

# FROM “BENEFITS” TO “GUARANTEES”: LOOKING AT RISK TRANSFERS IN LIFE ANNUITIES AND PENSION PRODUCTS

Ermanno Pitacco

University of Trieste - Italy

`ermanno.pitacco@econ.units.it`



June 24-26, 2013 Lyon

# Agenda

1. Motivation
2. Life annuities: once upon a time . . .
3. Guarantees and options
4. Building the post-retirement income
5. Some arrangements for the payout phase
6. The payment profile
7. Concluding remarks

# 1 MOTIVATION

Focus on:

- ▷ life annuities provided by occupational pension schemes
- ▷ purchased life annuities

Stressing the need for a shift from traditional actuarial methods to ERM approach, including the *product design*

Looking at risk transfers (annuity provider  $\Leftrightarrow$  annuitants) implied by each specific *guarantee / option* involved by the product

Implicitly allowing for both perspectives:

- annuitants
- annuity provider

Nothing original from a scientific point of view

This presentation only aims at providing a review of products available on the insurance and pension markets, with a special focus on risk transfer features

## 2 LIFE ANNUITIES: ONCE UPON A TIME ...

A long story: two milestones (in the actuarial context) follow

**Edmond Halley** (astronomer,  
mathematician, ...)

*His formula (1693):*

$$a_x = \sum_{h=1}^{+\infty} (1+i)^{-h} \frac{l_{x+h}}{l_x}$$



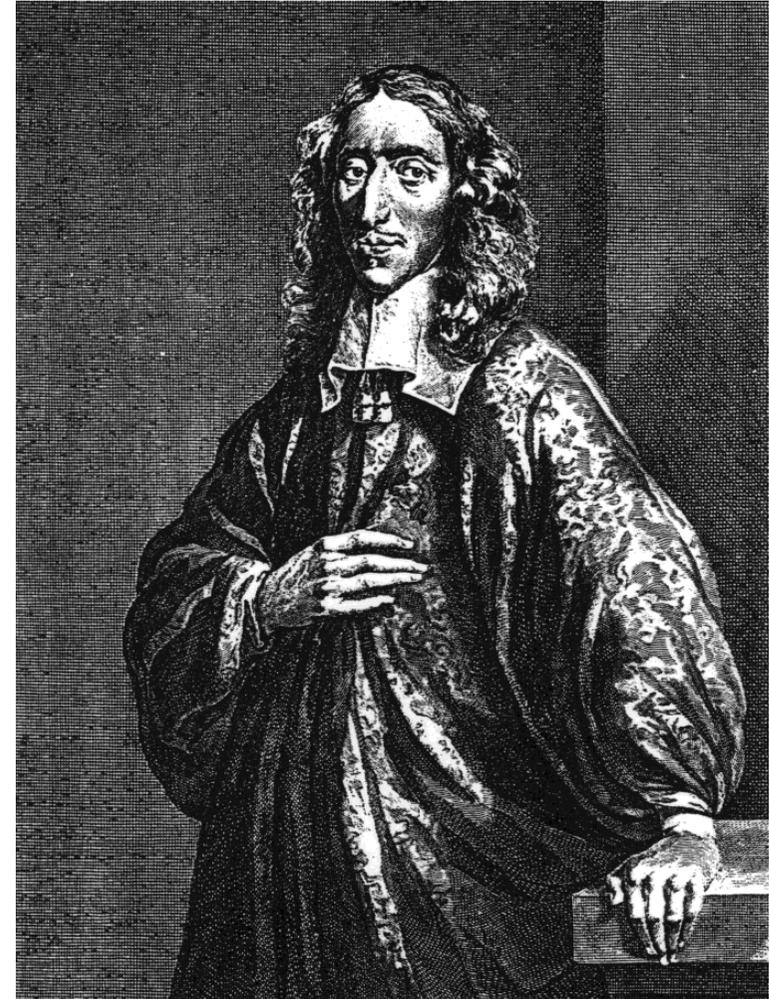
## Life annuities: once upon a time ... (cont'd)

**Jan de Witt** (Dutch prime minister)

His formula (1671):

$$a_x = \sum_{h=1}^{+\infty} a_h \frac{d_{x+h}}{l_x}$$

$$\left( = \mathbb{E}[a_{K_x}] \right)$$



See:

Haberman [1996], Hald [1987], Pitacco [2004]

## Life annuities: once upon a time ... (cont'd)

Both formulae still used in (basic) actuarial practice

Extension to the accumulation phase  $\Rightarrow$  deferred life annuity (with annual benefit  $b$ ):

$$P = b \frac{m|a_x}{\ddot{a}_{x:m}}$$

Note: technical basis chosen at time 0 (age  $x$ )

Under a “market” perspective, several arrangements (beyond the classical structures allowed for by the early formulae) for life annuities proposed to provide a post-retirement income

See, for example:

Kopf [1926], Poterba [1997]

for a historical perspective

### *Two features of the underlying biometric model*

- Deterministic
  - ▷ although relying on probabilities (see:  $\frac{d_{x+h}}{l_x}, \frac{l_{x+h}}{l_x}$ ), only *expected values of benefits* are finally addressed
  - ▷ possible impact of *risks originated by guarantees* (interest, mortality / longevity, etc) not (explicitly) accounted for
  - ▷ just implicit safety loading via adoption of prudential technical bases for premium calculation
- Static (implicitly)
  - ▷ the construction of the (period) life table  $l_x, l_{x+1}, \dots$  from observed mortality rates  $\hat{q}_{x+h}$  relies on the assumption that the age pattern of mortality will not change in the future

### *Remark*

Awareness of mortality trends and relevant impact on life annuities dates back to the beginning of the 20th century: see Nordenmark [1906]

### 3 GUARANTEES AND OPTIONS

#### *Risks in current scenarios*

Current biometric scenario:

- (a) mortality improvements  $\Rightarrow$  projected life tables
- (b) uncertainty in future mortality trend  $\Rightarrow$  aggregate risk

Other risk sources in the financial scenario

- ▷ volatility in financial markets
- ▷ unknown future trends in interest rates
- ▷ inflation ...

Focus on biometric risks, and in particular aggregate longevity risk (i.e. non-diversifiable via pooling inside the traditional insurance - reinsurance process)

See, for example:

Pitacco et al. [2009]

and references therein



## Guarantees and options (*cont'd*)

Many modern insurance and pension products designed as *packages*, whose items may be either included or not in the product actually purchased by the client

Interesting examples provided by:

- endowment insurance which can include various rider benefits and options
- Universal Life insurance
- Variable Annuities
- other insurance or financial products which eventually aim at constructing a post-retirement income
- presence of possible Long Term Care benefits in pension products (e.g. uplift of the annuity benefit in the case of LTC claim)

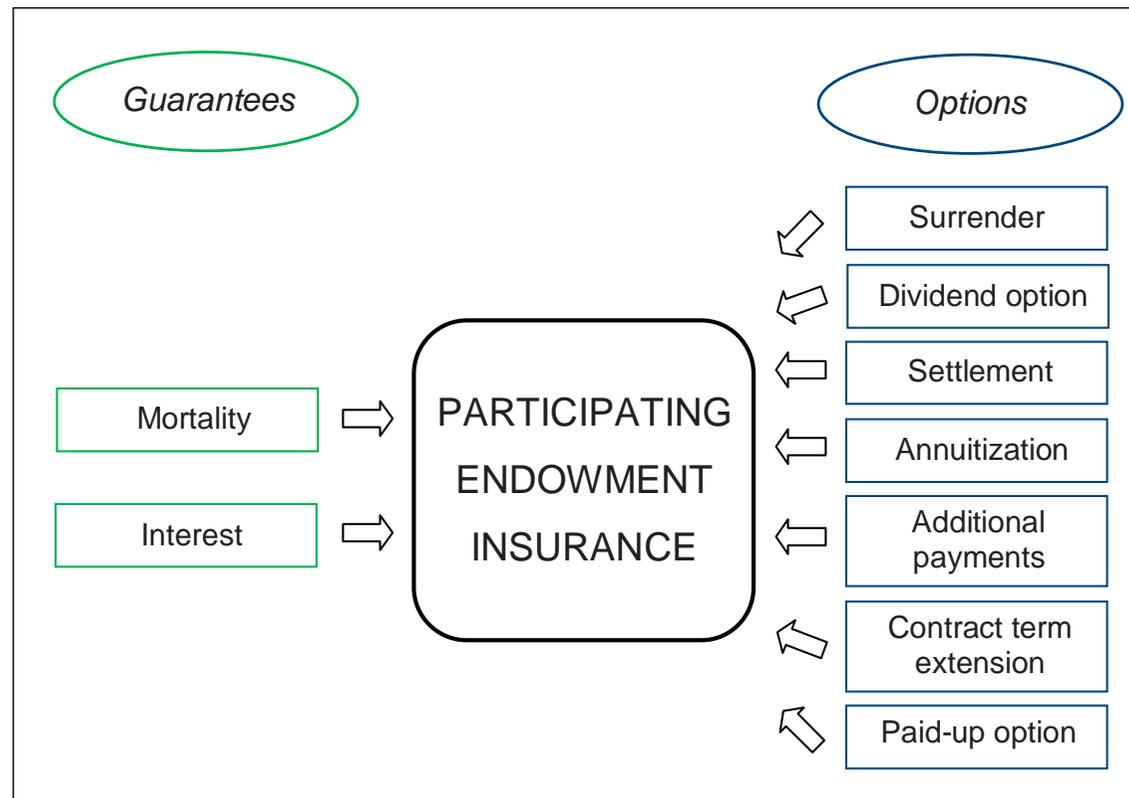
⇒ Look at insurance and pension products as *packages of guarantees and options*

See, for example:

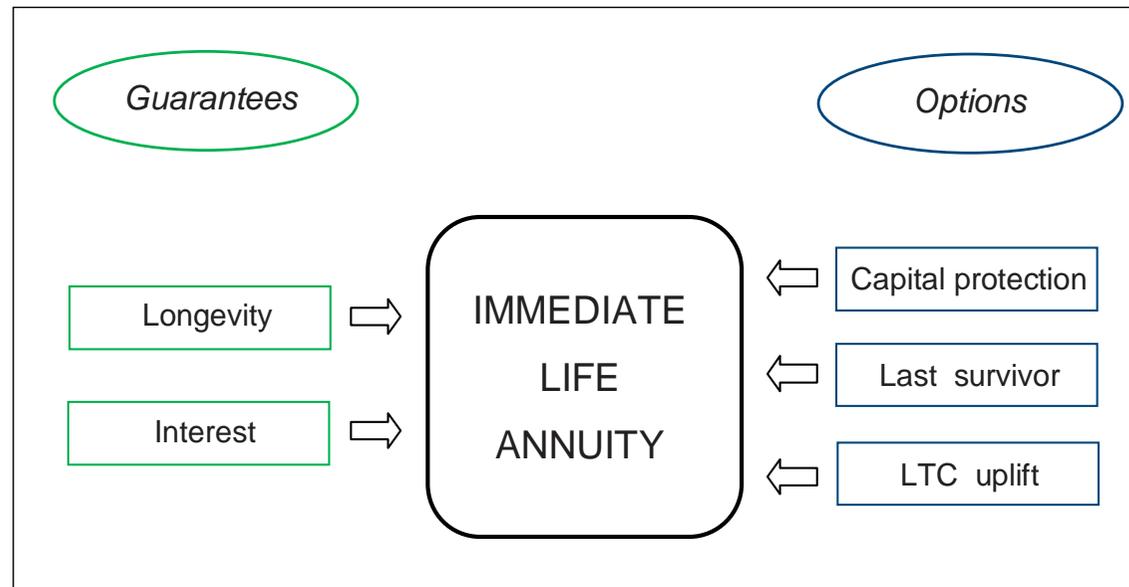
Black and Skipper [2000], Gatzert [2009], Hardy [2004], Pitacco [2012]  
and references therein

## Some examples

### Endowment Insurance

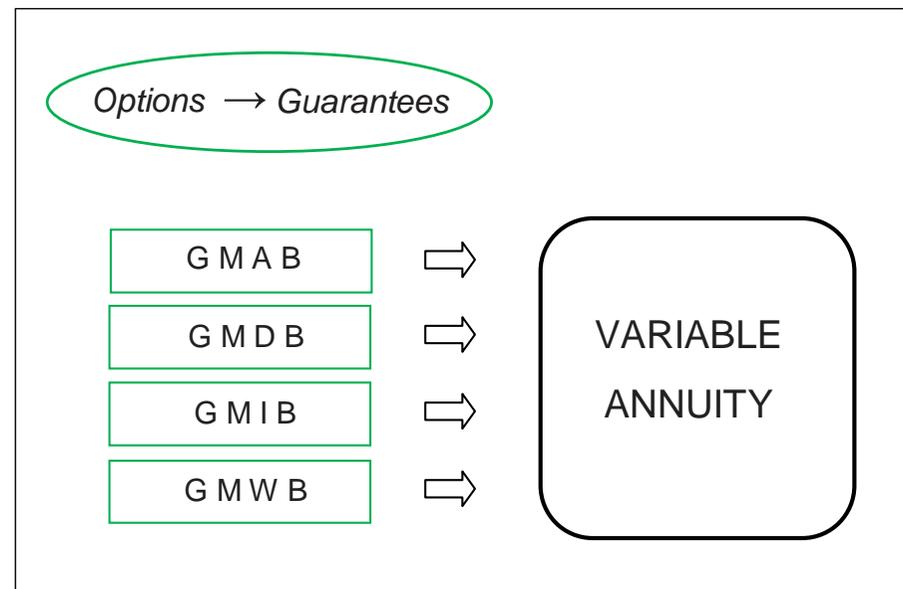


### Immediate Life Annuity



## Guarantees and options (cont'd)

In Variable Annuity products the presence of guarantees follows policyholder's choices



## 4 BUILDING THE POST-RETIREMENT INCOME

### *Introduction*

We describe various arrangements, involving either the accumulation phase, or the payout phase, or both

Various products are available on financial and insurance markets, each product with a specific guarantee structure (conventional life annuities either immediate or deferred, Variable annuities, withdrawal plans, etc.)

See:

Shapiro [2010]

This research provides an extensive literature review of post-retirement financial strategies

See also:

Pitacco et al. [2009], Rocha et al. [2011], Wadsworth et al. [2001]

and references therein, for general issues on life annuities

## Building the post-retirement income (cont'd)

We focus on guarantees provided by each arrangement

Risks taken by the intermediary, in particular the annuity provider (either insurer or pension fund) immediately identified looking at the guarantee structure

In the following figures:

$x$  = age at policy issue, or at entering the pension scheme

$x + r$  = age at retirement



Time at which the guarantee is stated



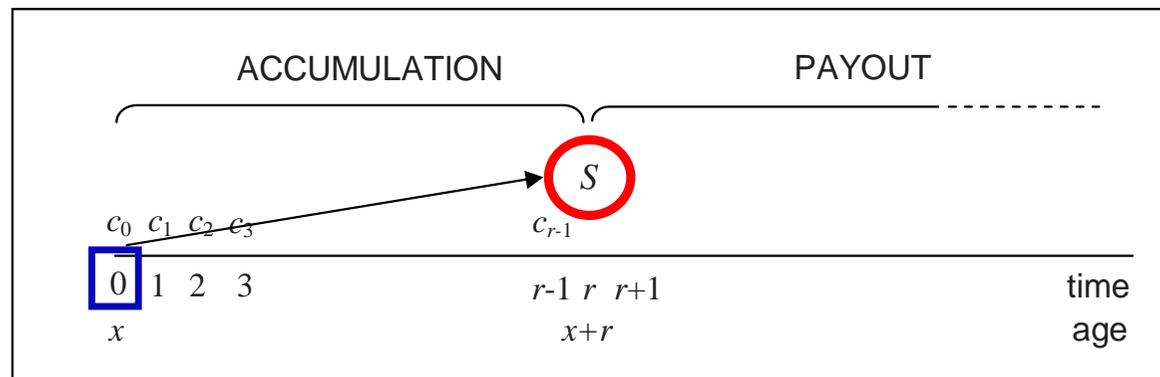
Ultimate object of the guarantee

## Building the post-retirement income (cont'd)

### Some basic structures

#### Structure 1 - Accumulation phase only

For any given sequence of contributions / premiums / savings  $c_0, c_1, \dots, c_{r-1} \Rightarrow$  amount  $S$  guaranteed



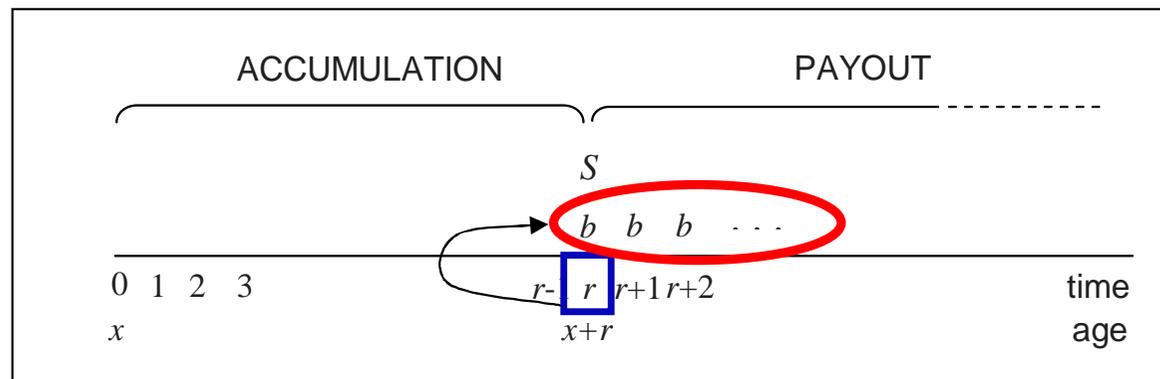
### Examples

- financial product: interest guarantee
- insurance product, e.g. pure endowment insurance or endowment insurance: interest guarantee and mortality guarantee

## Building the post-retirement income (cont'd)

### Structure 2 - Payout phase only

For any given amount  $S \Rightarrow$  annual benefit  $b$  guaranteed (assuming a flat payment profile)

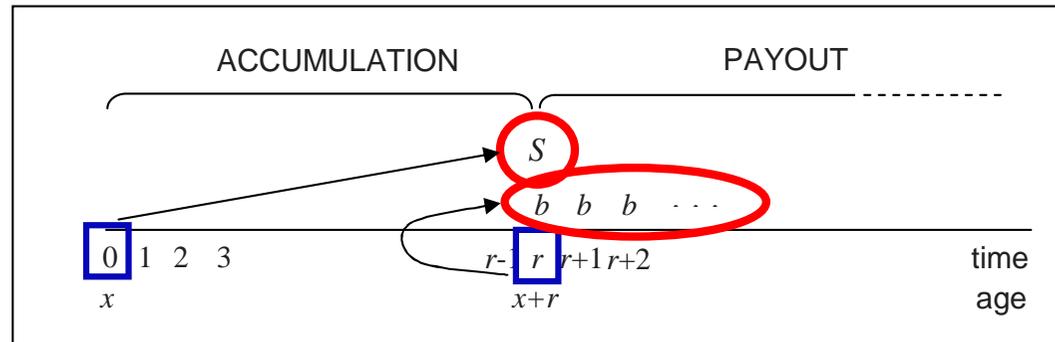


### Examples

- financial product: interest guarantee  $\Rightarrow$  annual benefit  $b$  guaranteed up to fund exhaustion (at a defined time)
- insurance product, i.e. a CAR immediate life annuity: interest guarantee and mortality guarantee  $\Rightarrow$  benefit  $b$  guaranteed lifelong  $\Rightarrow$  *longevity guarantee* (CAR = current annuity rate)

## Building the post-retirement income (cont'd)

Structure 3 - Accumulation phase + Payout phase (combining structure 1 and 2)



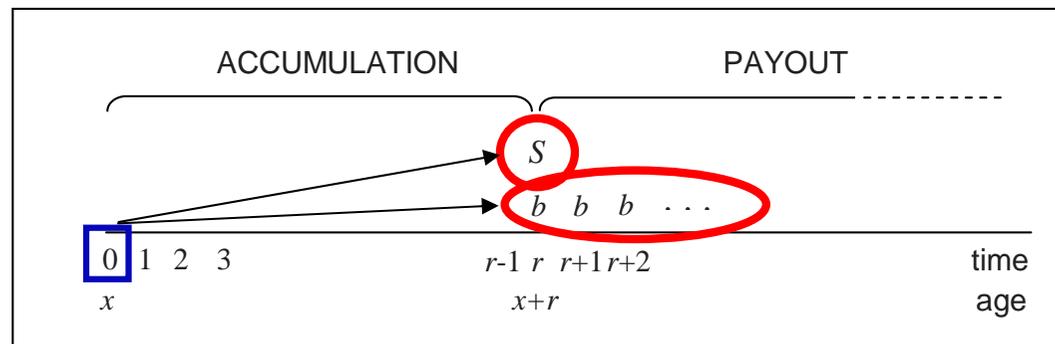
### Examples

- financial product for the accumulation phase: interest guarantee  $\Rightarrow S$  guaranteed
- insurance product, i.e. a CAR immediate life annuity for the payout phase: for any given  $S$ , interest guarantee and mortality guarantee  $\Rightarrow$  benefit  $b$  guaranteed lifelong

## Building the post-retirement income (cont'd)

### Structure 4 - Accumulation phase + Payout phase

All guarantees stated at time 0 (a challenge for the annuity provider !)



### Examples

- GAR deferred life annuity (GAR = guaranteed annuity rate)

#### Remark

Structure implied in particular by the classical actuarial formula

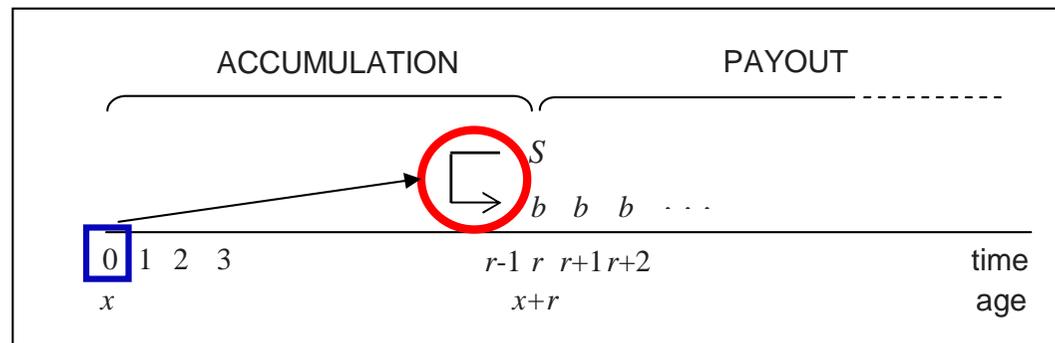
$$P \ddot{a}_{x:r|} = b {}_r| \ddot{a}_x, \text{ where } S = \ddot{a}_{x+r} = \text{mathematical reserve at time } r$$

- financial product with interest guarantee for the accumulation phase and GAR immediate life annuity for the payout phase

## Building the post-retirement income (cont'd)

### Structure 5 - Accumulation phase + Payout phase

Conversion rate stated at time 0



### Example

- financial product for the accumulation phase and immediate life annuity for the payout phase; guaranteed conversion rate

### Remark

In particular: GAO product, providing the options (at retirement):

- ▷ lump sum
- ▷ annuitization at CAR
- ▷ annuitization at GAR

## Building the post-retirement income (*cont'd*)

### *Remark 1*

Assume that the accumulation phase works according to the logic of single recurrent premiums (that is, a particular progressive funding of  $S$ )

Then, guarantees in both Structure 4 and Structure 5 can be weakened by linking the guarantee specification (the accumulation guarantee and/or the conversion rate) to each single recurrent premium

### *Remark 2*

Starting from the basic structures (see above) it is possible to conceive product design by moving in various directions; in particular:

- ▷ reducing the “scope” of some guarantees, viz the longevity guarantee
- ▷ designing a non-guaranteed product, allowing for the inclusion of one or more guarantees, chosen by the client  $\Rightarrow$  Variable Annuities and GMxB

See what follows

## Building the post-retirement income (cont'd)

### **Advanced Life Delayed Annuity (ALDA)**

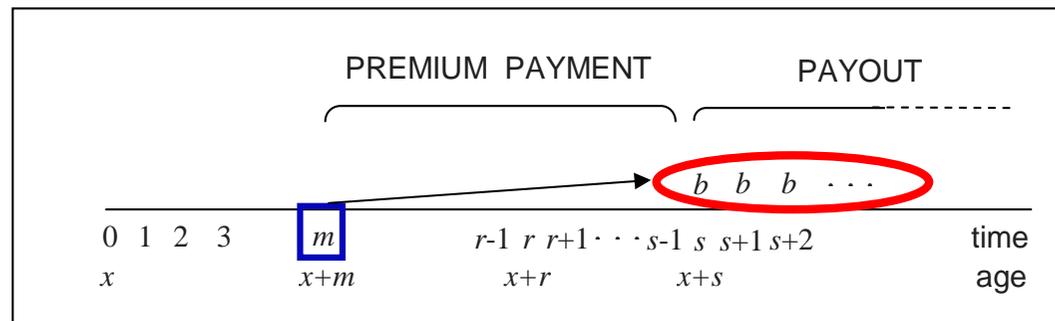
The premium payment period does not necessarily coincide with the (traditional) accumulation phase, being possibly shifted towards older ages

The payout period starts after retirement time (age 80 or 85, say)

⇒ withdrawal from a fund throughout the time interval  $(r, s - 1)$  to get post-retirement income

See:

Milevsky [2005], Gong and Webb [2010], Stephenson [1978]



## Building the post-retirement income (*cont'd*)

See Structure 4, adapted by shifting:  $0 \rightarrow m, r \rightarrow s$

Purposes of ALDA:

- to provide longevity insurance at old ages only (that is, insurance cover with a deductible)
- to pay an inflation-adjusted income
- to reduce premium amount (with respect to conventional deferred annuities)
- to enhance rates of voluntary annuitization

## Building the post-retirement income (cont'd)

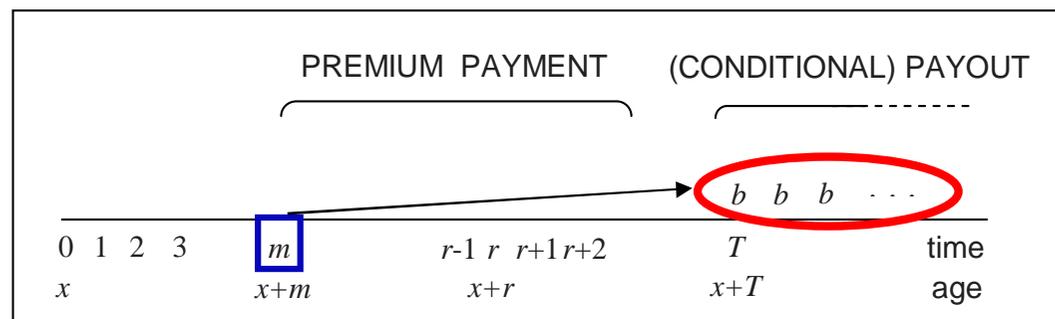
### ***Ruin Contingent Life Annuity (RCLA)***

The post-retirement income is provided by

- (1) withdrawal from a fund from time  $r$  onwards, up to (possible) exhaustion of the fund
- (2) a life annuity paid to the retiree from (random) time  $T$  of fund exhaustion because of “adverse” scenario
  - ▷ poor performance of the fund
  - ▷ long lifetime

See:

Huang et al. [2009]



## Building the post-retirement income (*cont'd*)

RCLA can be thought as

- (a) an ALDA with random delay  $T - r$ , and trigger given by the scenario
- (b) an insurance product generating annuitization as a worst case scenario

Pricing RCLA  $\Rightarrow$  need for constructing a pseudo-index, accounting for

- the behaviour of a market performance index
- a set of reasonable withdrawal rates throughout the payout phase

## Building the post-retirement income (*cont'd*)

### ***Variable Annuities (VA)***

An investment product (throughout the accumulation phase), then providing a post-retirement income

No guarantee is implicitly embedded

Various guarantees (GMxB = Guaranteed Minimum Benefit of type x) can be chosen by the client and then included

See, for example:

Bacinello et al. [2011], Kalberer and Ravindran [2009], Pitacco [2012] and references therein

Including guarantees logically results in structures we have defined above

In what follows we disregard the Guaranteed Minimum Death Benefit (GMDB)

## Building the post-retirement income (cont'd)

Let  $F_t$  denote the balance (fund value) at time  $t$

*Guaranteed Minimum Accumulation Benefit (GMAB)*  
(referring for simplicity to a single premium  $\Pi$ )

- *return of premiums*  $G_r^{[A]} = \Pi$
- *roll-up guarantee*  $G_r^{[A]} = \Pi (1 + i')^r$
- *ratchet guarantee*  $G_r^{[A]} = \max_{t_h < r} \{F_{t_h}\}$

where  $t_h, h = 1, 2, \dots$  are stated times

- *reset guarantee*  $G_r^{[A]} = F_{\max\{t_j: t_j < r\}}$   
where  $t_j, j = 1, 2, \dots$  are the stated reset times

See Structures 1, 3 and 4:

$$S \geq G_r^{[A]}$$

## Building the post-retirement income (cont'd)

### *Guaranteed Minimum Income Benefit (GMIB)*

Provides a life annuity, i.e. a lifelong post-retirement income

Two possible arrangements

(1) Amount to annuitize; then

$$b^{[I]} = \frac{1}{\ddot{a}_{x+r}^{[CAR]}} \max\{F_r, G_r^{[I]}\}$$

where  $G_r^{[I]}$  can be defined as  $G_r^{[A]}$

See Structure 3:

$$S \geq G_r^{[I]}$$

## Building the post-retirement income (cont'd)

(2) Annuitization rate; then

$$b^{[I]} = F_r \max \left\{ \frac{1}{\ddot{a}_{x+r}^{[CAR]}}, \frac{1}{\ddot{a}_{x+r}^{[GAR]}} \right\}$$

Guarantee aka GAO

See Structure 5

In principle, the two guarantees can be combined; in practice, resulting product very expensive, because of insurer's huge risk

(3) Amount & annuitization rate; then

$$b^{[I]} = \max\{F_r, G_r^{[I]}\} \max \left\{ \frac{1}{\ddot{a}_{x+r}^{[CAR]}}, \frac{1}{\ddot{a}_{x+r}^{[GAR]}} \right\}$$

See Structure 4

## Building the post-retirement income (*cont'd*)

### *Guaranteed Minimum Withdrawal Benefit (GMWB)*

Guaranteed benefits even in the case of fund exhaustion because of

- ▷ poor investment performance
- ▷ long lifetime

The guarantee affects both

- benefit amount
- benefit duration
  - (i) fixed
  - (ii) fixed provided that the retiree is alive
  - (iii) lifelong

Guaranteed duration (iii)  $\Rightarrow$  logical structure of RCLA

## 5 SOME ARRANGEMENTS FOR THE PAYOUT PHASE

Basic features of the life annuity product

1. The life annuity relies on the mutuality mechanism; hence:
  - (a) amounts released by the deceased annuitants are shared among the annuitants still alive  $\Rightarrow$  mortality credits
  - (b) on the annuitant's death, her / his estate not credited with any amount (no bequest available)
2. A life annuity provides an “inflexible” income (annual amounts cashed by the annuitant must be in line with the payment profile, as stated by policy conditions, or by pension plan rules)

Features 1(b) and 2: possibly perceived as disadvantages  $\Rightarrow$  weaken propensity to immediately annuitize the whole amount available at retirement.

## Some arrangements for the payout phase (cont'd)

Disadvantages can be mitigated:

- ▷ purchasing a particular product (life annuity + other benefits)
- ▷ adopting a specific annuitization strategy

### ***Life Annuity with a Guarantee Period***

Temporary annuity-certain (throughout the guarantee period)  
+ deferred life annuity

	Guarantee period		
	0	5	10
$x + r = 65$	18 070	18 131	18 386
$x + r = 70$	15 265	15 376	15 832

*Single premium at retirement age;  $b = 1\,000$*

## Some arrangements for the payout phase (cont'd)

### ***Value-Protected Life Annuity (i.e. with “capital protection”)***

In case of early death of the annuitant  $\Rightarrow$  difference (if positive) between single premium and cumulated benefits paid to the annuitant is paid to the beneficiary

Usually, capital protection expires at some given limit age

	Limit age		
	70	75	80
$x + r = 65$	18 596	19 213	19 807
$x + r = 70$	15 265	16 062	16 936

*Single premium at retirement age;  $b = 1\,000$*

## Some arrangements for the payout phase (cont'd)

### *Remark*

In both the products *Life Annuity with a Guarantee Period* and *Value-Protected Life Annuity* the extra-premium is small or very small, depending on the extension of the rider benefit

Obvious reason: the mortality in the age intervals involved is small or very small

Under the annuity provider's perspective: capital protection (i.e. a death benefit) does not provide an effective hedge against the (aggregate) longevity risk

*Natural hedging* of the aggregate longevity risk (both *across LOBS* and *across time* as well) remains a difficult issue !

## Some arrangements for the payout phase (cont'd)

### Life Care Annuity

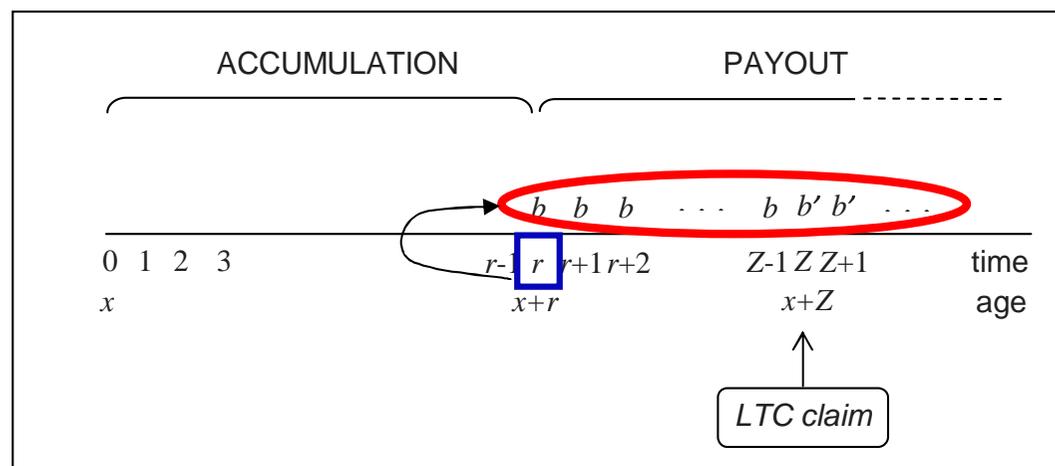
A health-related product: in the case of Long Term Care need  $\Rightarrow$  shift from the basic benefit  $b$  to  $b'$  ( $b' > b$ )

See, for example:

Warshawsky [2007], Zhou-Richter and Gründl [2011]

and references therein

Life annuity with benefit  $b' - b \Rightarrow$  logical structure of RCLA conditional on health status (but different financial structure !)



## Some arrangements for the payout phase (*cont'd*)

Purpose: to reduce the prevailing risk feature of the stand-alone LTC annuity

A further option: financing the uplift via reduction of the basic benefit  
⇒ *Enhanced pension*

See, for example:

Haberman and Pitacco [1999]

and references therein

### ***Packaging LTC annuity and ALDA***

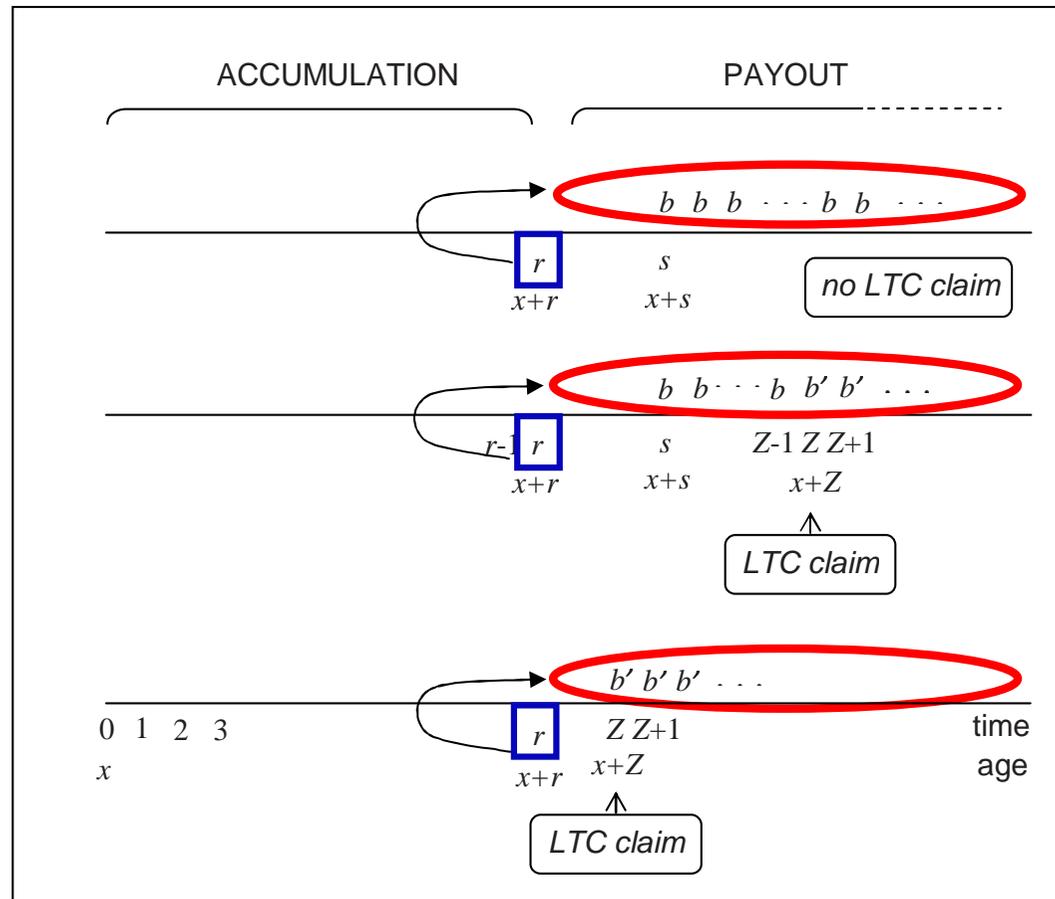
Insurance package including:

- (1) LTC annuity
- (2) deferred life annuity (e.g. from age 80), while the insured is not claiming LTC benefits

See following Figure

Another example of product design aiming at a reduction of the prevailing risk feature of the stand-alone LTC annuity

## Some arrangements for the payout phase (cont'd)



## Some arrangements for the payout phase *(cont'd)*

### *Remark*

When a Life Care annuity or a LTC annuity is involved, a specific type of aggregate longevity risk is taken by the annuity provider, inherent the lifetimes of elderly people claiming for LTC

Various theories concerning the relation between trend in expected total lifetime and trend in expected healthy lifetime

See, for example:

Olivieri and Ferri [2003]  
and references therein

## Some arrangements for the payout phase (*cont'd*)

### ***Progressive annuitization***

See:

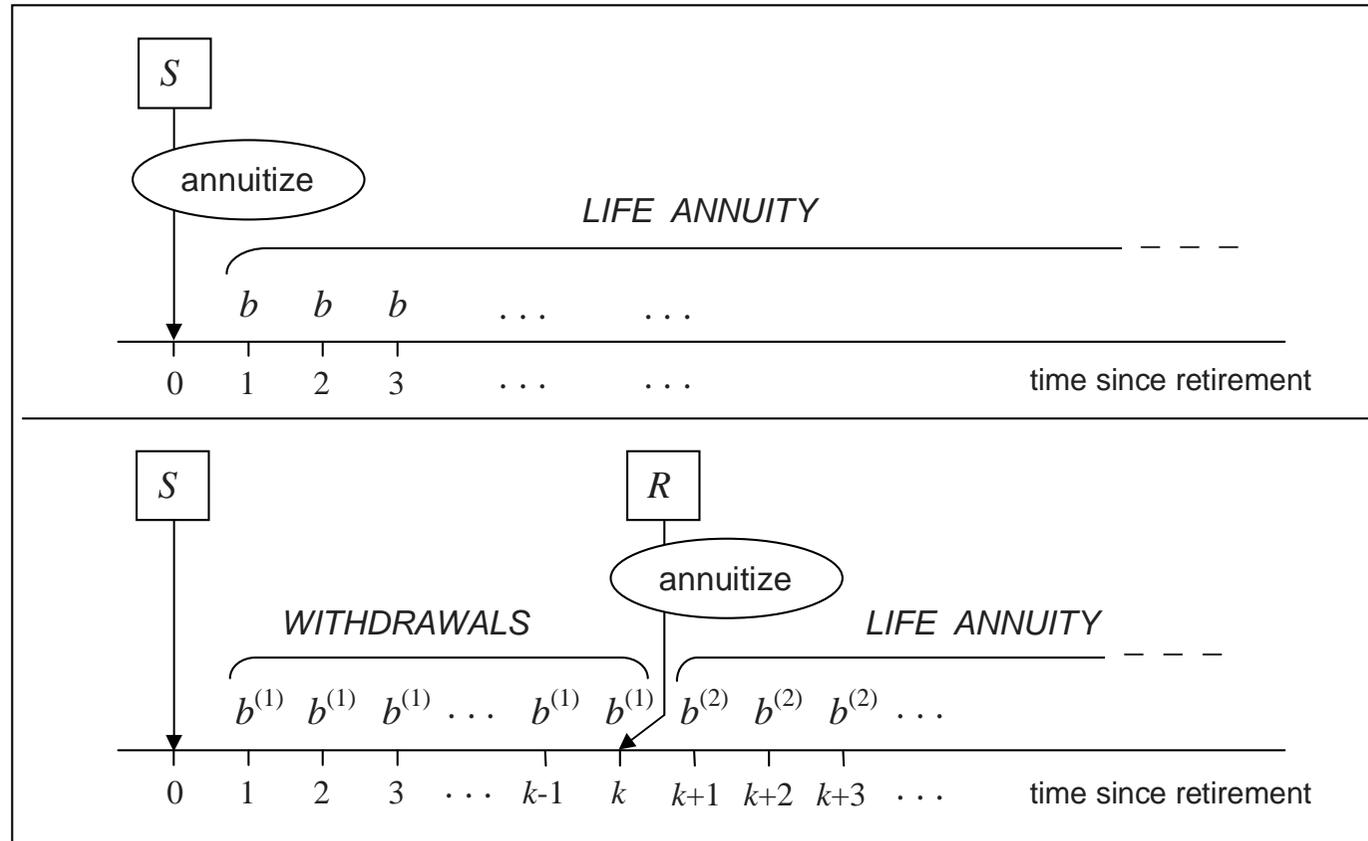
Blake et al. [2003], Horneff et al. [2008], Milevsky and Young [2002]

Assume that, at time of retirement, amount  $S$  available to the retiree

The retiree can choose between two alternatives:

- (1) to purchase an immediate life annuity, with annual benefit  $b$  (i.e. to annuitize amount  $S$ ); see Figure, upper panel
- (2) to leave amount  $S$  in a fund, and then
  - (a) withdraw the amount  $b^{(1)}$  at times  $h = 1, 2, \dots, k$  (say, with  $k = 5$  or  $k = 10$ )  $\Rightarrow$  temporary withdrawal process
  - (b) convert at time  $k$  the remaining amount  $R$  into an immediate life annuity with annual benefit  $b^{(2)}$   $\Rightarrow$  delayed annuitization (provided she / he is alive); see Figure, lower panel

## Some arrangements for the payout phase (cont'd)



*Immediate annuitization versus delayed annuitization*

## Some arrangements for the payout phase (cont'd)

Advantages of delay in the purchase of the life annuity:

- in the case of death before time  $k$ , the fund available constitutes a bequest
- more flexibility gained, as the annuitant may change her / his income profile modifying the withdrawal sequence (however, with possible change in the fund available at time  $k$ )

Disadvantages:

- a higher interest rate than that provided by the annuity, to recover the absence of mortality credits (i.e. absence of mutuality)
- risk of a shift to a different life table in the pricing basis  
⇒ conversion rate at time  $k$  possibly less favorable to the annuitant
- if  $k$  is high, difficult to gain the required yield avoiding too risky investments

## Some arrangements for the payout phase (cont'd)

Interest rate  $g(k)$  needed to recover mortality credits lost in  $(0, k)$   
( $i = 0.02$ )

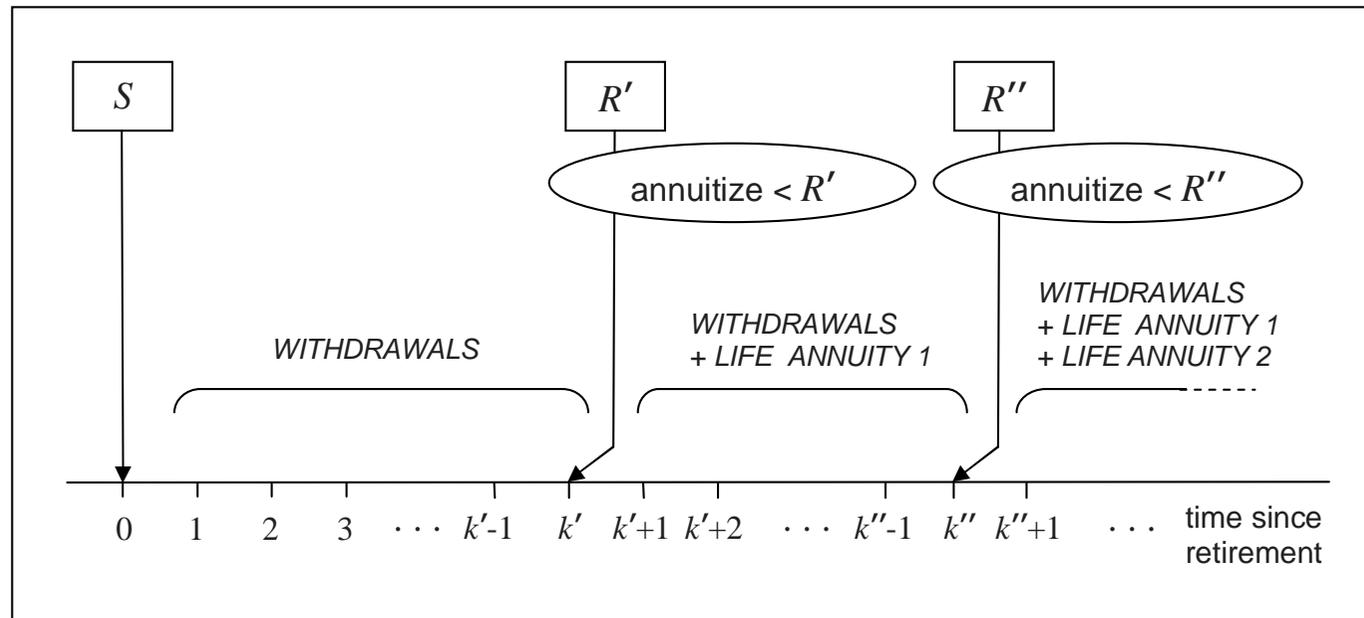
$k$	$g(k)$
5	0.02748
10	0.03009
15	0.03336
20	0.03718

Interest rate  $g(k) \Leftrightarrow b^{(1)} = b^{(2)} = b$

Delayed annuitization  $\Rightarrow$  trade-off between mortality risk and financial risk (and longevity risk as well, because of possible change in the annuitization rate)

## Some arrangements for the payout phase (cont'd)

A more general arrangement:



*Staggered annuitization*

## 6 THE PAYMENT PROFILE

So far we have focussed on *level annuities*  $\Rightarrow$  income which is constant in nominal terms

A number of models of “varying” annuities have been derived, mainly with the purpose of protecting the annuitant against the loss of purchasing power because of inflation

In particular:

1. *Fixed-rate escalating annuities (or constant-growth annuities)*
2. *Index-linked annuities*
  - 2.a Inflation-linked annuities
  - 2.b Equity-indexed annuities
3. *Investment-linked annuities*
  - 3.a With-profit annuities (UK)
  - 3.b Annuities with profit participation mechanisms
  - 3.c Unit-linked annuities

## The payment profile (*cont'd*)

Participation mechanisms (3.b) can involve both financial and mortality experience

Possible problem: poor mortality experience because of unexpected increase in longevity  $\Rightarrow$  aggregate longevity risk

In traditional life annuity and pension design, the longevity risk is borne by the annuity provider

Alternative product design  $\Rightarrow$  transfer part of the longevity risk to the annuitants  $\Rightarrow$  definition of a *longevity-linked life annuity*

### **Sharing the (aggregate) longevity risk**

Formally: *Adjustment process*  $\Rightarrow$  benefit  $b_t$  due at time  $t$ :

$$b_t = b_0 \alpha_t^{[m]}$$

with  $\alpha_t^{[m]}$  = coefficient of adjustment over  $(0, t)$ , according to mortality trend measure  $[m]$

Coefficient  $\alpha_t^{[m]}$  can incorporate investment profit participation  
 $\Rightarrow$  longevity loss can be offset by investment profit

Various interesting contributions regarding practicable models for the adjustment process and the measure  $[m]$

See:

Denuit et al. [2011], Goldsticker [2007], Kartashov et al. [1996], Lüty et al. [2001], Piggott et al. [2005], Richter and Weber [2011], Rocha et al. [2011], Sherris and Qiao [2011], van de Ven and Weale [2008], Wadsworth et al. [2001]

See also:

Olivieri [2013]

to be presented at this Colloquium

## 7 CONCLUDING REMARKS

Actuarial mathematics and technique traditionally focussed on “benefits” in terms of the relevant expected present value (  $\Rightarrow$  basically, a deterministic approach)

Risks implied by *guarantees* and *options* provided by policy conditions and pension plan rules usually disregarded (or, at least, not explicitly accounted for)

Current scenarios (market volatility and uncertainty in longevity dynamics)  $\Rightarrow$  careful consideration of risks inherent in the life annuity and pension structures

Purpose of this presentation: to focus (according to ERM guidelines) on *risk identification* and possible risk transfers between annuitants and annuity provider

# References

A. R. Bacinello, P. Millossovich, A. Olivieri, and E. Pitacco. Variable annuities: a unifying valuation approach. *Insurance: Mathematics & Economics*, 49(3):285–297, 2011. doi: 10.1016/j.insmatheco.2011.05.003

K. Black, Jr. and H.D. Skipper, Jr. *Life & Health Insurance*. Prentice Hall, 2000

D. Blake, A. J. G. Cairns, and K. Dowd. Pensionmetrics 2: stochastic pension plan design during the distribution phase. *Insurance: Mathematics and Economics*, 33(1): 29–47, 2003

M. Denuit, S. Haberman, and A. Renshaw. Longevity-indexed life annuities. *North American Actuarial Journal*, 15(1):97–111, 2011

N. Gatzert. Implicit options in life insurance: An overview. *Zeitschrift für die gesamte Versicherungswissenschaft*, 98(2):141–164, 2009

R. Goldsticker. A mutual fund to yield annuity-like benefits. *Financial Analysts Journal*, 63(1):63–67, 2007

G. Gong and A. Webb. Evaluating the advanced life deferred annuity - an annuity people might actually buy. *Insurance: Mathematics and Economics*, 46(1):210–221, 2010

## References (cont'd)

S. Haberman. Landmarks in the history of actuarial science (up to 1919). Department of Actuarial Science and Statistics, City University, London. Actuarial Research Paper No. 84, 1996. Available at:

[http://www.cass.city.ac.uk/\\_\\_data/assets/pdf\\_file/0010/37198/84-ARC.pdf](http://www.cass.city.ac.uk/__data/assets/pdf_file/0010/37198/84-ARC.pdf)

S. Haberman and E. Pitacco. *Actuarial Models for Disability Insurance*. Chapman & Hall / CRC, 1999

A. Hald. On the early history of life insurance mathematics. *Scandinavian Actuarial Journal*, (1):4–18, 1987

M. R. Hardy. Options and guarantees in life insurance. In J. Teugels and B. Sundt, editors, *Encyclopedia of Actuarial Science*, pages 1216–1225. Wiley, 2004

W. J. Horneff, R. H. Maurer, and M. Z. Stamos. Optimal gradual annuitization: quantifying the costs of switching to annuities. *The Journal of Risk and Insurance*, 75(4): 1019–1038, 2008

H. Huang, M. A. Milevsky, and T.S. Salisbury. A different perspective on retirement income sustainability: The blueprint for a ruin contingent life annuity (RCLA). *Journal of Wealth Management*, 11(4):89–96, 2009

## References (cont'd)

T. Kalberer and K. Ravindran, editors. *Variable Annuities. A global perspective*. Risk Books, 2009

V. Kartashov, R. Maurer, O. S. Mitchell, and R. Rogalla. Lifecycle portfolio choice with systematic longevity risk and variable investment-linked deferred annuities. National Bureau of Economic Research, Cambridge, MA. Working Paper No. 17505, 1996. Available at: <http://www.nber.org/papers/w17505>

E. W. Kopf. The early history of the annuity. *Proceedings of the Casualty Actuarial Society*, 13(27):225–266, 1926. Available at: <http://www.casact.org/pubs/proceed/proceed26/26225.pdf>

H. Lüty, P. L. Keller, K. Binswangen, and B. Gmür. Adaptive algorithmic annuities. *Mitteilungen der Schweizerischen Aktuarvereinigung*, 2:123–138, 2001

M. A. Milevsky. Real longevity insurance with a deductible: introduction to advanced-life delayed annuities (ALDA). *North American Actuarial Journal*, 9:109–122, 2005

M. A. Milevsky. *The calculus of retirement income*. Cambridge University Press, 2006

## References (cont'd)

M. A. Milevsky and V. R. Young. Optimal asset allocation and the real option to delay annuitization: it's not now-or-never. Pensions Institute, London. Discussion paper PI-0211, 2002

N. V. E. Nordenmark. Über die Bedeutung der Verlängerung der Lebensdauer für die Berechnung der Leibrenten. In *Transactions of the 5th International Congress of Actuaries*, volume 1, pages 421–430, Berlin, 1906

A. Olivieri. Longevity risk and related issues for life annuities and pension business. Presented at AFIR/ERM - PBSS - LIFE Colloquium, Lyon, 2013

A. Olivieri and S. Ferri. Mortality and disability risks in Long Term Care insurance. *IAAHS Online Journal*, (1), 2003. Available at:

[http://www.actuaries.org/IAAHS/OnlineJournal/2003-1/Mortality\\_and\\_Disability\\_Risks\\_in\\_Long\\_Term\\_Care\\_Insurance.pdf](http://www.actuaries.org/IAAHS/OnlineJournal/2003-1/Mortality_and_Disability_Risks_in_Long_Term_Care_Insurance.pdf)

J. Piggott, E. A. Valdez, and B. Detzel. The simple analytics of a pooled annuity fund. *Journal of Risk and Insurance*, 72(3):497–520, 2005. doi: 10.1111/j.1539-6975.2005.00134.x

E. Pitacco. From Halley to “frailty”: a review of survival models for actuarial calculations. *Giornale dell'Istituto Italiano degli Attuari*, 67(1-2):17–47, 2004

## References (cont'd)

E. Pitacco. From “benefits” to “guarantees”: looking at life insurance products in a new framework. CEPAR Working Paper 2012/26, 2012. Available at:

[http://www.cepar.edu.au/media/103403/lecturetext\\_pitacco.pdf](http://www.cepar.edu.au/media/103403/lecturetext_pitacco.pdf)

E. Pitacco, M. Denuit, S. Haberman, and A. Olivieri. *Modelling Longevity Dynamics for Pensions and Annuity Business*. Oxford University Press, 2009

J. M. Poterba. The history of annuities in the United States. Working Paper 6001, National Bureau of Economic Research, 1997. Available at:

<http://www.nber.org/papers/w6001>

A. Richter and F. Weber. Mortality-indexed annuities: Managing longevity risk via product design. *North American Actuarial Journal*, 15(2):212–236, 2011

R. Rocha, D. Vittas, and H. P. Rudolph. *Annuities and Other Retirement Products. Designing the Payout Phase*. The World Bank, Washington DC, 2011

A. F. Shapiro. Post-retirement financial strategies from the perspective of an individual who is approaching retirement age. Technical report, Society of Actuaries’ Pension Section, 2010. Available at:

<http://www.soa.org/research/research-projects/pension/research-post-retire-fin.aspx>

## References (cont'd)

M. Sherris and C. Qiao. Managing systematic mortality risk with group self pooling and annuitisation schemes. ARC Centre of Excellence in Population Ageing Research. Working Paper No. 2011/4, 2011. Available at SSRN:

[http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1791162](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1791162)

J. B. Stephenson. The high-protection annuity. *The Journal of Risk and Insurance*, 45 (4):593–610, 1978

J. van de Ven and M. Weale. Risk and mortality-adjusted annuities. National Institute of Economic and Social Research. London. Discussion Paper No. 322, 2008. Available at:

[http://www.niesr.ac.uk/pdf/290808\\_110826.pdf](http://www.niesr.ac.uk/pdf/290808_110826.pdf)

M. Wadsworth, A. Findlater, and T. Boardman. Reinventing annuities. Presented to the Staple Inn Actuarial Society, 2001. Available at:

[http://www.sias.org.uk/siaspapers/listofpapers/view\\_paper?id=ReinventingAnnuities](http://www.sias.org.uk/siaspapers/listofpapers/view_paper?id=ReinventingAnnuities)

## References (cont'd)

M. J. Warshawsky. The life care annuity - A proposal for an insurance product innovation to simultaneously improve financing and benefit provision for long-term care and to insure the risk of outliving assets in retirement. Georgetown University - Long-Term Care Financing Project. Working Paper No. 2, 2007. Available at:

<http://ltc.georgetown.edu/forum/2warshawsky061107.pdf>,

T. Zhou-Richter and H. Gründl. Life care annuities - Trick or treat for insurance companies? Technical Report, 2011. Available at SSRN:

<http://ssrn.com/abstract=1856994>

*Many thanks for your kind attention*