Solvency assessment within the ORSA framework

Issues and quantitative methodologies

Julien Vedani

ISFA, Université Claude Bernard
Université Lyon 1

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Joint work with L. Devineau (R&D Principal - Milliman Paris)

Outlines

1 Introduction

2 Notion of multi-year solvency

3 Parametric solutions to assess the Overall Solvency Needs

4 Application

5 Conclusion

Own Risk and Solvency Assessment - Quantitative issues

Article 45 - Solvency II directive:

As part of its risk-management system every insurance undertaking and reinsurance undertaking shall conduct its own risk and solvency assessment. That assessment shall include at least the following:

(a) *the overall solvency needs taking into account the specific risk profile, approved risk tolerance limits and the business strategy of the undertaking;*

(b) *the compliance, on a continuous basis, with the capital requirements and with the requirements regarding technical provisions;*

Own Risk and Solvency Assessment - Quantitative issues

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(b) *the compliance, on a continuous basis, with the capital requirements and with the requirements regarding technical provisions;*

- Introduces two highly quantitative issues inherent to the ORSA process
  - Definition and assessment of the Overall Solvency Needs, required capital amount to withstand a solvency constraint coherent with the Risk Appetite
  - Development a process to guarantee the continuous compliance with the regulatory requirements linked to Pillar I

- Main contributions
  - Practical formalization of the multi-year solvency concept
  - Development of implementation tools for the Overall Solvency Needs assessment adapted to life insurance
  - Implementation on a standardized life insurance product

- Selected literature on the subject

[1] BROADIE M., DU Y., MOALLEMI C.C., Efficient Risk Estimation via Nested Sequential Simulation, 2010

[2] DECUPERE S., Agrégation des risques et allocation de capital sous Solvabilité II, 2011

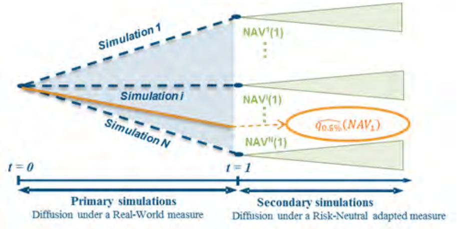
[3] EISELE K.-T., ARTZNER P., Multiperiod insurance supervision: Top-down models, 2011

[4] BONNIN F., PLANCHET F., JUILLARD M., Calculs de best estimate de contrats d'épargne par des formules fermées - Application à l'ORSA, 2012

Regulatory notion of solvency

Let NAV_t be the Net Asset Value at date t, SCR_t be the Solvency Capital Requirement at date t and SR_t be the Solvency Ratio at date t (random variables as soon as $t > 0$)

- Solvency constraint in a one-year framework (regulatory approach)
  - Constraint: $RegulatorySolvency \Leftrightarrow \mathbb{P}[NAV_1 \geq 0] \geq 99.5\%$
  - Practical Issue: assessment of the NAV 0.5% - empiric quantile at 1y (requires the use of Nested Simulations)



- Monte-Carlo approach, two levels of simulation required in the case of an Internal Model to assess $SCR_0 = NAV_0 + K / : K = -VaR_{0.5\%}\left(\delta_1 NAV_1\right)$

Notion of multi-year solvency

ORSA framework – Various interpretation of the multi-year solvency

- Multi-year adaptation of the regulatory constraint (constraint on economic bankruptcy)
  - Question: level of own funds at $t = 0$ required to withstand economic bankruptcy on the whole horizon $[1, T]$ with a p threshold
  - Constraint: $(SC1) \Leftrightarrow \mathbb{P}[\cap_{0<t\leq T}\{NAV_t \geq 0\}] \geq p$

$$RequiredCapital_{SC1} = NAV_0 + K / : K = Argmin_X \left(\mathbb{P}\left[\cap_{0<t\leq T}\left\{NAV_t + \frac{X}{\delta_t} \geq 0\right\}\right] \geq p \right)$$

- Adjustment of the underlying risky variable (constraint on solvency shortfalls)
  - Question: level of own funds at $t = 0$ required to ensure the coverage of at least a level α of the regulatory capital on the whole time horizon with a p threshold
  - Constraint: $(SC2) \Leftrightarrow \mathbb{P}\left[\cap_{0<t\leq T}\left\{\frac{NAV_t}{SCR_t} \geq \alpha\%\right\}\right] \geq p$

$$RequiredCapital_{SC2} = NAV_0 + K / : K = Argmin_X \left(\mathbb{P}\left[\cap_{0<t\leq T}\left\{\frac{NAV_t + \frac{X}{\delta_t}}{SCR_t} \geq \alpha\%\right\}\right] \geq p \right)$$

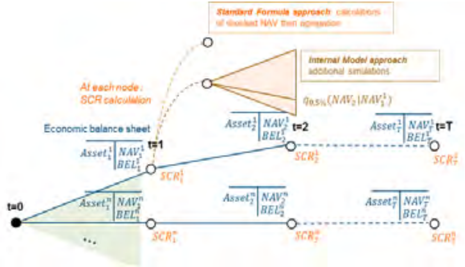
Notion of multi-year solvency

Major practical issue: difficulties to assess the required capital

- In practice this assessment would require multi-year projections of *NAV* / *SCR* empirical outcomes
  - Three levels of Nested Simulations in an Internal Model framework (we add the time-dimensions)
  - If a Standard Formula is considered, only two levels of simulations

→ Unusable in practice, requires the development of proxy methodologies

In this presentation we address the ($SC2$) implementation issue and propose two proxy methodologies adapted to a life insurance



Elementary Risk Factors

Consider the simulations used to assess Monte-Carlo outcomes of *NAV* and *SCR*

- Primary simulations
  - Real World simulations of various economic drivers (stock index, ZCB on various maturities. . .)
  - Relevant to try and synthesize the information embedded in each drivers' diffusion in a minimal number of simplified factors
  - → Notion of Elementary Risk Factors (ERF)
- Elementary risk factor := simple tools that enable one to trace the evolution of the modeled risks on each one-year period $[t-1, t]$
  - Consider a stochastic stock index denoted S_t^i at date t and for scenario i, and a ZCB denoted $ZCB^i(t, m)$ at date t, for scenario i and maturity m

Stock risk factor associated with simulation i and period $[t-1,t]$ → $${}^{S}\varepsilon_t^i = \frac{X_t^i - \hat{E}[X_t]}{\hat{\sigma}(X_t)} \;;\; and \;\; X_t^i = \ln\left(\frac{S_t^i}{S_{t-1}^i}\right)$$

ZC risk factor associated with simulation i, maturity m and periode $[t-1,t]$ → $${}^{ZC}\varepsilon_t^i(m) = \frac{Y_t^i(m) - \hat{E}[Y_t(m)]}{\hat{\sigma}[Y_t(m)]} \;;\; Y_t^i(m) = \frac{ZCB^i(t,m)}{ZCB^i(t-1,m+1)} - \frac{1}{ZCB^i(t-1,1)}$$

Synthesized ZC risk factor associated with simulation i and period $[t-1,t]$ → $${}^{ZC}\varepsilon_t^i = \frac{1}{M}\sum_{m=1}^{M} {}^{ZC}\varepsilon_t^i(m)$$

Parametric proxies

Basic idea: The *NAV* can be approximated by its conditional expectation given the σ-field generated by the ERF

*H*0: This conditional expectation is a polynomial form of the ERF

- Two distinct methodologies can be intuited, the Curve Fitting (CF) and Least Squares Monte-Carlo (LSMC)
- **Same principle, calibration of a polynomial function of the ERF that approximates the *NAV* at each date t.**
- Concept opposition between the Net Asset Value and the Net Present Value of margins (NPV)

The CF (resp. LSMC) methodology consists in the implementation of a multiple linear regression on a small (resp. large) number of $N\hat{A}V_t t$ (resp. $NPV_t t$) outcomes

Parametric proxies: 1-year framework

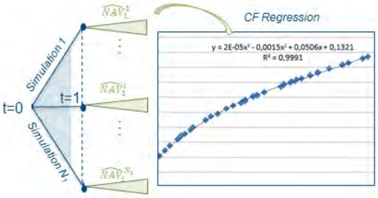
Implementation sequence - case of two risk factors (stocks and IR - level risk)

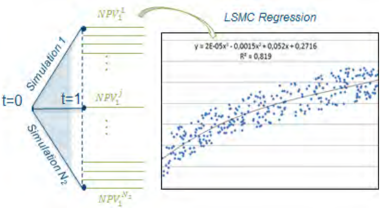
Standard multilinear regression: Calibration of the optimal set of regressors $X_t = (I, {}^1X_t^n, .., {}^kX_t^n)$, with ${}^iX_t = {}^s\varepsilon_t^{x_i}.{}^{ZC}\varepsilon_t^{y_i}$ then determination of $\hat{\beta}_t = ({}^I\hat{\beta}_t, {}^1\hat{\beta}_t, .., {}^k\hat{\beta}_t)'$, the OLS parameters' estimator of the multiple regression $Y_t = X_t.\beta_t + u_t$ where $Y_t = \hat{NAV}_t / NPV_t$.

The underlying assumption of both the CF and LSMC methodologies at each date t can be written

$$\exists \beta_t, \mathbb{E}\left[Y_t \,|X_t\right] = X_t.\beta_t$$

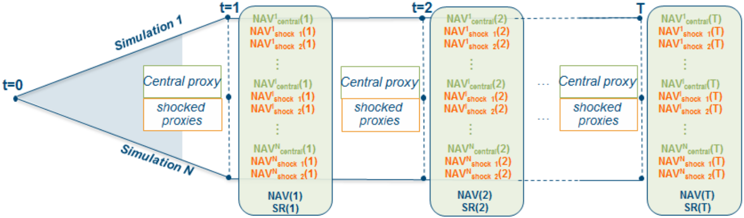
- Illustration at 1 year to replicate the central Net Asset Value
  - CF calibration on N_1 $\hat{NAV}_1$ outcomes
  - LSMC calibration on N_2 NPV_1 outcomes ($N_1 << N_2$)





Parametric proxies: multi-year framework

- Adaptation at *T* years to obtain empirical distributions of *NAV* and *SR*
  - Calibration of one polynomial proxy per considered Standard Formula shock (Stock, IR, spread,. . .) and per projection year
  - Standard Formula aggregation of the approximated *NAV* (central and marginally shocked) to assess the *SCR* and then the *SR*



Theoretical comparison

Equivalence of the optimal parameters

Remark: The $\hat{NAV}$ outcomes considered in the CF framework are Monte-Carlo approximations of the real NAV outcomes (unknown random variable in practice)
One can consider three distinct multi-linear regressions

- **CF regression** - $\hat{NAV}_t = \beta^1.X_t + u_t$, assumption $\mathbb{E}[\hat{NAV}_t \,|X_t] = X_t.\beta^1$
- **LSMC regression** - $NPV_t = \beta^2.X_t + v_t$, assumption $\mathbb{E}[NPV_t \,|X_t] = X_t.\beta^2$
- **Underlying regression** - $NAV_t = \beta.X_t + v_t$, assumption $\mathbb{E}[NAV_t \,|X_t] = X_t.\beta$

Under these assumptions

Result 1: $\beta^1 = \beta^2 = \beta$

Theoretical comparison

Comparison under the asymptotic OLS assumptions

Curve Fitting:
N_1 primary simulations
Secondary tables of P simu.
Algorithmic complexity $N_1.P$

Question?

For a similar asymptotic efficiency: $N_1.P = N_2$?

LSMC:
N_2 primary simulations
1 secondary scenarios
Algorithmic complexity N_2

Recall the three multi-linear regressions

- **CF regression** - $\hat{NAV}_t = \beta^1.X_t + u_t$, assumption $\mathbb{E}[\hat{NAV}_t \,|X_t] = X_t.\beta^1$
- **LSMC regression** - $NPV_t = \beta^2.X_t + v_t$, assumption $\mathbb{E}[NPV_t \,|X_t] = X_t.\beta^2$
- **Underlying regression** - $NAV_t = \beta.X_t + v_t$, assumption $\mathbb{E}[NAV_t \,|X_t] = X_t.\beta$

Equalizing the asymptotic speeds of convergence of the OLS estimators β^1 and β^2 and denoting $\mathbb{E}[\mathbb{V}[NPV_t|\mathcal{F}_t]] = \sigma^2_{NPV_t}$, one obtains

Result 2: $N_2 = N_1.P.\left(\frac{1+\frac{\sigma_w^2}{\sigma^2_{NPV_t}}}{1+P.\frac{\sigma_w^2}{\sigma^2_{NPV_t}}}\right) \leq N_1.P$

Implementation framework

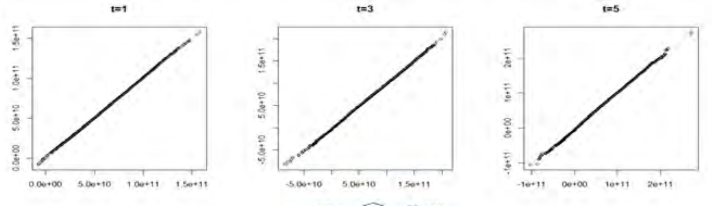
Test on a realistic but simplified framework

- Two major financial risks
  - Stock (level)
  - IR (level)

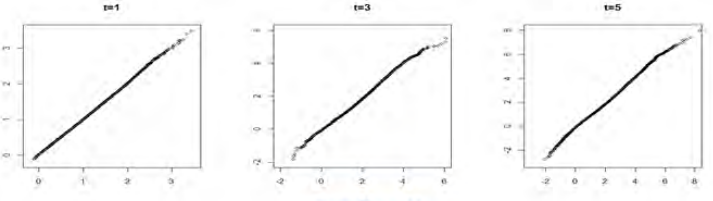
Three implementations

- Reference implementation
  - Projection through 5y and MC calculation of 5000 joint outcomes (central / shocked *NAV*)
  - Secondary tables of 500 scenarios
- Curve Fitting calibration
  - 100 primary scenarios x 500 secondary scenarios
- LSMC calibration
  - 50 000 primary scenarios (x 1 secondary scenario)

Results Curve Fitting

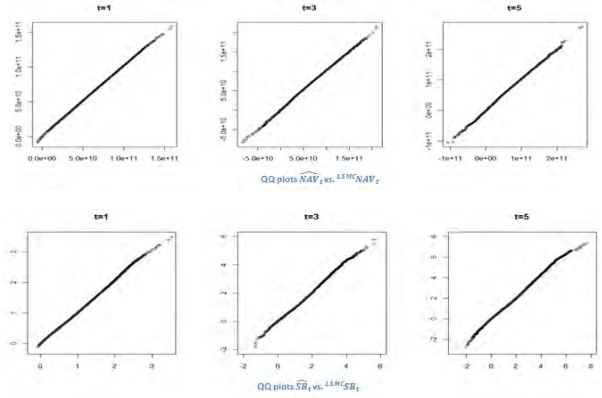


QQ plots $\widehat{NAV}_t$ vs. ${}^{CF}NAV_t$



QQ plots $\widehat{SR}_t$ vs. ${}^{CF}SR_t$

Results LSMC



Example of proxy obtained for the central NAV at $t = 5$

$$\begin{aligned} {}^{LSMC}NAV_5 = I + \alpha_1 {}^{LSMC}NAV_4 + \alpha_2 {}^{LSMC}NAV_4 . {}^{s}\varepsilon_5 + \alpha_3 {}^{s}\varepsilon_5 + \alpha_4 {}^{ZC}\varepsilon_3 + \alpha_5 {}^{s}\varepsilon_5{}^{2} + \alpha_6 {}^{ZC}\varepsilon_5{}^{2} \\ + \alpha_7 {}^{ZC}\varepsilon_5 . {}^{ZC}\varepsilon_4 + \alpha_8 {}^{ZC}\varepsilon_5 . {}^{ZC}\varepsilon_3 + \alpha_9 {}^{ZC}\varepsilon_5 . {}^{ZC}\varepsilon_2 + \alpha_{10} {}^{ZC}\varepsilon_5 . {}^{ZC}\varepsilon_1 + \alpha_{11} {}^{ZC}\varepsilon_1 \end{aligned}$$

Overall Solvency Needs assessment

Relative differences between the assessed required capitals

Multi-year solvency constraint	Curve Fitting	LSMC
$(SC2) : \mathbb{P}\left[\cap_{0<t\leq T}\left\{\frac{NAV_t}{SCR_t} \geq 110\%\%\right\}\right] \geq 85\%$	11.9%	9.3%

One can also consider a constraint on solvency shortfalls that does not take path-dependence into account

Multi-year solvency constraint	Curve Fitting	LSMC
$(SC2bis) : \forall 0 < t \leq T, \mathbb{P}\left[\frac{NAV_t}{SCR_t} \geq 110\%\right] \geq 85\%$	7.9%	6.7%

Remark : During the application we have observed that the relative differences between the $\hat{NAV}$ and the approximated NAV (central / shocked) can partially be explained by the sampling bias introduced by the MC calculations.

Conclusion

- Development of proxy methodologies with theorical limits but great practical interest to assess future values of the central and shocked *NAV*
- The use of CF or LSMC allows to consider the most complex multi-year solvency metrics with fast and satisfactory empirical results

- Future developments
  - Test of the efficiency comparison formula on simple financial instruments
  - Proxy recalibration frequency issue
  - Study of the sample bias impact
  - Assessment of theoretical results in an heteroskedastic framework