Calculation of Risk Adjusted Loss Reserves based on Cost of Capital

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Posthuma Partners, The Hague

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Outline

Introduction
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Risk Adjusted Loss in IFM
  Calculation of Risk Adjusted Loss
  Loss-reserving model
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Examples
  TPL Motor
  Fire (real estate) portfolio

Conclusions
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Risk Adjusted Loss based on Cost of Capital

- Demanded by IFRS and by Solvency II
Risk Adjusted Loss based on Cost of Capital

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- Method for calculating the market value of insurance liabilities
Risk Adjusted Loss based on Cost of Capital

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- Method for calculating the market value of insurance liabilities
- Idea: the party taking over the portfolio wants to be compensated for the missed remuneration of the resilient capital, and for the expected payments it needs to make
Risk Adjusted Loss in IFM
Calculation of Risk Adjusted Loss

Let $B(t_1, t_2)$ denote the future loss payments in $[t_1, t_2]$ and suppose we have

$$E[B(t_1, t_2)] = \int_{t_1}^{t_2} b(s) \, ds$$

and

$$P(B(t_1, \infty) \geq V(t)) = 0.005.$$
Calculation of Risk Adjusted Loss

Let $B(t_1, t_2)$ denote the future loss payments in $[t_1, t_2]$ and suppose we have

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- a function \( b(t) \) such that
  \[
  \mathbb{E} B(t_1, t_2) = \int_{t_1}^{t_2} b(s) \, ds
  \]

- a function \( V(t) \) such that
  \[
  \mathbb{P}(B(t, \infty) \geq V(t)) = 0.005.
  \]

Note At time \( t \) the insurer must have a minimal resilient capital of \( V(t) \). This part of his capital cannot be used to yield more than the risk-free rate.
Calculation of Risk Adjusted Loss

Now suppose the liability is transferred to a third party. Denote

- $R_f$ risk free rate
- $R_t$ total rate = $R_{coc} + R_f$
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We will regard the third party’s minimum price $E(t)$ as a fund which is (partly) available for investment, and from which the future losses are paid. This fund changes in a short time period $\Delta t$ in the following way:
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- The Resilient Capital $V(t)$ generates a risk-free interest $R_f \cdot \Delta t$. 
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- The rest $E(t) - V(t)$ generates the total interest $R_t \cdot \Delta t$.
- The loss payment $-b(t) \cdot \Delta t$. 
Calculation of Risk Adjusted Loss

This leads to the following differential equation for $E(t)$:

$$\frac{dE}{dt} = (E(t) - V(t))R_t(t) + V(t)R_f(t) - b(t).$$
Calculation of Risk Adjusted Loss

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$$\frac{dE}{dt} = (E(t) - V(t))R_t(t) + V(t)R_f(t) - b(t).$$

In the end all liabilities have been winded down; no further loss payments are foreseen and no further resilient capital is needed. To determine a “fair price” we therefore take the boundary condition $E(\infty) = 0$. 
Calculation of Risk Adjusted Loss

Solving this equation, we find

\[ E(t) = \int_t^\infty (V(s)R_{CoC} + b(s)) \exp \left( - \int_t^s R_t(u) \, du \right) \, ds \]

and in particular, we find the Risk Adjusted Loss \( E(0) \).
Interpretation

\[ E(t) = DFL(t) + DMR(t) \]

The fund \( E(t) \) consists of reimbursements for:
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\( DFL(t) \): future loss payments discounted at \( R_t \) (total rate)

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**DMR(t):** missed remuneration on Resilient Capital also discounted at \( R_t \) (total rate)

\[ DMR(t) = \int_{t}^{\infty} V(s) R_{coc} \exp \left( - \int_{t}^{s} R_t(u) \, du \right) \, ds. \]
A brief recapitulation of the IFM loss-reserving model to answer the question of how to find $b()$ and $V()$
Picture of the model

1. incurred
   → development duration
   ↓ loss
   ↓ period
   ↓ known
   ↓ unknown
   ↓ Σ

2. paid
   → development duration
   ↓ loss
   ↓ period
   ↓ Σ

duration function

Σ

total loss
exposure

time series analysis
Initial definitions

We will use the following notation to define our model for the paid and the incurred run-off tables.
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\[ Y_{1k}^{(1)} \] indicates the incremental incurred.
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Initial definitions

We will use the following notation to define our model for the paid and the incurred run-off tables.

- \( l \) indicates the loss period.
- \( k \) indicates the development period.
- \( Y_{lk}^{(1)} \) indicates the incremental incurred.
- \( Y_{lk}^{(2)} \) indicates the incremental paid.

Our goal is to model the vector \( (Y^{(1)}, Y^{(2)}) \), including all future values.
Auxiliary variables

We start with auxiliary independent Gaussian random variables:

\[
Z_{lk}^{(1)} \sim N \left( \mu_{lk}^{(1)}, V_{lk}^{(1)} \right)
\]

\[
Z_{lk}^{(2)} \sim N \left( \mu_{lk}^{(2)}, V_{lk}^{(2)} \right)
\]
Auxiliary variables

We start with auxiliary independent Gaussian random variables:

\[ Z_{lk}^{(1)} \sim \mathcal{N} \left( \mu_{lk}^{(1)}, \sigma_{lk}^{(1)} \right) \]

\[ Z_{lk}^{(2)} \sim \mathcal{N} \left( \mu_{lk}^{(2)}, \sigma_{lk}^{(2)} \right) \]

Now, define the event

\[ R = \left\{ \sum_k Z_{lk}^{(1)} = \sum_k Z_{lk}^{(2)} \quad (\forall l) \right\}. \]

This says that for each loss period, the total amount incurred equals the total amount paid.
Final step

Finally we define the incremental losses by

\[ Y^{(1)} \sim Z^{(1)} \mid R \]
\[ Y^{(2)} \sim Z^{(2)} \mid R \]

This means that \((Y^{(1)}, Y^{(2)})\) is normally distributed, and that the row sums of the two tables are always equal.
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This means that \((Y^{(1)}, Y^{(2)})\) is normally distributed, and that the row sums of the two tables are always equal.

Means and covariances of \((Y^{(1)}, Y^{(2)})\) are functions of \(\mu_{lk}^{(i)}\) and \(V_{lk}^{(i)}\). 
Product structure of parameters

To reduce the number of parameters we write:

\[ \mu_{lk}^{(i)} = w_l \exp((X\beta)_l)\pi_k^{(i)} \]

- \( w_l \) exposure
- \( X\beta \) time series of ultimate loss ratio
- \( \pi_k^{(i)} \) fraction of ultimate loss paid in \( k \)
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\[ V_{lk}^{(i)} = \sigma^{(i)} w_l \exp((X\beta)_l)\tilde{\pi}_k^{(i)} \]
Product structure of parameters

To reduce the number of parameters we write:

- $\mu_{lk}^{(i)} = w_l \exp((X\beta)_l)\pi_k^{(i)}$
  - $w_l$ exposure
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  - $\pi_k^{(i)}$ fraction of ultimate loss paid in $k$

- $V_{lk}^{(i)} = \sigma^{(i)} w_l \exp((X\beta)_l)\tilde{\pi}_k^{(i)}$

Parameters $\pi_k^{(i)}$ and $\tilde{\pi}_k^{(i)}$ are summarized via some function $f(; \theta)$. For example Weibull pdf, Beta pdf or more advanced functions.
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1. It is flexible in aggregating various data sets and in handling aggregation levels of the input data.
2. It handles loss triangles with missing cells with ease.
Our joint model for paid and incurred losses performs very well on actual data. Moreover, our model has several important advantages in managing “difficult” data:

1. It is flexible in aggregating various data sets and in handling aggregation levels of the input data.
2. It handles loss triangles with missing cells with ease.
3. It incorporates in a proper way negative data coming from negative adjustments to losses.
Our joint model for paid and incurred losses performs very well on actual data. Moreover, our model has several important advantages in managing “difficult” data:

1. It is flexible in aggregating various data sets and in handling aggregation levels of the input data.
2. It handles loss triangles with missing cells with ease.
3. It incorporates in a proper way negative data coming from negative adjustments to losses.
4. Future premium risk is added by extending the exposure measure, which in an integrated way produces future loss in force (Solvency II requires the loss risk on one-year future premium).
Examples
Two lines of business have been modeled in IFM to calculate the Risk Adjusted Loss Reserving based on Cost of Capital:

1. TPL Motor injuries (long tail)
2. Fire Real estate (short tail)

Excel sheet data available in:
www.posthuma-partners.nl (See Library > Examples in Menu)
Examples loss triangles CAS Denver CLRS2012.xls
TPL Motor portfolio

Data:
- Incurred (paid + case reserves)
- Paid triangles

Loss periods:
- 1998Q1 - 2011Q3

One-year future premium:
- 2011Q4 - 2012Q3 $ 10,000
TPL Motor portfolio

![TPL Motor portfolio diagram](image-url)
## TPL Motor portfolio

<table>
<thead>
<tr>
<th></th>
<th>loss on earned premium</th>
<th>including one-year future premium ($ 10,000)</th>
<th>loss on one-year future premium (\€ 10,000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR: Loss reserve best estimate</td>
<td>$ 21,204</td>
<td>$ 26,593</td>
<td>$ 5,389</td>
</tr>
<tr>
<td>Time value of money</td>
<td>$ (1,696)</td>
<td>$ (2,061)</td>
<td>$ (365)</td>
</tr>
<tr>
<td>Risk margin based on CoC 7%</td>
<td>$ 5,713</td>
<td>$ 6,695</td>
<td>$ 982</td>
</tr>
<tr>
<td>E(0): Risk adjusted loss</td>
<td>$ 25,221</td>
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<td>DMR(0): Discounted Margin on resilient Capital</td>
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Table: TLP Motor, Loss reserving outcomes including future premium (\$ 10,000)

Time value of money based on zero-risk yield curve Dutch Supervisor
# TPL Motor portfolio

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| Risk share E(0) | 26% | 25% | 20% |

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TPL Motor portfolio

![Graph showing the relationship between CoC-rate and Risk Adjusted Loss](image-url)
Fire (real estate) portfolio

Data:
- Incurred (paid + case reserves)
- Paid triangles

Loss periods:
- 2003Q1 - 2011Q3

One-year future premium:
- 2011Q4 - 2012Q3 $10,000
Fire (real estate) portfolio
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| Time value of money based on zero-risk yield curve Dutch Supervisor |

Table: Fire insurance (real estate), Loss reserving outcomes including future premium ($10,000).
## Fire (real estate) portfolio

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Risk share E(0) = 5/7

8% 10% 11%

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Fire (real estate) portfolio
Conclusions
Take-home message
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- Both Solvency II and IFRS ask for a Risk Adjusted Loss on a Cost of Capital-basis
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- In line with the intuition behind this method we proposed a calculation method for this Risk Adjusted Loss.

\[
E(t) = \int_t^\infty (V(s)R_{CoC} + b(s)) \exp \left( - \int_t^s R_t(u) \, du \right) \, ds
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- This method (and other methods) requires a stochastic model of future payments. IFM is such a model
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- This method (and other methods) requires a stochastic model of future payments. IFM is such a model
- The examples showed that our calculation method in combination with IFM leads to intuitive results
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