The coefficient α of the combination being equal to: $\alpha = \frac{1}{1+M} = \frac{1-\pi}{1-\pi+\delta}$

In example 5.1, this coefficient α is equal to 0.62, showing once again that DM is closer to DB than DC.

In general, the convex parameter α of a DM will be greater than 0.5 if:

$$\frac{1-\pi}{1-\pi+\delta} > \frac{1}{2}$$

or:

$$\pi + \delta < 1$$

or by using (5.1):

$$\pi < \frac{D}{1+D}$$

$$\delta < \frac{1}{1+D}$$

Relation (7.1) allows us to generalize the DM approach by choosing other values of the convex coefficient α. It is already easy to see that a DB scheme corresponds to the case α=1 and a DC scheme to the case α=0.

In general, a convex invariant risk sharing will be based on the following rule:

$$\alpha.\delta + (1-\alpha).\pi = \text{constant} \qquad (7.2)$$

where the coefficient α is chosen between 0 (DC) and 1 (DB).

This coefficient α can be seen as a measure of the importance given to the retirees; $1-\alpha$ being the measure of the importance given to the contributors.

In case of a demographic shock, the new contribution and replacement rates become then solutions of the following equations :

$$\alpha.\delta_2 + (1-\alpha).\pi_2 = \alpha.\delta_1 + (1-\alpha).\pi_1$$

$$\pi_2 = D_2.\delta_2$$

So we obtain the new rates as function of the convex coefficient :

$$\delta_2 = \delta_1 . \frac{\alpha + (1-\alpha).D_1}{\alpha + (1-\alpha).D_2}$$
$$\pi_2 = \pi_1 . \frac{D_2}{D_1} . \frac{\alpha + (1-\alpha).D_1}{\alpha + (1-\alpha).D_2} \tag{7.3}$$

For instance , if $\alpha = 0.5$ (equal weight between contribution rate and replacement rate), we obtain:

$$\delta_2 = \delta_1 . \frac{1 + D_1}{1 + D_2}$$
$$\pi_2 = \pi_1 . \frac{D_2}{D_1} . \frac{1 + D_1}{1 + D_2} \tag{7.4}$$

The following property gives a link between this convex parameter α and the risk sharing coefficient ρ introduced in section 6 (formula (6.4)):

<u>Property 7.1. :</u>

The risk sharing coefficient ρ given by (6.4) and the convex parameter α given by (7.2) are linked by the relation:

$$\rho = \frac{1}{D_1} . \frac{\alpha}{1-\alpha} \tag{7.5}$$

<u>Proof :</u>

The replacement rate expressed in terms of the risk sharing coefficient ρ is given by (6.13):

$$\delta_2 = \delta_1 . \frac{D_1.(1+\rho)}{D_2 + \rho.D_1}$$

On the other hand , the replacement rate in terms of the convex parameter α is given by (7.3):

$$\delta_2 = \delta_1 . \frac{\alpha + (1-\alpha).D_1}{\alpha + (1-\alpha).D_2}$$

So the equivalence condition is :

$$\frac{\alpha + (1-\alpha).D_1}{\alpha + (1-\alpha).D_2} = \frac{D_1(1+\rho)}{D_2 + \rho D_1}$$

which gives after simple computations :

$$\rho = \frac{1}{D_1} . \frac{\alpha}{1-\alpha}$$

The same development with the contribution rate generates the same relation.

<u>Remark:</u>

Relation (7.5) shows that if we look at a pension system on a multi period model with successive changes in the dependence ratio and therefore successive applications of the automatic adjustment,

the two coefficients α and ρ cannot remain simultaneously constant across time, apart from the two limit situations of a DB or a DC. For instance, a DM scheme is characterized by a constant $\alpha = \frac{1}{1+M}$, (see (7.1)), but its risk sharing coefficient given by (6.9) will change on time together with the contribution rate. On the other hand, the DE system is defined by a constant risk sharing coefficient (ρ =1) (6.11), but its convex parameter will change on time (given by : $\alpha = \frac{D}{1+D}$).

8. Conclusion

NDC and classical DB social security systems can be seen as extreme solutions in the risk sharing between retirees and contributors. The "Musgrave "technique (using a replacement rate expressed in terms of salaries, net of pension contributions) is an intermediate solution that has been considered in the proposition of reform based on a points system for Belgium. This approach seems to bring more solidarity and equity between the generations than in a pure DB or DC scheme, where only one generation bears the risk. However, this Musgrave system is but one example of mix between DB and DC and is based on a solidarity level that can be challenged. In this paper, we introduce two ways to extend these three systems into a continuous family of systems with an automatic adjustment mechanism based on some constraint.

In section 6 we propose a first family based on the intuitive request of a fixed ratio between the variation of the contribution rate and the variation of the replacement rate; in section 7 we introduce a convex mix between the replacement rate and the contribution rate. The Musgrave rule is a particular case of this last philosophy. These different formulas allow the State to decide the level of solidarity to be injected in the social security.

In this paper we have considered only a deterministic model on two periods of time. Future extension will examine stochastic models with more than two periods and the stability issue of these adjustments.

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