

# Formulating appropriate utility functions and personal financial plans

Anthony Asher

School of Risk & Actuarial Studies  
UNSW Business School,  
University of New South Wales  
Sydney NSW 2052, Australia  
email: [a.asher@unsw.edu.au](mailto:a.asher@unsw.edu.au)

Adam Butt

Research School of Finance, Actuarial Studies and Applied Statistics  
ANU College of Business and Economics  
Australian National University  
Canberra

Ujwal Kayande,

Department of Business Administration  
Faculty of Business & Economics  
University of Melbourne

Gaurav Khemka

Department of Actuarial Science  
Bond University

## Abstract

Without inflation protected pensions, people need decision making tools (financial calculators) to make informed decisions about savings and investment for retirement. For investment, they need a framework to trade off risk and return. This paper examines the assumptions underlying some of the common utility functions in the financial literature and suggests ways of making them more consistent with the behavioural and happiness literature. In particular, frictional costs are introduced to explain loss aversion. The results are illustrated in a way that could perhaps be presented to users of financial calculators to elicit their preferences and assist in making more coherent decisions.

**Key words:** Loss aversion, retirement planning, dynamic programming, utility theory

**JEL classification:** D14, G11, J26

## 1 Introduction

The last half century has witnessed a dramatic shift in the financial world of the elderly. Firstly, life expectancies at older ages have increased significantly: the latest Australian Life Tables (Australian Government Actuary, 2014) show that life expectancy at 65 has risen from about 12 to 18 years for men and from 17 to 22 years for women. Not all of this increased life expectancy is healthy, which means that people need income support for longer, even if jobs were available and they want to work. Howse (2006) reports almost a decade of unhealthy life expectancy in the UK in 2001, which must be funded in some way.

Secondly, responsibility for the management of post retirement finances has shifted perceptibly from state and employers to individuals. This is most noticeable in the move from Defined Benefit (DB) plans to Defined Contribution (DC) schemes, particularly in Anglophone countries. These shifts are perhaps best explained as a response to longevity – as argued by Munnell et al (2007), who does not find the other possible explanations for the scaling back of DB plans to be persuasive. In other OECD countries, the European Commission (2012) reports on the scaling back of state benefits to respond to demographic changes, of which increased longevity is a major part.

The consequence is that many more people need to make the relatively complex decisions necessary to provide for when they are not going to be able to work, and when state-provided benefits appear inadequate, or are less than their desired standard of living. Lusardi (2008) however finds that many people are financially illiterate and take no interest in obtaining financial advice or of addressing the question of retirement savings. Even when they seek advice, the advice provided can be inconsistent, biased and counter-productive (Gokhale and Kotlikoff (2002), Inderst and Ottaviani (2012), and Hackethal et al (2012) respectively). Merton (2014) suggests that the confluence of additional individual responsibility and inappropriate advice represents a crisis in financial well-being.

Lusardi and Mitchell (2013) do find that people are better prepared for retirement when they are more financially literate and have received advice. Hershey and Walsh (2000) in a laboratory setting find a dramatic improvement in the quality of retirement savings decisions after minimal training, suggesting the possibility that interventions can be successful. There appears, however, to be little written about what constitutes helpful advice, and even less on the role and design of decision support systems (in financial calculators particularly) to assist in the making of appropriate decisions. In particular, we raise the question as to whether an appropriately structured web-based calculator (i.e., a decision support system) could be used to improve decisions about savings and investment, both before and after retirement.

This paper forms part of a broader project aimed at designing such a calculator. Related work by project members includes Butt and Khemka (2014), which has shown the costs of focussing only on the lump sum available at retirement, and Kayande et al (2009), which investigates the conditions under which people can be best motivated to use decision support systems. The objective of the present paper is to consider the extent to which the standard utility functions in the literature can be better related to individual's preferences. It draws on the financial and economic literature on the financial lifecycle, such as Blake et al (2011), with broader work on happiness and discounting, as summarised by Clark et al (2008) and Frederick et al (2002), as well as the psychology theories of Stanovich and West (2000) and Kahneman (2011). From the actuarial literature, it extends Thomson's (2003a; 2003b) work

on the elicitation of risk aversion in the choice of investments for retirement. Future research will develop a prototype calculator with alternative formulation of utility functions that will be tested on users, who are Australian superannuation fund members and financial advisors. The utility functions need to be plausible and presentable in ways that are accessible to users so that they can choose investment and consumption strategies that match their risk aversion. It is expected that the utility functions need to be as simple as possible to limit the number of questions users need to answer.

The next two sections provide an outline of the decisions that need to be made over the lifecycle and describe the parlous current state of advice available from web-based calculators and financial advisors. Sections 4 and 5 discuss the elicitation of utility functions and the anomalies exposed by behavioural economics, and suggest that many of these can be explained by the psychological theory of two decision systems: fast and slow. It is also suggested that asymmetric loss aversion can be explained and modelled by frictional costs associated with losses of income. The inclusion of frictional costs also allows for a reduction in utility arising from a decline in consumption relative to previous periods.

Section 6 discusses potential specifications of the utility function, and Section 7 provides an illustrative example of the modelling. Section 8 makes some comparisons with methods and heuristics currently in use and section 9 concludes.

## **2 A general model for lifecycle decision making**

For an individual who is privately funding at least part of their retirement income by defined contribution means (and may also perhaps receive a government-provided age-based pension), the retirement experience can be presented in a timeline as shown in Figure 1. All aspects about the timeline are affected by the financial preferences of the individual, which in its simplest form is to provide an appropriate level of spending throughout life. The quantification of these objectives will be discussed in more detail in Section 6. The accumulation phase represents the period where contributions are being made to the retirement savings account, whilst the withdrawal phase represents the period where an income is being drawn to fund retirement. There may be overlap between these two phases in a transition to retirement period, where the individual may move to part-time work and supplement their income from labour by withdrawing from the retirement savings account.

[Figure 1 here]

Decisions at each stage require choices involving a trade-off between leisure and consumption over the lifetime, and between risk and return. There are four main categories of decisions:

- i) *Personal earnings* depend on education and training, choice of career, industry and employer, and then on working hours and the age of retirement, which may be phased. Time may be taken off for various reasons relating to family particularly. Future earnings will also be contingent on the evolution of the relevant labour markets.
- ii) *Consumption* needs to be constrained in earlier years if people want to supplement state benefits in later years. The major choices include whether to plan for an increasing or decreasing pattern of consumption; household or family structure including the timing and expenses incurred in having and educating children, and sometimes in supporting

other family members; and the extent of any drop in spending in retirement. Whether or not to purchase one's home may also be classified as a consumption decision, although it also has significant investment implications.

- iii) *Investment strategy* needs to be selected to create an appropriate balance between risk and reward.
- iv) *Insurance products* need to be chosen to protect against death and disability whilst working and against excess longevity in retirement.

In making these decisions, a number of environmental factors need to be taken into account and modelled in the any calculator:

- a) Availability and cost of products which can be used by individuals to provide a retirement income;
- b) The current and future regulatory and tax environment (including all aspects of the retirement income system and other relevant components such as personal income tax);
- c) The distribution of future values of key economic variables and asset class returns;
- d) The distribution of future income from labour to be earned by the individual;
- e) The current wealth of the individual;
- f) The current and future health of the individual and associated medical expenses; and
- g) The timing of death of the individual.

We may consider (a) – (c) above to be market factors and (e) – (g) to be individual factors, with (d) depending on both. Some factors, such as the current wealth of the individual will be known at the commencement of modeling, and can hence be thought of as inputs rather than assumptions. Other factors are assumptions about the future and may be modelled deterministically or stochastically. Future investment returns and mortality rates lend themselves to stochastic modelling, while changes to legislation seem to be beyond any sensible attempts to model mathematically. In between these factors, health status, labour market income and household changes (especially having children divorce) can be modelled, but may be worth the effort given the degree of control over the outcomes that will be exercised by the users at the time.

The four groups of decisions cover somewhat more than the three things that Merton (2014) suggests should concern a client: “retirement income goals, how much she is prepared to contribute from her current income, and how long she plans to work” — although he subsequently mentions investment risks. While he may have oversimplified, his analogy of describing these decisions as automobile pedals seems an apt description of them as the controls that people can exercise to decide on their financial futures. Staying with the analogy, it is not necessary for the driver of the financial plan to understand the environmental factors, which may well be kept under the hood.

As noted by Lusardi and Mitchell (2013) such a model will be less useful to those who will rely completely on government funding in retirement. It also has less relevance to those who are likely to live off investment income in retirement. These include the independently wealthy and many who work in family businesses and farms, where outside investments may be minimal and complete retirement unlikely. The historical focus of the advice industry has been on the independently wealthy groups. This perhaps explains the lack of development of resources by to support the middle-income employee (who could actually benefit from the thinking described in this section). This is suggested by the results in Hershey and Walsh

(2000), where students with just six hours of training performed better than accountants with experience in financial advice: the financial advice required by the wealthy and self-employed is more tax driven than concerned with how much to save.

### 3 Current advice

There is *prima facie* evidence that the financial advice available to members of DC retirement funds is inadequate in many countries. Ciccotello and Wood (2001), Dorfman and Adelman (2002), Kotlikoff (2006) and Turner (2010) have found huge variations in the advice given by US website calculators and personal financial advisors. A preliminary survey of different Australian websites suggests similar differences exist here.

[Figure 2 here]

We suggest that many of the calculators available on the internet are not fit for the intended purpose. A common error is illustrated in the screenshots in Figure 2. The same information for an individual aged 60 has been provided for both charts, but the left chart has a “Conservative” asset allocation, whilst the right chart has a “High Growth” asset allocation. The calculator simply assumes a higher expected return for the “High Growth” asset allocation without acknowledging the additional risk, and providing some warning about the likelihood of underperformance.

Numerous examples exist in the literature of what appear *prima facie* to be suboptimal asset allocations as a direct consequence of poor financial advice. For example, Bodie (2003) reports that the advice on investment options and asset allocations provided to self-directed retirement plans in the US is logically flawed and dangerously misleading. The advice provided by financial services firms and investment advisory services leads to “a strong bias in favour of investing retirement savings in the stock market without insurance against a market decline”. Mullainathan et al (2012) find that financial advisors fail to undo behavioural biases and misconceptions of the clients. Instead their portfolio recommendations “reflect either biases that are in line with the financial interests of the advisers (e.g., returns-chasing portfolio) or run counter to their interests (e.g., a portfolio with company stock or very low-fee index funds)”. This is not to mention the role of conflicts of interest. The Australian Securities and Investments Commission (2014) report on the quality of life insurance advice found over a third of the advisors they questioned were giving advice that was illegally conflicted.

One of the consequences of inconsistent advice is underinsurance, or inappropriate insurance. Gokhale and Kotlikoff (2002) investigate underinsurance in the US and suggest that “66 percent ... of poverty among surviving women and 37 percent ... of poverty among surviving men resulted from a failure to adequately insure survivors.” They also find almost no relationship between the actual amounts of life insurance families have, and the amount of insurance that the authors believe is appropriate. They therefore find many families are over-insured. In considering the reasons for this problem, they suggest that “questionable financial advice, inertia, procrastination, and the unpleasantness of thinking carefully about one’s death are the likely culprits.” One relatively easy solution to this is to automatically provide life and disability insurance through retirement funds. Not only does this remove the unpleasantness of thinking about death, it is more cost effective, and provides greater coverage as much cover can be given “free of underwriting”. Hubener et al (2013) show how life insurance

needs can be generated relatively easily from financial lifecycle modelling, given age, income and family status. While family status is not available to Australian superannuation funds, it is available to retirement funds in many countries. The calculator envisaged in this product will also give life insurance advice, and would perhaps have an application for retirement funds.

#### **4 Eliciting utility functions**

In order to assist making the retirement decisions set out above, the envisaged calculator will need to elicit user's preferences for consumption and leisure over time – in the face of the environmental contingencies that are modelled. To be used in modelling, these preferences have to be formulated as a utility function. The form and parameterisations of the utility function are however critical to the investment strategies and insurance products that can be recommended.

Starmer (2000) traces the standard economic approach of expected utility back two hundred years to Bernoulli, but little progress has been made in eliciting coherent utility functions from people. Research into behaviour finds that while some individuals (and families) try to maximise their utility (self-interest), there is evidence of inertia and procrastination and less than perfect outcomes (Mitchell and Utkus, 2003). Mullainathan and Thaler (2000) summarise behavioural finance findings in suggesting that people's decisions are subject to bounded rationality, bounded willpower and bounded self-interest (by which they mean altruism).

It is therefore not surprising that, while calibrated to household data, lifecycle models explain a small proportion of the heterogeneity in populations even after allowing for almost all the factors that might be expected to predict savings and investment. Hubener et al (2013), who produce a comprehensive model, are not able to reproduce more than 14% of the squared error in their US data. Their utility functions and parameterisations may therefore not be appropriate to be applied normatively to the construction of advice for particular individuals or families.

Thomson (2003 a and b) and Thomson and Reddy (2013) make the case for eliciting utility directly from investors (or fund trustees) by asking them to rank uncertain lump sum benefits at retirement. While it is possible to elicit relatively consistent functions from most users, (Thomson 2003b) warns that the results are dependent on how the context is described and the questions framed. He quotes private correspondence with French, whose view is: "Subjects do not possess a utility function internally which they need to estimate accurately. Rather during the analysis, they construct a function and the construction process guides the evolution of their ... preferences." (921)

Thomson's results do not however address post-retirement investment. This is considerably more complex as each investment decision creates a range of possible returns for each year in the future, and the pensioner is able to choose different consumption patterns that may or may not depend on returns generated. Rabin (2000) warns against naïve extensions of utility preferences from modest to large stakes, and similar caution needs to be applied when increasing complexity of any form is introduced.

## 5 Two decision making systems

Before addressing the problem, it may be helpful to consider French's construction process. Stanovich and West (2000) provide one defence of the psychological theory of two different systems of decision-making. To simplify, *System 1* is instinctive, responds rapidly, requires little effort, is quick to make judgments and necessarily works off rules of thumb. *System 2* is slower, reasoned and requires high effort to engage. The difficulty of engaging system 2 means that we all have a tendency to make snap judgements that we can regret because we did not take the time and effort to engage our deeper thinking processes. Epstein (1994) traces the idea back to Freud's distinction between the conscious and the unconscious, but it is not difficult to see it in St Paul's: "So then with the mind I myself serve the law of God; but with the flesh the law of sin<sup>1</sup>" and many other distinctions made between intuition, reason and will. Kahneman (2011) has popularised the theory, which has gained greater credibility by the findings of neuroscience: "There is imaging evidence that the mere anticipation of a high-effort task mobilizes activity in many areas of the brain, relative to a low-effort task of the same kind" (451).

Frederick et al (2002), in a review of time discounting and preferences research, outline the behavioural anomalies that bedevil the construction of coherent utility functions. While they do not make the connection, the theory of two separate decision-making systems provides a useful tool to interpret some of their empirical findings. System 1 decisions will be more appropriate where the differences in outcome are not material, are likely to be outweighed by the cost of information retrieval and cognition, or the results of rules of thumb are unlikely to be improved by cogitation. These conditions do apply to a number of decisions that otherwise appear inconsistent with other choices people make:

- People often use hyperbolic discount rates, meaning that the rate of discount is high for short durations but reduces over longer durations. (Frederick et al, 2002: 360). In contrast, people also normally express preferences for increasing rather than reducing consumption patterns when given choice over a series. (363) If we see the choice set of decisions involving short durations as limited, with minimal differences in outcome, it can make sense to enjoy the quick win rather than make the effort to remember to take the opportunity later.
- Similar explanations could apply to the fact that people make different decisions about smaller amounts of money. (363) In this case, they may turn down apparently attractive odds because the effort of participating is not worthwhile.
- The two systems provide a good explanation for people's tendency to make prior financial commitments in order to prevent themselves from overspending. (366) By making the effort to pre-commit, they can allow freer rein to system 1 to make impulse decisions.
- Prior commitments are a type of mental accounting, where money is not regarded as fungible but allocated to different accounts for different uses. (373) In retirement planning, Merton (2014) for instance, suggests three retirement accounts providing

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<sup>1</sup> Romans 7:25 King James Version

guaranteed, conservative and higher return accounts. If the proceeds from these accounts are used for different expenditure, system 2 has less future work in deciding on whether one can afford the more luxury items.

- Implicit rates of discount appear to be more realistic when people are also asked to consider uncertainties (382), which would require the use of system 2.
- The two systems can also throw light on some of the differences made by framing. (375) Expressing a problem as one where thinking and effort is required is much more likely to bring the logic of system 2 into play.

Frederick et al (2002) also describe a variety of “multiple-self models” (375) where one self is myopic and the other long-sighted, and discussions about temptation, which also fit readily into the two systems view.

If the two systems view is largely correct, it is clear that retirement advice has to be addressed to system 2, and that the apparent anomalies in rational behaviour that arise from system 1 have to be by-passed. This can be achieved by ensuring that the recipient of advice is adequately engaged and able to give adequate consideration to the matter in hand. The advice will also need rational justification in order to be acceptable to system 2. There is no suggestion in the literature that system 2 is invariably correct in its logic, nor even that it is not subject to systematic bias, but merely that it is susceptible to reason. There is however reason to believe that better outcomes can be achieved.

One possible consequence of repeated application of the system 1 approach to decision making, however, is that individuals then develop sub-optimal mental models that are stubbornly applied when they make decisions. If encouraged to use system 2 type decision making, i.e., a more reasoned and cognitively elaborate approach, individuals then simply access and use their sub-optimal mental model to make decisions (Kayande et al 2009), thereby simply perpetuating the errors of the system 1 approach. These findings imply that individuals require assistance not only in moving toward a system 2 approach to decision making, but also in shedding their persistent sub-optimal mental models of retirement related decisions.

## 6 Alternative specifications of utility

Putting aside, for the moment, the question of how to activate system 2 (which will be a function of the design of the calculator and is a subject for future research), we need first to consider how to construct coherent utility functions that will appeal to it. The standard formulation of constant relative risk aversion (CRRA) utility at time zero in the lifecycle literature, given for instance in Cocco et al (2005), is the equation:

$$U_0 = \sum_1^T \beta^{t-1} (\prod_{j=0}^{t-2} p_j) \{p_{t-1} \frac{1}{1-\gamma} (C_t)^{1-\gamma} + b(1 - p_{t-1}) \frac{1}{1-\gamma} (D_t)^{1-\gamma}\} \quad (7.1)$$

where  $\beta$  represents the time preference rate;  $C_t$  the consumption in period  $t$ ;  $p_t$  the probability of survival in the year  $t$  to  $t+1$ ;  $\gamma > 0$  is the coefficient of risk aversion;  $b$  the intensity of the bequest motive, and  $D_t$  the bequest.

Hubener et al (2013) incorporate the trade-off between leisure and consumption by introducing a time budget that is divided into working hours and commuting times, household

maintenance that allows for care of children, and leisure. They also jointly consider the utility of the couple, which would lead to the equation:

$$U_0 = \sum_1^T \beta^{t-1} (\prod_{j=0}^{t-2} p_j) \{p_{t-1} \frac{1}{1-\gamma} (\frac{c_t}{\phi_t} l^\alpha)^{1-\gamma} + b(1 - p_{t-1}) \frac{1}{1-\gamma} (D_t)^{1-\gamma}\} \quad (7.2)$$

Here,  $\phi$  represents an allowance for the family state (which in Hubener et al (2013) is contingently modelled using empirical transition rates) and  $l^\alpha$  represents the utility of the hours of leisure with  $\alpha < 1$  to allow for declining marginal utility.

This section discusses each of these parameters in turn.

## 6.1 Time preference

Frederick et al (2002, 359) summarise an earlier debate on the extent to which coherent reasons can be found for allowing for the subjective time value of money. As they point out, one needs some sense of personal continuity through time if one's future welfare is to be relevant at all. The stronger the continuity, the lower the discount factor. The standard assumptions in the lifecycle literature are for complete continuity, usually with an expectation of returns greater than the rate of discount, which then imply deferment of consumption. This is consistent with results reported in Frederick et al (2002) of a general preference to prefer increasing patterns of consumption to declining ones.

One can argue that this is consistent with the standard design of life insurance and pensions annuities, which might be an appropriate anchoring point from a normative perspective. Guaranteed annuities are invariably level in nominal terms or indexed to price or wage inflation. Many would agree with Shafir et al (1997) that those that are level in nominal terms are designed or purchased by individuals who have underestimated the inflation risk and suffer from money illusion. If nominal pensions are ignored, it seems that there is widespread agreement that benefits should be not be decreasing in real terms — as confirmed by Beshears et al (2014). There is a further debate as to whether real should be defined in terms of average wages or by average prices. In general the former is to be preferred on grounds of lifestyle relative to the working population, the latter on grounds of affordability. This is again confirmed by average behaviour in retirement. Calibration of the lifecycle models invariably find rates of time preferences lower than expected returns (Cocco et al, 2005; Hubener et al, 2013).

For purposes of the illustration in this paper, we have used a rate of time preference equal to the increase in average wages, and performed modelling in this paper on a “real” basis. By making this assumption, we assume that the discount factor is independent of the investment decision of the individual.

If desired, it would, however, be possible to increase the discount rate to provide some anticipation of the investment return, which is normally what is done in the pricing of

investment linked or with profit annuities.<sup>2</sup> A common argument, made in industry for this approach, is that product design should be prepared to accept that consumption does decline at a slow rate during retirement. Fisher et al (2008) find a rate of decline of about 1% per year. An unanswered question is whether this decline for individuals is a result of a sudden decline in health, or a general reduction in energy levels. If it is health related, it introduces another contingency, and would suggest an alternative utility function based on health status.

## 6.2 Survival

Survival rates are not used in all calibrations of survival models. We have found no discussion in the literature for whether they should be included or not, but there may be arguments for their removal, and the use of life annuities in modelling.

- If a life annuity is purchased, a reduction in the amount of the annuity appears to be double counting the possibility of not being alive.
- Utility functions held to make decisions about uncertain states of the world. Being dead vs being alive are not different states of a person's world: the one does not exist.
- "Future me" would have a legitimate complaint against "present me" for overspending when future consumption has to be curtailed in proportion to survival probabilities.

The consequence of removing the survival factors would be to produce a plan that aims to level consumption throughout life as found by Beshears et al (2014), although this will result in the drawing down of capital in retirement until a specific maximum possible age of life.

In this paper, however, we have incorporated survival and consequently produced consumption patterns that decrease at very high ages. Whether this is acceptable for users will be investigated in our subsequent research.

## 6.3 Risk aversion

Starmer (2000) surveys some of the many alternatives to the standard expression of risk aversion. Perhaps the main problem identified is the asymmetry in the weights people apply to losses and gains, which can be described as the loss aversion addressed directly by Cumulative Prospect Theory, formulated by Tversky and Kahneman (1992). While loss aversion is being used widely, there is as yet no consensus on how to incorporate within utility functions.

- The standard constant relative risk aversion (CRRA) produces a constant ratio invested in risky assets over the lifetime (Dybvig and Liu, 2010).
- The log of the income over and above that necessary for "protected consumption" produces a lower allocation to risk around retirement (Kingston and Weng, 2014).

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<sup>2</sup> TIAA CREF variable annuities, for instance, are determined at 4%, which is somewhat higher than real interest rates have been for some time. See <https://www.tiaa-cref.org/public/pdf/How-to-make-changes-to-annuity-income.pdf>

- A CRRA function plus a higher elasticity of inter-temporal substitution produces a declining risk proportion over time (Blake, Wright and Zhang, 2008).
- A utility function kinked at protected wealth creates investment risk seeking until the satiation threshold (Blake, Wright and Zhang, 2011).

One approach is suggested by the Clark et al (2008) review of the current literature on happiness. Prominent in their discussion is the role played by comparisons of current consumption with alternative benchmarks: both people's own previous consumption and the consumption of reference groups: "happiness is indeed negatively related to others' incomes and to own past income."<sup>(57)</sup> Comparisons with reference groups will be considered later when considering investment returns, but comparisons with one's own previous consumption would require an additional term to be added to the utility function. Frederick et al (2002) note that standard utility functions assume the independence of utility period by period, and note that there is evidence that people prefer a smoother income, which is effectively to make the same point. The idea of smoothness and reference levels has been incorporated into the literature as in Kingston and Weng (2011) and van Bilsen (2013), but we would like to investigate an approach which attempts to explain loss aversion in rational terms.

While adding another term does go against parsimony, loss aversion can be explained rationally and simply by introducing frictional costs. The need to reduce consumption plausibly requires costs not present when deciding how to spend additional income. It may be necessary to move to a smaller home selling furniture at a loss, or investigate less expensive items of consumption – and to make all such decisions under time pressure. Assets may lie idle because it is no longer possible to afford running or maintenance costs. These costs can be modelled by applying a penalty when income declines.

Applying a penalty creates more risk aversion in the utility function, and will address the problem raised by Rabin (2000), who questions the plausibility of very convex functions.

## **7 An illustrative example**

This section provides an illustrative example of the procedure that could be used to generate optimal retirement outcomes to assist in decision making. It is an extension of the work undertaken in Butt and Khemka (2014), and reconciles a backward looking smoothing function with a forward looking value function that incorporates a measure to limit the losses imposed by the smoothing function. These losses occur to extent smoothing delays reducing consumption when returns are low.

### **7.1 Decisions modelled**

We consider an individual throughout adult life. The decisions optimised by the model are the investment strategy (short selling or margin borrowing is not allowed), contributions pre-retirement and withdrawal amounts post-retirement. The decision on contribution amounts and withdrawal amounts are equivalent to a decision on consumption.

## 7.2 Assumptions

The following assumptions are as per Butt and Khemka (2014). Further details can be found there.

- Individuals work full-time up until exact age 65 for a constant salary, where work will cease completely and retirement will occur permanently.
- Retirees maintain all investment, longevity and other risks in retirement.
- Mortality rates are determined by reference to the male rates in the Australian Life Tables 2005-07 (Australian Government Actuary, 2009). Mortality is assumed to be zero up until age 65 as life insurance is not at this point considered and the effect of including it would be trivial. Individuals live until a maximum age of 110, at which point they are assumed to die.
- No fees or tax are allowed for. No external sources of income (e.g. a government-sponsored pension) are allowed for.

Individuals are assumed to have a choice between two asset classes: equities and cash. The future distribution of returns of these asset classes takes a non-parametric approach using past data.

The data used are the daily, rolling annual returns on the UBS AU Bank Bill All Maturities Index (Cash) and S&P/ASX200 Accumulation Index (Equities) from 1 January 1993 to 31 December 2012. The data were first sorted by the difference between the equity and the cash returns (we call this the equity premium). Then the data were separated into buckets of width 1% equity premium (104 buckets). To increase the granularity of the stratified data, a limit of 5% range of equity return was imposed on each bucket (provided that the bucket size exceeded 20 observations). Buckets that had more than 20 observations with the range of equity returns exceeding 5% were further sub-divided into two (giving equity premium width of 0.5% in the new buckets). The final stratified data were composed of 145 buckets, with the observation chosen from each bucket being based on the median equity premium in that bucket. The number of data points in each bucket is used to weight the observation in the calculation of the utility function (see below).

The asset class returns above are expressed on a real basis, with the inflation measure chosen being that of Average Weekly Earnings (AWE; calculated by the Australian Bureau of Statistics, Catalogue No. 6302.0). An earnings inflation measure is chosen rather than a price inflation measure to reflect the desire of individuals to maintain the living standards of the population rather than simply maintaining purchasing power. It also ensures no adjustments to pre-retirement income or lifetime consumption are required in the model. Since AWE are only calculated on a quarterly basis, the daily increases in AWE are assumed to be constant between quarters in order to generate the daily, rolling annual increases in AWE required by the process described above.

It should be noted that a consequence of the approach described above is that no information about current economic conditions is taken into account when making decisions. Most interest rate models incorporate both momentum and mean reversion factors, and Asher (2007) summarises research which suggests that non-linear models can be justified for equity markets. It is intended that future work will investigate the materiality of these alternatives.

Should the impact of current economic conditions be incorporated into future work, this would require an economic model that allows explicit autocorrelation in economic variables from year to year, and at least one economic factor at the start of the year to be treated as a state variable governing mean reversion.

### 7.3 Method

The CRRA function described above is used, although with an adjustment to include a penalty that applies where the current period's consumption is lower than the past period's consumption. Bequest motives are not allowed for.

A dynamic programming approach is used to calculate the optimal decisions at integer ages for a series of retirement savings account balances. Dynamic programming requires optimisation to be performed by a backwards iterative process. The following series of equations defines the dynamic programming process for the objective function chosen. The utility  $v_i$  for a single observation  $i$  is defined as follows:

$$v_i(x-1, y_k(x-1)) = v_i'(x-1, y_k(x-1)) + V(x, y_i(x, y_k(x-1)))(1 - q(x_j)) \quad (8.3.1)$$

$$v_i' = \left[ \frac{P^{1-\rho}}{1-\rho} + \beta \times \min \left( 0, \frac{P^{1-\rho}}{1-\rho} - \frac{P_p^{1-\rho}}{1-\rho} \right) \right] \left[ 1 - \frac{q(x)}{2} \right] \quad (8.3.2)$$

$$V(110, y_i) = 0 \quad (8.3.3)$$

where:

- $x$  is age
- $y_k$  are the other state variables at the start of the current period (i.e. retirement savings account balance and past period consumption)
- $y_i$  are the other state variables and the end of the current period (i.e. retirement savings account balance and past period consumption)
- $v_i'$  is the utility contribution from consumption in the current period
- $V$  is the utility contribution from consumption in future periods, and is obtained by bilinear interpolation of  $V(x, y_i(x, y_k(x-1)))^{\rho-1}$ .
- $q(x)$  is the mortality rate
- $P$  is the consumption in the current period
- $P_p$  is the consumption in the previous period
- $\rho$  is the coefficient of risk aversion
- $\beta$  is the coefficient of penalty for consumption decrease

## 7.4 Results

We illustrate results for  $\rho = 5$  and  $\beta = 0.1$ . For clarity of the results, we assume that before retirement, the individual earns an overall income from labour of \$72,800, which approximates AWE.

We first show the optimal asset allocation and consumption amounts for a range of balances and where the prior consumption amount equals zero (i.e., the penalty never applies) in Figures 3A and 3B. We then show the optimal asset allocation and consumption amounts for a range of balances and where the prior consumption amount equals \$100,000 (i.e. the penalty always applies) in Figures 4A and 4B.

[Figures 3A, 3B, 4A and 4B here]

There is minimal difference between the results if a consumption penalty is applied. The results broadly follow those described in Butt and Khemka (2014). After retirement, the allocation to equities is not impacted by the balance amount or age, and the consumption amount is a proportion of the balance both pre- and post-retirement (as proven by Samuelson, 1969). The amount of consumption as a proportion of the balance increases with age as the future expected life decreases, at rates of 4.4%/5.6%/7.7%/11.3% for ages 65/75/85/95.

We note, however, that the consumption decrease penalty creates a discontinuity and has the effect of creating two optimal consumption amounts: one where the optimal consumption is lower than the previous period consumption and one where the optimal consumption is higher than the previous period consumption. For example, at age 65 and with a balance of \$842,400, the optimal consumption is around \$37,600 where the penalty applies and around \$36,900 where the penalty does not apply (with transition occurring where the prior period consumption is between \$36,900 and \$37,600). Under the utility function with a penalty, individuals will aim to maintain the consumption if they can, delaying small reductions in consumption in the reasonable hope that investment markets may recover. This appears to be a sensible response and may accord with some common advice, and occurs without incorporating any mean reversion. The penalty has minimal impact on the choice of asset allocation.

Before retirement there is a clear decrease in the allocation to equities with age, which is consistent with the argument of Bodie, et al. (1992) that investors should reduce their allocation to equities as their human capital (i.e. their future ability to earn an income through labour) decreases. The reduction in equity as balance increases relative to human capital can be understood in the same context. However, at extremely high balances this is offset by the boundary condition that consumption can only increase to the value of income earned by labour.

We cannot directly use the results in Figures 3A – 4B to investigate decisions over time, as choices are determined by the balance and consumption pathway that an individual takes as they age. To investigate this further, we perform 10,000 simulations on a forward looking basis starting for an individual aged 25 with a starting balance of \$0. Decisions made are based on the optimal decisions outlined above where the consumption penalty is applied, with linear interpolation performed where necessary. Objective values are calculated on the same

basis as described above. We investigate the distribution of balance and allocation across lifetime for these simulations for a variety of starting ages and balance amounts.

There are two ways that we could present the results to a user of the calculator. The first approach is to present results that are percentiles of that particular year's results. These are shown in Figures 5A – 5C, which generate relatively smooth profiles. The results however hide the volatility of income that the individual will actually experience. A second approach is to present results that represent a single simulation, and are percentiles based on the aggregate utility experienced in each simulation. These are shown in Figures 6A – 6C and look more like what an individual might actually experience given volatility in investment markets.

[Figures 5A – 5C and 6A – 6C here]

The results again demonstrate the appropriateness of a reduction in equity allocation with age, with sharp reductions occurring between ages 40 – 65 depending on balance level. As observed in Figure 4A there is little impact of age or balance on the post-retirement equity allocation.

The upward sloping consumption curves indicate that individuals will expect to contribute more in earlier years to take advantage of the compounding of returns in excess of wage growth. In Figure 5C we can see that the median and lower percentile outcomes have a smooth transition of consumption from pre to post-retirement. The higher percentiles experience a jump in consumption at retirement, due to the upper bound of consumption pre-retirement being the labour income earned, with no such upper bound existing after retirement. Those enjoying the higher returns may also take earlier retirement or leave bequests.

In all of Figures 5A – 5C, median consumption tends to trend upward until around age 80, after which point consumption tails off due to the reducing probability of being alive after this date, and hence the lowering of the balance in attempt to experience utility whilst still alive.

The results in Figures 6A – 6C are more volatile than in Figures 5A – 5C as they represent individual simulation (path) results – for this reason on the median and 5/95 percentiles are shown. Of particular interest are the consumption results in Figure 6A for a 65 year old. The median consumption path is similar to the 5<sup>th</sup> percentile result until around age 77, after which it increases (relatively) until it is greater than the 95<sup>th</sup> percentile result from age 90. Paths relatively close to the median show can however show almost the opposite results depending on the incidence of investment returns, and illustrate sequencing risk.

## **8 Comparison with common approaches**

In this section, we comment on common approaches to advice in the light of our findings to date.

## 8.1 Elicitation of utility

In Australia, investment risk must be presented to members as “the estimated number of years in a 20 year period ... that negative net investment returns will be incurred”<sup>3</sup>. Advice to members is therefore presented in this context and the risk is compared with expected returns from different asset allocations. The approach can be criticised firstly on the grounds that it is not well understood by members (Bateman et al., 2014). Even if it were understood, short term investment fluctuations have a remote relationship with the level and volatility of retirement income at younger ages.

As retirement approaches, the relationship is closer, but it is unlikely that anyone (using system 2) would trust a post-retirement investment strategy chosen by a utility function, without some idea of the consequent variations in consumption. Nor should they. It is suggested that illustrations of paths given in figures 6A – 6C give some idea of how possible variations in output could be communicated. Sample paths could be given for alternative formulations of risk aversion parameters, and users asked to choose that set of paths that they regard as most acceptable.

This is clearly more complicated than the elicitation process developed by Abdellaoui et al. (2008) and Thomson and Reddy (2013). It is an open question as to whether the alternatives can be readily explained to users in such a way that they respond meaningfully. This is to be a main subject of future research.

## 8.2 Other heuristics

A utility maximisation approach is not consistent with a number of the heuristics currently being used by advisors and implicit in the web calculators available.

- “Bucket” asset allocation. (See Australian Super<sup>4</sup> p14, Pfau and Cooper (2014) and Merton, 2014.) This approach allocates low risk assets to fund “protected consumption” and invests the balance in higher risk assets. It effectively assumes that it is possible to lose all of the higher risk assets, and so probably leads to under consumption (as they will not all be lost) and too low an allocation to higher returns. It is possible that our research may produce a simplified algorithm very like this approach.
- Assets run out at life expectancy (with or without buffer): This is the strategy implicit in the illustrations of Figure 2, that money will “run out”. The welfare loss is seldom traded off against current consumption, and no allowance is made for the likelihood that all but the completely myopic person would reduce consumption some years before assets are completely exhausted.
- Evaluation of outcomes using probability of shortfall rather than a plausible utility function: It is widely recognised that probability of shortfall is an incoherent risk

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<sup>3</sup> Australian Prudential Regulation Authority Reporting Standard SRS 700.0 Product Dashboard

<sup>4</sup> <http://www.australiansuper.com/layouts/Download.aspx?ID=c01bc384-2513-4742-a1f6-3ca224cca4f0>

measure giving no weight to welfare losses once a shortfall occurs. The same approach is implicitly taken when users are encouraged to take greater risks to meet shortfalls in target expenditure – as in the calculator shown in Figure 2. This ignores the costs of greater shortfalls that come with greater risk. Butt and Khemka (2014) quantify the loss of this approach using the standard utility function.

- Targeting retirement spending or insurance cover without consideration of the impact on current consumption: This is again the assumption illustrated in Figure 2. Savings recommendations can even exceed current income!

It is left to future research to investigate more fully the effect of these heuristics on the utility obtained by the individual, as compared with a utility optimisation approach.

## **9 Summary and conclusion**

Coherent and accessible financial advice requires some complex calculations that need to take uncertainty and a number of details about users and preferences into account. In order to construct usable financial calculators, the standard utility functions in the finance literature need to be recast in order to elicit preferences.

In this paper, we make a start on explicitly bringing the utility functions used in positive research to the normative question of financial advice. We suggest a reconciliation of some of the main anomalies found in behavioural finance with coherent utility functions, and in particular suggest using a penalty when consumption declines. In terms of the elicitation of risk aversion, we suggest that users will need to understand the volatility of consumption over their lifetime. We present graphs that illustrate such volatility.

This paper presents only a first step at considering the issues involved in setting up a usable financial calculator. Further research will be needed on how to best elicit preferences from users, how to best represent output of the model to users, and also to extend the basic illustrative model provided here to incorporate additional factors such as tax and social security and additional realism in investment modelling.

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## Figures

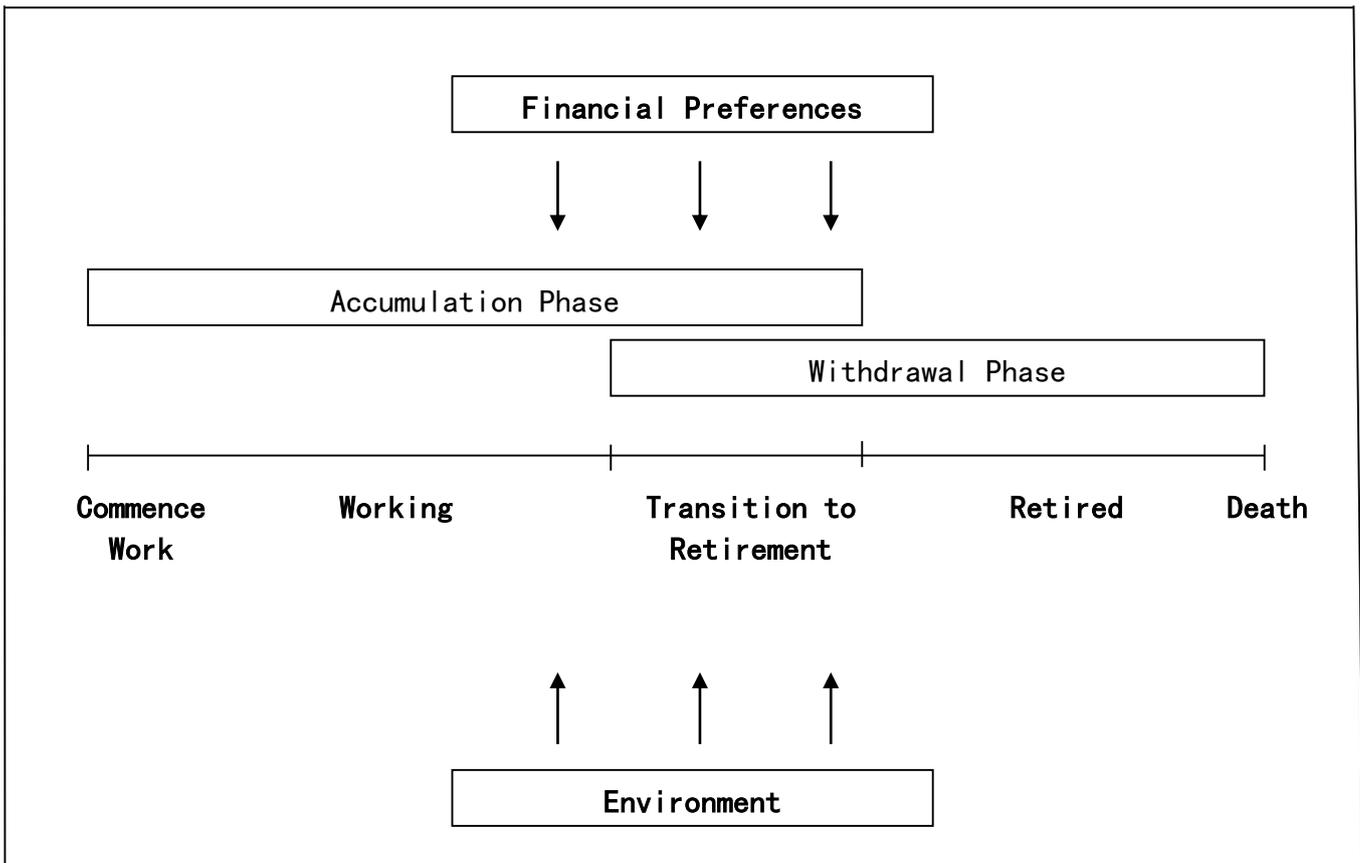


Figure 1: Retirement decisions



Figure 2: Typical Web Calculator (Source: Association of Superannuation Funds of Australia Retirement Projector – accessed from [www.superguru.com.au](http://www.superguru.com.au) in December 2014)

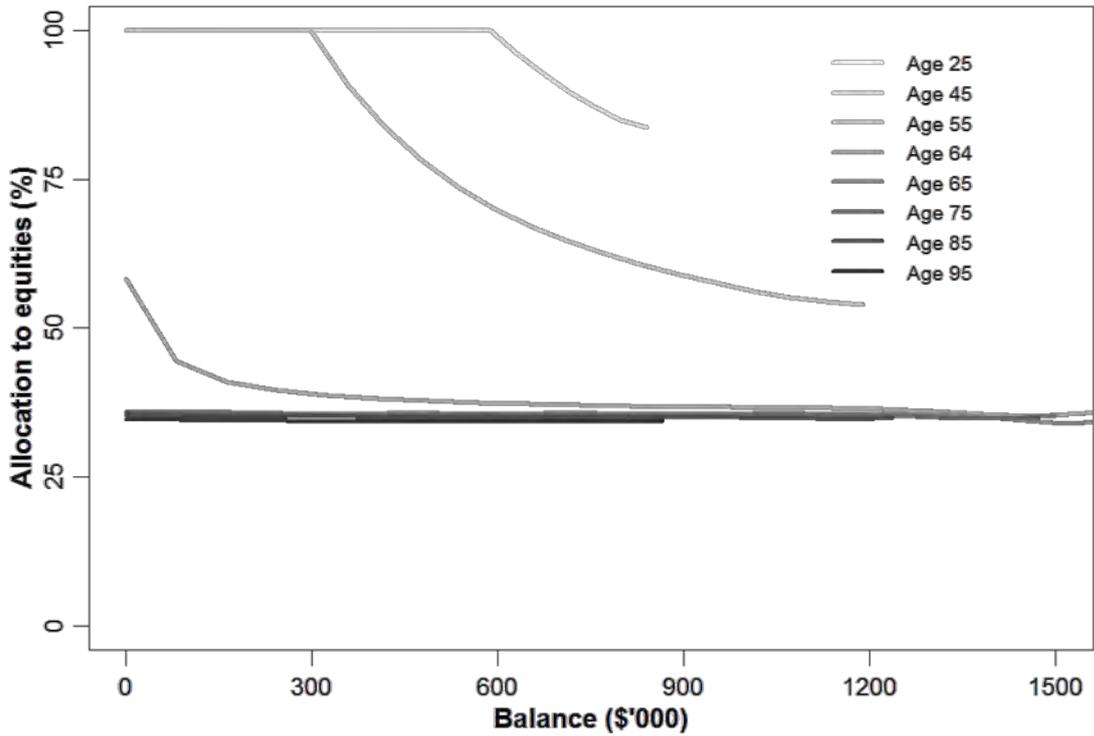


Figure 3A: Optimal allocation to equities (no consumption decrease penalty)

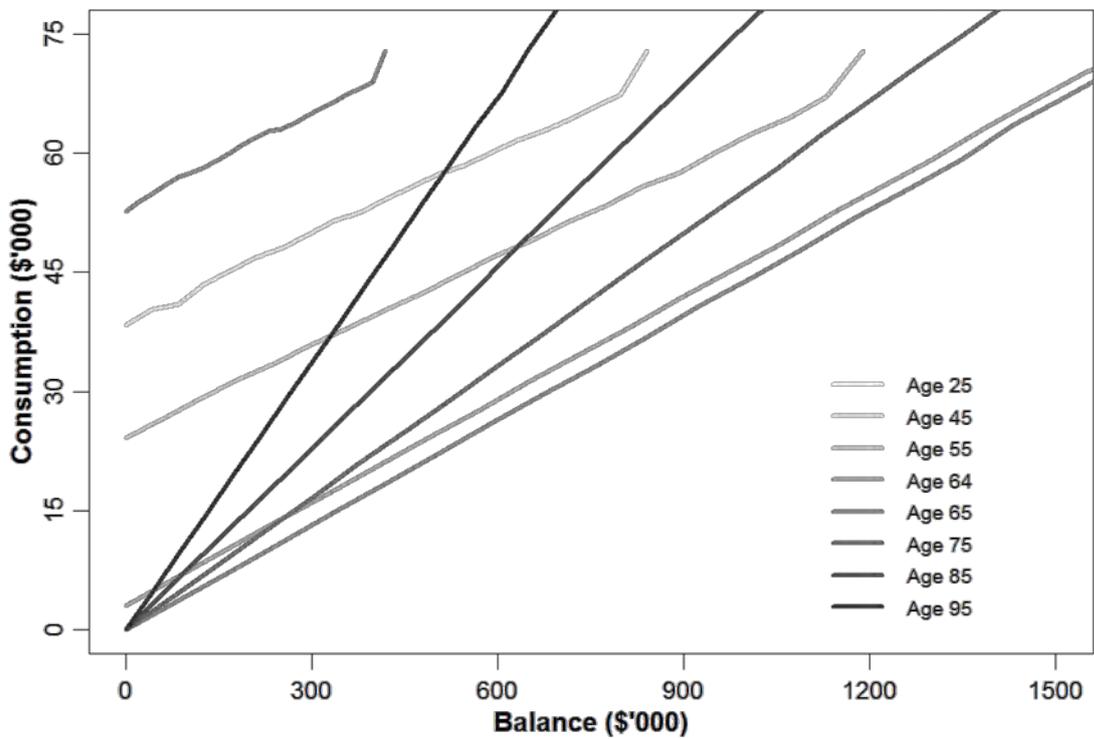


Figure 3B: Optimal consumption (no consumption decrease penalty)

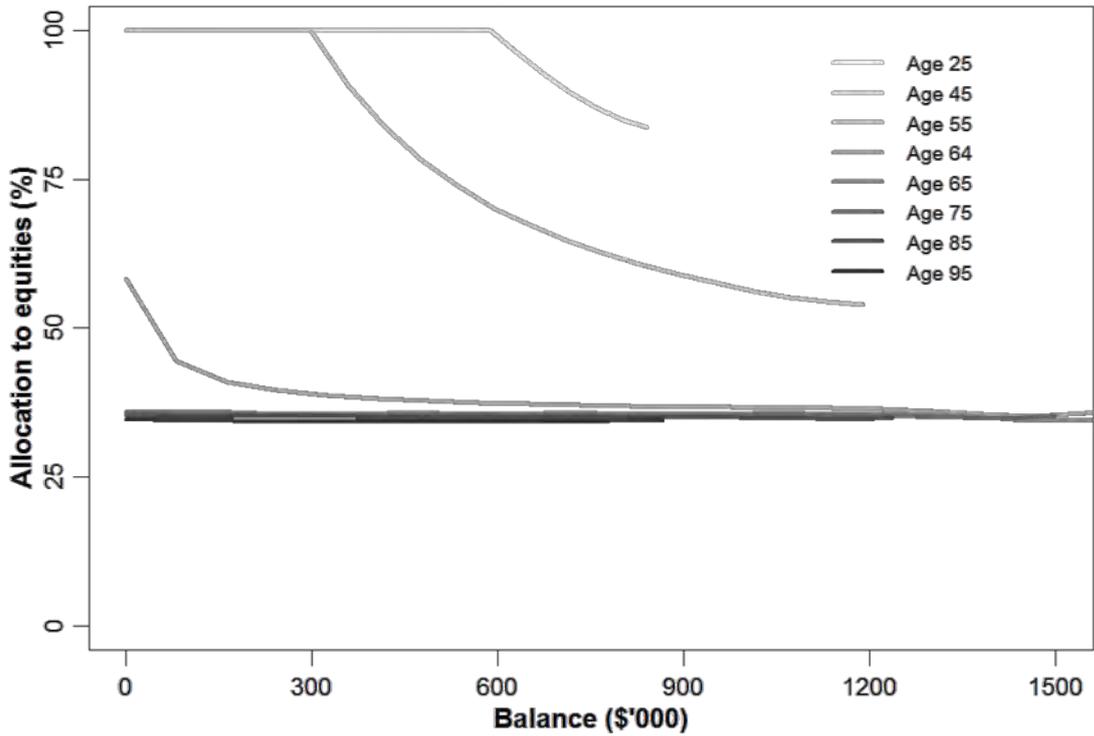


Figure 4A: Optimal allocation to equities (consumption decrease penalty applied)

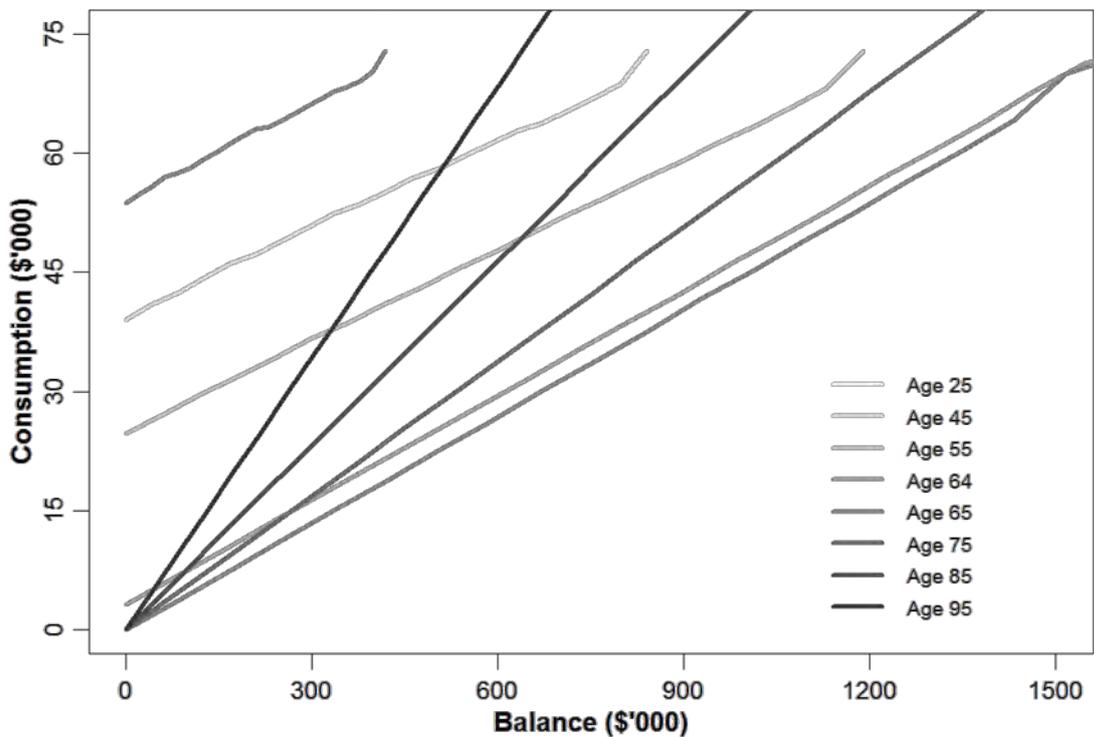


Figure 4B: Optimal consumption (consumption decrease penalty applied)

### Projected Outcomes for Age 65 Balance \$750,000

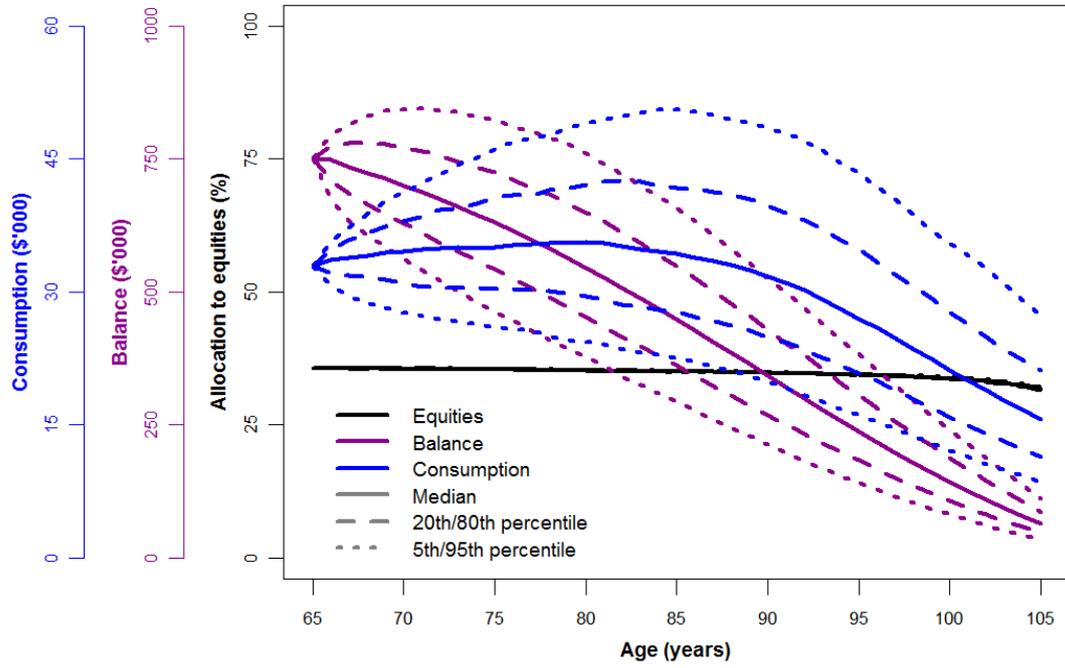


Figure 5A: Percentiles of future results for Age 65 / Balance \$750,000 (by year)

### Projected Outcomes for Age 45 Balance \$100,000

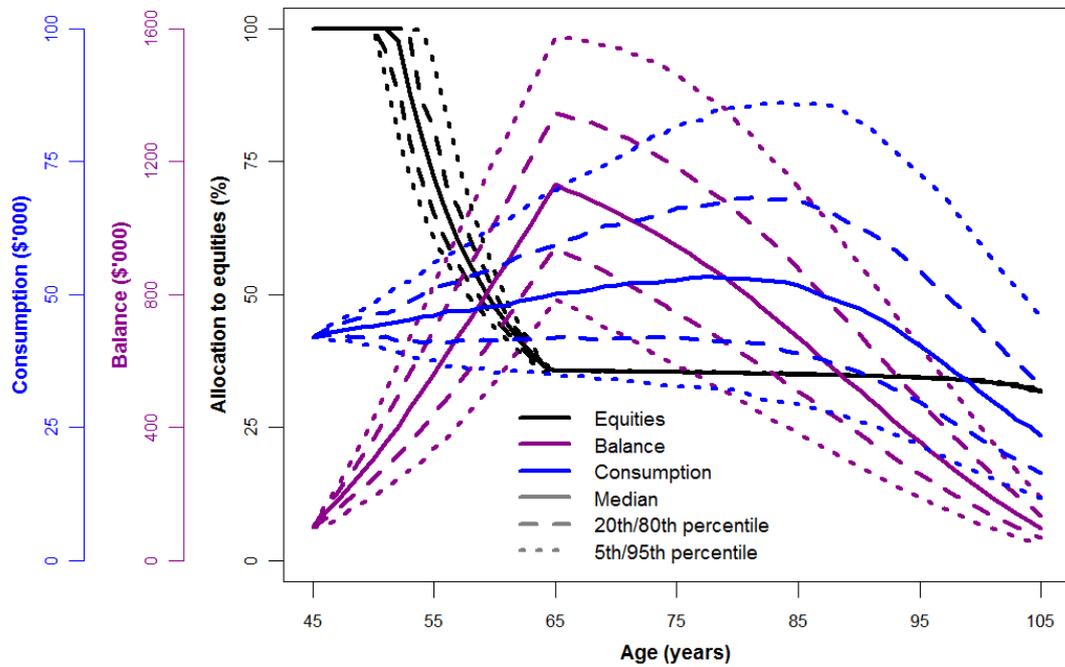


Figure 5B: Percentiles of future results for Age 45 / Balance \$100,000 (by year)

### Projected Outcomes for Age 25 Balance \$0

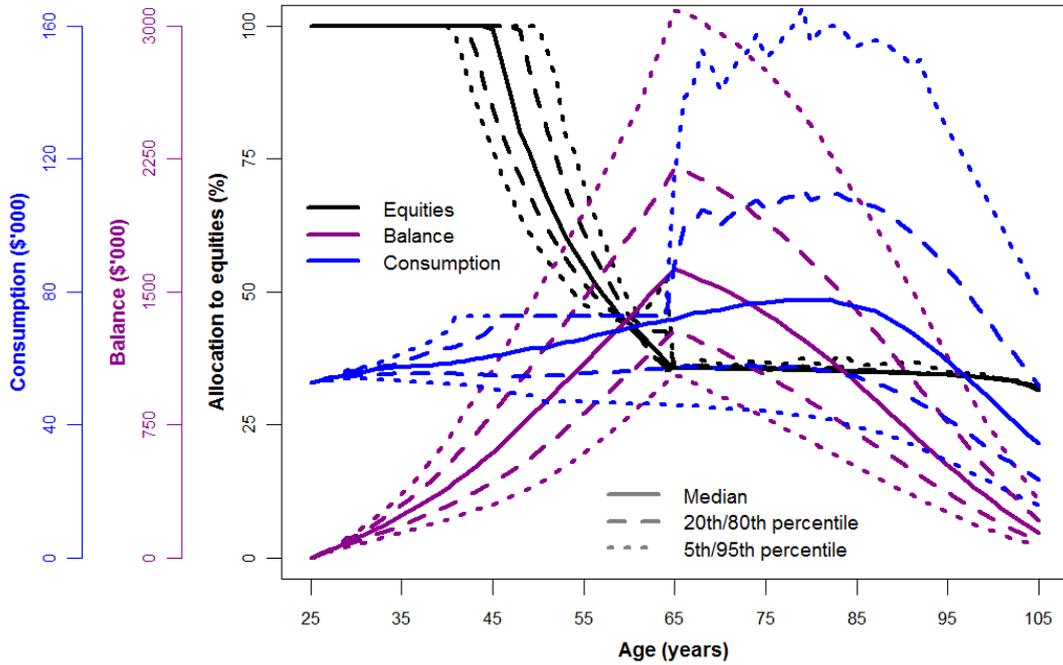


Figure 5C: Percentiles of future results for Age 25 / Balance \$0 (by year)

### Projected Outcomes for Age 65 Balance \$750,000

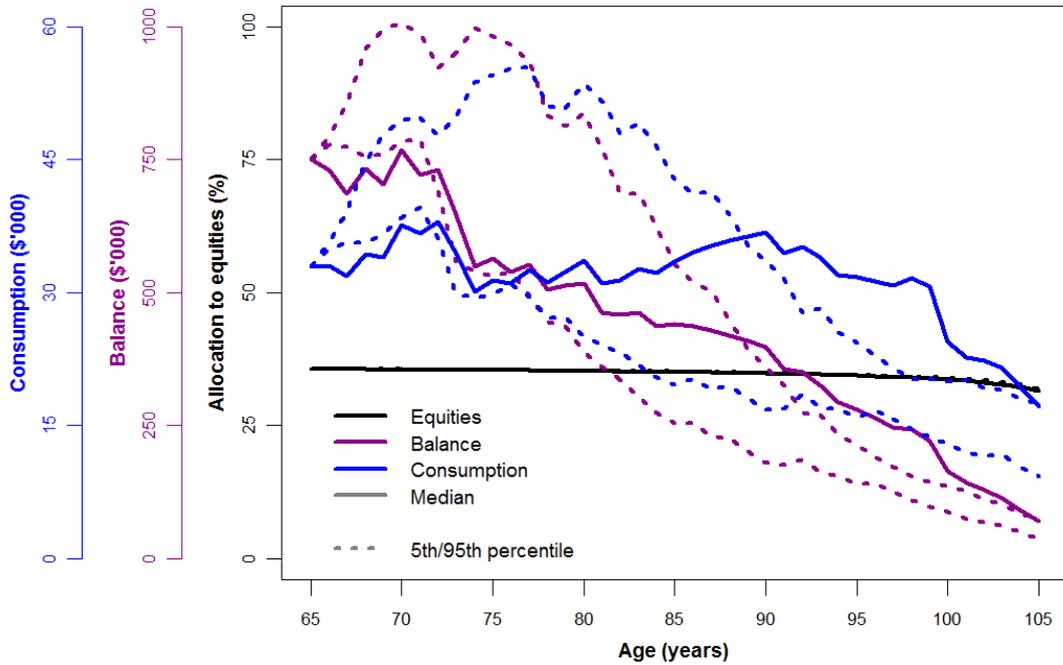


Figure 6A: Percentiles of future results for Age 65 / Balance \$750,000 (by path)

### Projected Outcomes for Age 45 Balance \$100,000

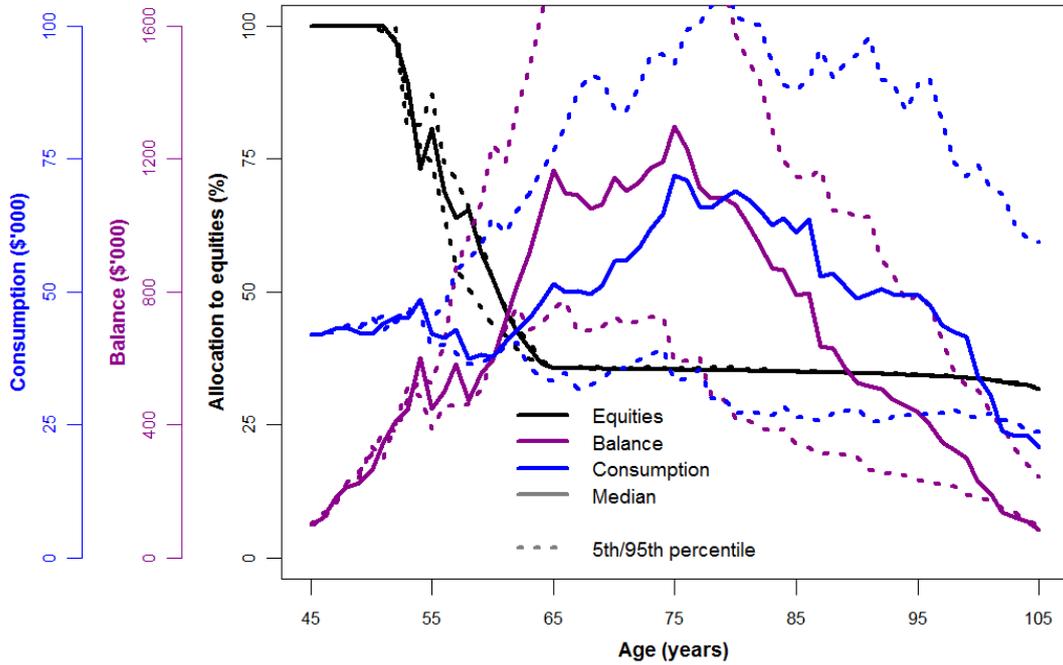


Figure 6B: Percentiles of future results for Age 45 / Balance \$100,000 (by path)

### Projected Outcomes for Age 25 Balance \$0

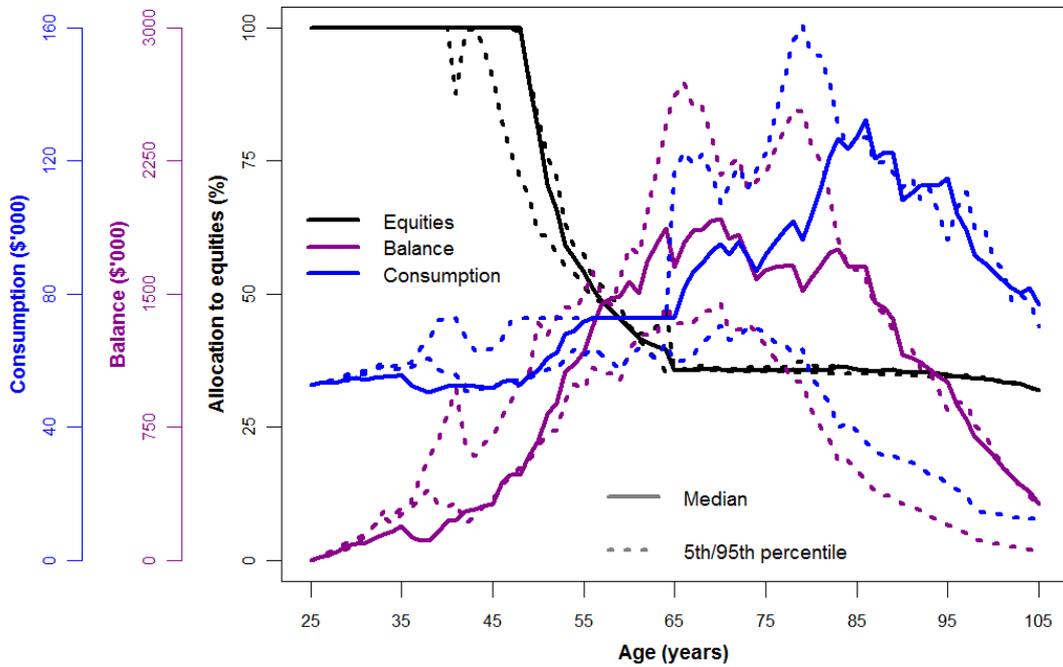


Figure 6C: Percentiles of future results for Age 25 / Balance \$0 (by path)