Stochastic Simulation of Individual Retirement Accounts in Mexico
(Replacement Rates Comparison)

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Abstract

Reforms in 1997 changed Mexican Social Security System from once designed for being financed through collective capitalization, but in fact it converted to a Pay-as-you-go (PAYGO) system; into a system of individual retirement accounts. Those accounts are managed by pension fund administrators AFORE (for its acronym in Spanish) and every worker may choose one of them. Due variability of markets, regulations, portfolio constraints imposed by the government, AFORE's investment politics and other variables; there are no confident pension estimations at retirement.

Hence, the aim of this paper is give elements to workers for a better choose of AFORE: which has higher probability to give higher pensions at retirement age. The comparison between the administrators is through replacement rates distributions and the probability of not having enough resources to finance the minimum pension guaranteed (MPG) which is subsidy by the Mexican government.

For one replacement rate scenario, an entire labour life is simulated, for this: AFORE's monthly investment returns are modeled by multivariate normal distribution whose parameters are fitted with the maximum likelihood method, the advantage of this model is that keeps the correlation structure between the administrators; for unemployment periods of time, a Markov Chain is used, their parameters represents the contribution density in average that has been seen during the individual account regime; finally at the retirement age is calculated pension and replacement rate by dividing the cumulated fund with an actuarial factor and the last salary respectively. Cases where the MPG is granted can be seen as a Bernoulli random variable, so in order to estimate the probability of exercising the MPG is improved the maximum likelihood estimator. The simulations indicates that probably only two market participants could be the administrators who can give to the workers replacement rates, in average, above the recommendation of International Labour Organization (ILO); in contrast, the others administrators are among the recommendation and the minimum pension guaranteed.

Keywords: Pension funds, individual retirement accounts, replacement rates, contribution density, multivariate normal distribution, minimum pension guaranteed.

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Chapter 1

The Mexican Pension Fund System

The purpose of Pension Fund Systems is protecting workers income and their families in any circumstance, disability, old age and death. In Mexico the major social security institute for workers in private sector is the IMSS, its benefits and pension system have sum importance for the Mexican population.

Originally, the IMSS pension system was designed to be a collective fund financed from contributions from workers, employers and the state, characterized as a partially-funded defined-benefit scheme, accumulating the reserves needed to finance the cost of future pension liabilities. However, in practice it operated as a *Pay-as-you-go* (PAYGO) scheme.

The lack of actuarial reserves for decades made necessary reforms of pension finance scheme. These reforms were in the same line of similar pension reforms which have been previously implemented in other Latin American countries, particularly in Chile, where the new system had been successfully operating since 1981. Reforms in Mexican Social Security Law were made in July 1997, the *Pay-as-you-go* scheme changed for a fully funded with individual accounts system.

2.1 The 1997 Reform

Under the new Social Security Law, private sector workers have a defined contribution system with individual pension accounts managed by specialized companies denominated AFOREs *(for its acronym in Spanish*). These are private entities that compete for funds through interest paid on account and commissions. Each worker may choose his or her personal AFORE. The AFOREs invest retirement savings in the market by using specialized investment societies denominated SIEFOREs, whose main objective is to invest the resources in order to keep their real value. There is also a supervising institution, the CONSAR who guarantees the pension resources are properly managed.

Table 1.1: AFOREs

Afirme Bajío	**Invercap**
Azteca	**Metlife**
Banamex	**PENSIONISSSTE**
Bancomer	**Principal**
Banorte	**Profuturo GNP**
Coppel	**SURA**
Inbursa	**XXI**

Administrators in operation in 2012, source: CONSAR

At moment of study, there were fourteen AFOREs and five SIEFOREs where the worker savings are invested in function of the age. Inversions for younger worker can manage financial instruments with higher risk; meanwhile inversions for older workers are more conservatives.

- Basic SIEFORE 1: 60 years old and older.
- Basic SIEFORE 2: 46 to 59 years old.
- Basic SIEFORE 3: 37 to 45 years old.
- Basic SIEFORE 4: 27 y 36 years old.
- Basic SIEFORE 5: 26 and younger.

Contributions

The individual account is subdivided into tree subaccounts: Retirement account RCV, Volunteer contributions subaccount and Housing subaccount which is managed by INFONAVIT). Contributions are bimonthly and correspond to the next table.

Table 1.2 : Contributions
(Salary Percentage)

Subaccounts	Employer	Employee	Government	Total
RCV	5.15%	1.125%	0.225%	6.5%+ Social Quota*
Volunteer contributions	-	-	-	-
INFONAVIT	5%		-	5%

Percentage of contributions into the individual account, Source: CONSAR.

*Into the RCV subaccount Social quota depends of salary level, the contribution is per worked day and the amount is indexed by the Mexican inflation index IPC. The next table shows the Social coat amount for every salary level when the law was published in 1997.

Table 1.3: Social Quota

Salary	Social Quota per worked day (MXP)
1 Minimum wage	$3.87077
1.01 to 4 Minimum wages	$3.70949
4.01 to 7 Minimum wages	$3.54820
7.01 to 10 Minimum wage	$3.38692
10 to 15 Minimum wage	$3.22564

Source: Mexican Social Security Law

Requirements

At least 1250 recognized labour weeks and 60 years old. If it is recognized more than 750 weeks the integral medical insurance is granted

Pension at Retirement

The new regime consists in contributions into the account from the employer, employee and the government during labour years. The pension at retirement depends of the cumulated fund less commissions. In case the employee accomplishes the retirement requirements an insurance company has to be chosen for purchasing a pension and survival insurance, if the fund is not enough to finance at least the minimum pension guaranteed, that pension is granted. In case the employee does not accomplish the requirements of time of contributions at the age of 65 years old, the employee receives the total amount of the account. The problem is the uncertain amount of these benefits because depends on employee's contribution behavior during his labour period. In later sections it will discuss that point.

Chapter 2

Simulation of Individual Accounts

Since AFOREs began in operation in 1997 one of the more important doubts for every employee is the election of the administrator of his or her savings. For workers information the regulator institution CONSAR publish monthly investment returns, commissions, and several information of the administrators. For comparing between administrators there is a *net investment return index* NRI and consists on moving averages of monthly returns less commissions in 60 months (5 years) for making and more stable index, however volatility is present in investments and benefits at retirement are unknown. Another comparison it would be scoring by which administrator generates higher replacements rates, unfortunately due the system is a recent implementation; there are no historical values that bring us information about which administrator is better to choose. One way to solve this issue is estimating future pensions with actual information. For general account, the pension at retirement age is the maximum between the cumulated fund divided into an actuarial factor and the MPG. After algebra, the pension for each administrator can be seen as follows[3]:

$$P = max\left(\frac{\sum_{\boldsymbol{t=1}}^{\boldsymbol{n}} \boldsymbol{C}_t \prod_{\boldsymbol{j=t}}^{\boldsymbol{n}} (1+r_j)(1-c_j)}{\left(\frac{13}{12}\right)\left(\ddot{\boldsymbol{a}}_x^{(12)} + \ddot{\boldsymbol{a}}_y^{(12)} - \ddot{\boldsymbol{a}}_{xy}^{(12)}\right)}, MPG\right)$$

Hence, the replacement rate is:

$$\Delta\% = \frac{P}{\boldsymbol{S}_n}$$

Where

$\boldsymbol{C}_t$: Contributions at time t

r_t: Real investment return at time t

c_t: Commissions at time t

$\boldsymbol{S}_n$: Last Salary

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[3] The actuarial factors $\ddot{\boldsymbol{a}}_x^{(12)}, \ddot{\boldsymbol{a}}_x^{(12)}, \ddot{\boldsymbol{a}}_{xy}^{(12)}$ were calculated with the mortality dinamic table EMSSA-09 as said by the Mexican Insurance Institution CNSF "*Circular Única de Seguros*", chapter 18.7.

Mostly of studies that analyze and estimate replacement rates assumes continuous contributions and fixed investments returns. The weak point is not consider the randomly nature of variables that affect the individual account, in real life is difficult the stability in such variables. That is the case of the actuarial valuations of the pension regime of the IMSS in 2009 and (Durán & Peña, 2011), where one of the assumptions is fixed real investment returns. In previous pension formula, the contributions and investment returns are stochastic processes that have direct impact in replacement rates. Therefore, the aim of the paper is to analyze the risk of this scheme with a probabilistic perspective.

2.1. Modeling affecting variables of Individual Account

The interacting factors in the generation of the replacement rates are several and have such complexity, because there are interconnection between economic, demographic variables and from the system that affect dynamics of the individual accounts. However, to reach equilibrium between realism and simplicity, in simulations are considered the following variables.

- Salary history
- Contribution density.
- Real investment returns

2.1.1. Salary history

During the labour life the salary changes in response of abilities, productivity and other characteristics. The comparison among AFOREs might be independent of salary history, this argument allow the using of fixed percentage of salary growth as in Actuarial Valuations of IMSS where considered annual real salary growth of 1%. Additionally, for representing average workers, the salary is considered as the Mexican average and one standard deviation; it means 4.2 and 8.8 minimum Mexican wages respectively.

2.1.2. Contribution Density

Exist periods in time that for any reason the worker is out of the formal labour market. The ratio between the quantity of real contributions and possible contributions is called contribution density. The case when contribution density is equal to one is complicated in real life and more in developing countries like Mexico. Benefits at retirement are affected directly from contributions; the low density cases will accumulate fewer interests, hence less pension. Without doubt, low density cases are more probably to not accomplish the retirement requirements and only the amount saved in their accounts will granted which is insufficient for maintaining life level due less quantity contributions.

The contribution process indicates whether or not the worker is active or inactive respectively. Let $\{X_t\}_{t=0}^{\infty}$ the stochastic process of contributions where:

$$X_t = \begin{cases} 1, & if\ worker\ contributes\ at\ time\ t \\ 0, & otherwise \end{cases}$$

For simulating the contribution process X_t taking into account contribution density it was considered two distinct models

1. Markov Chain Model.
2. Bayesian Model.

Markov Chain Model

For the individual account context it was considered a non homogeneous markov chain that depends of worker age x.

$$P_x = \begin{pmatrix} 1-q_x & q_x \\ p_x & 1-p_x \end{pmatrix}$$

Where $P_x(i,j) = \mathbb{P}[X_{x+1} = i | X_x = j] \quad i,j \ \in \{0,1\}$

Theorically, this model should reflex better a labour trajectory with non-mortality and non-invalidity assumptions. For including those variables multiple decrements might be used inside the markov chain, however, because the purpose of the paper is comparing the administrators for any kind of worker, the mortality and invalidity assumptions are excluded. The disadvantage of the non homogeneous chain is that requires at least ninety parameters that correspond to each age from twenty to sixty five; moreover the information quality regarding new hires and layoffs for each year and worker profile could be subjective. In counterpart, homogeneous markov chain reduces parameters to only two.

$$P = \begin{pmatrix} 1-q & q \\ p & 1-p \end{pmatrix}$$

Since the markov chain is regular, it means, with positive probability can be hired or fired, the matrix P^n-which represents the probability transitions in labour market in n periods of time– tends to a matrix W, which has both rows equal, in mathematical notation it means:

$$\lim_{n \to \infty} P^n \to W \,, \;\; W = \begin{pmatrix} \delta & 1-\delta \\ \delta & 1-\delta \end{pmatrix}$$

This result is useful to approximate the labour trajectory process in a long term with a $Bernoulli(\delta)$ distribution where parameter δ corresponds to contribution density.

Bayesian Model

As previous model, given the contribution density the process of contributions has a Bernoulli distribution at time n, $X_n|\delta$ ~ $Bernoulli(\delta)$, just the parameter δ has *a priori* distribution $Beta(\alpha, \beta)$ with $\alpha, \beta > 0$.

$$\pi(\delta) = \frac{\delta^{\alpha-1}(1-\delta)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1} du}$$

So, the predictive distribution $\hat{f}(X_{n+1}|X_1,,,,X_n)$ ~ $Bernoulli(\delta_{n+1})$

Where,

$$\delta_{n+1} = \left(\alpha + \sum_{t=1}^{n} X_t\right) \Big/ (\alpha + \beta + n)$$

This process elevates probabilities of continuing hired or in unemployment giving more realism to the process. The renovation of δ_{n+1} change the percentage of contributions to do in future, in consequence, there will be cases where no contributions are made and others where almost always contributes. Hence, there is less control directly for contribution density for simulations given a worker profile. In conclusion, for modeling contribution density is chosen the Bernoulli model, it requires just one parameter and keeps the percentage of contributions faithfully during labour history, also is convergent with other markov chains which look the same proportion of contributions.

The contribution density in average during the individual accounts regime has been 70% for men and 66% for woman. Data were given from the regulator institution CONSAR to the Mexican Social Security Institution IMSS for the Actuarial report of labour risk premium 2010. For the model, the percentages of 70% and 66% mentioned above are the parameters δ. Additionally, contribution density of 100% is also modeled for analyzing differences between workers whom never interrupt work and the average labour population. Markov chains for market sectors also have been used for analyze Mexican Pension system, see (Lagarda & Mandujano, 2008) parameters are estimated from National Employment Survey, replacement rates and contribution density are estimated for distinct salary, education and market sector levels. In our case, historical contribution density is used for estimating replacement rate.

2.1.3. Real investment returns

During labour life, the contributions into the individual account are invested in different funds depending worker age. The investment politics are regulated for evade possible losses due market volatility. For funds corresponding to younger persons, more investments are allowed in equity and derivatives instruments. The argument is that younger people are more risk taking than older. At moment of study there were five funds according to the following ages[4]:

- SIEFORE 1: 60 years old and older.
- SIEFORE 2: between 46 and 59 years old.
- SIEFORE 3: between 37 and 45 years old.
- SIEFORE 4: between 27 and 36 years old.
- SIEFORE 5: 26 years old and younger.

The different investment politics in the fund is reflected in their monthly volatility as can be seen in the next charts.

[4] Nowadays Siefore 4 and 5 are fusionated.

Chart 1: Monthly Real Investment Returns SIEFORE 1

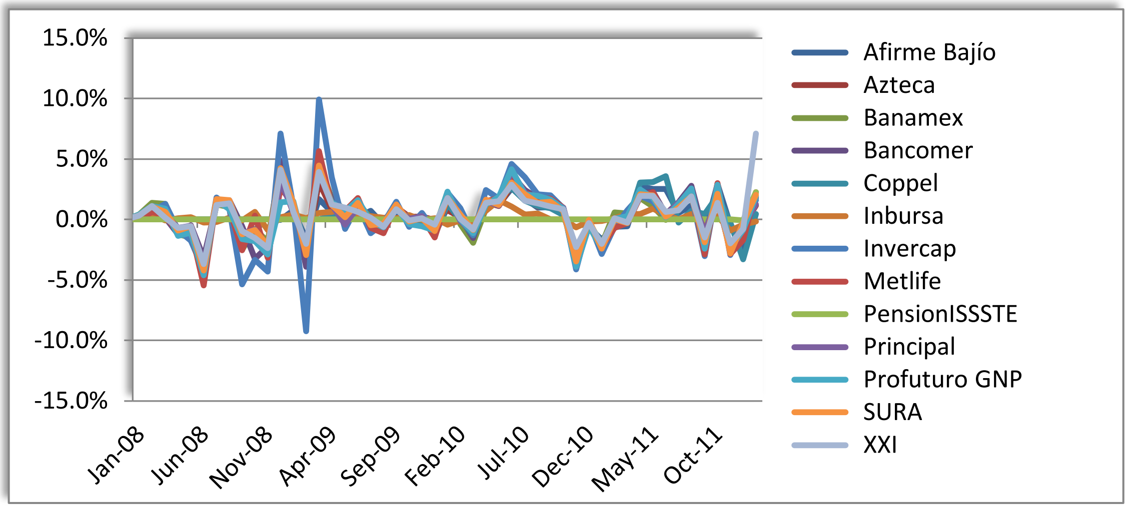
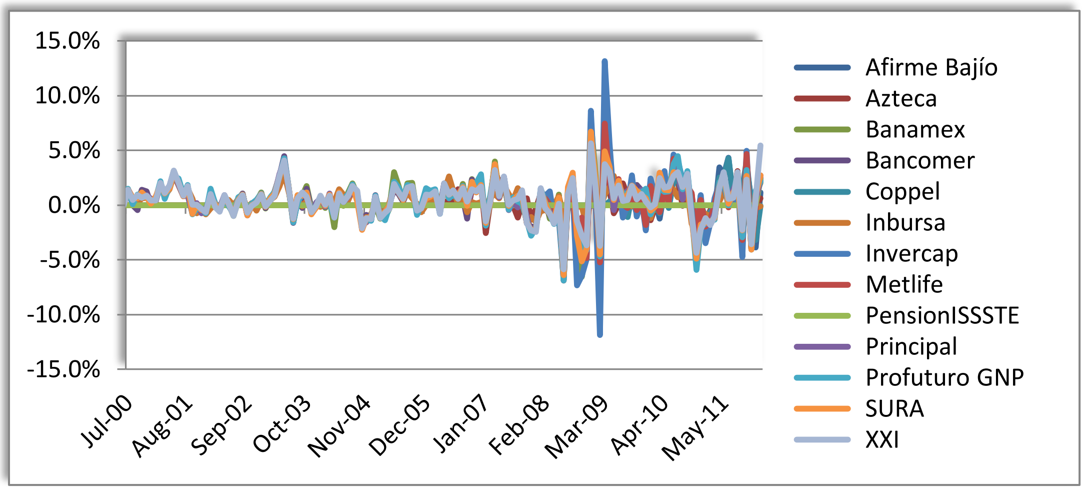


Chart 2: Monthly Real Investment Returns SIEFORE 2



Source: Own elaboration with data published by regulator institution CONSAR and Mexican Central Bank

Chart 3: Monthly Real Investment Returns SIEFORE 3

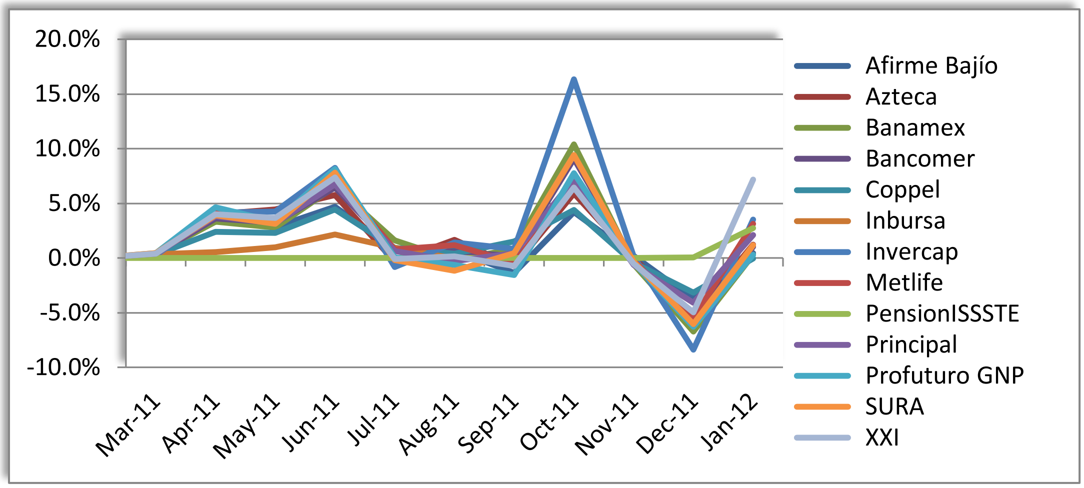
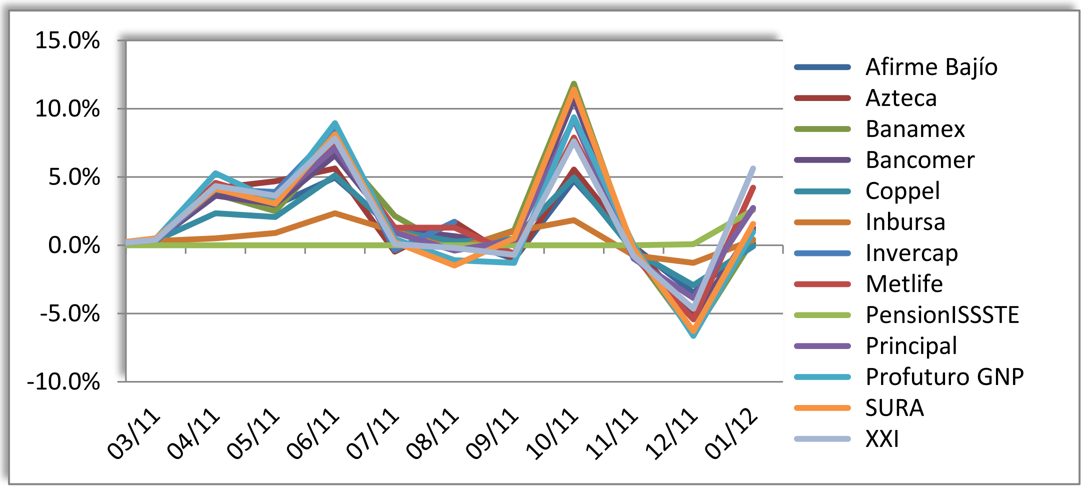
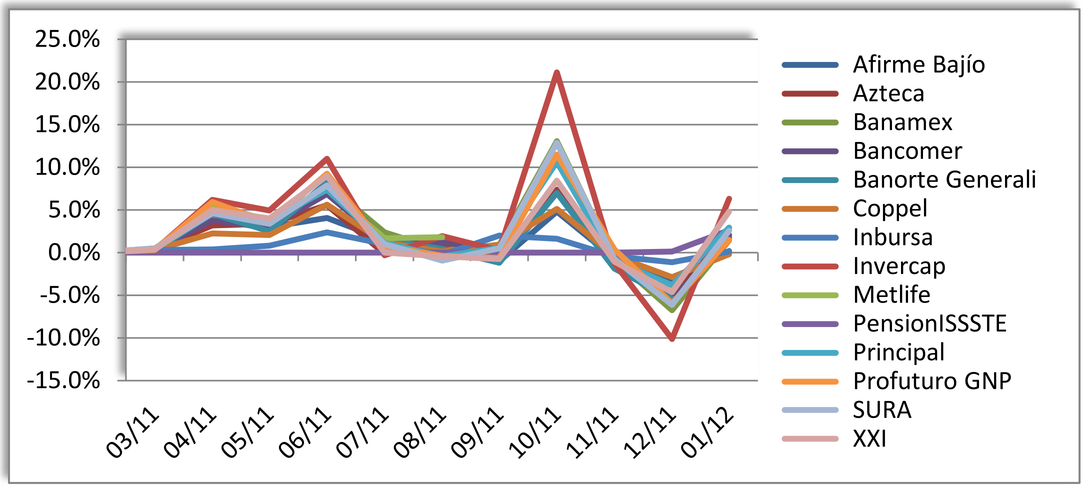


Chart 4: Monthly Real Investment Returns, SIEFORE 4



Source: Own elaboration with data published by regulator institution CONSAR and Mexican Central Bank

Chart 5: Monthly Real Investment Returns SIEFORE 5



Source: Own elaboration with data published by regulator institution CONSAR and Mexican Central Bank

The previous charts show the strong correlation between the administrators, the investment constraints and inflation affects equally to all of them. In financial crisis the relationship structure impacts with negative returns in administrators with more investment risk, meanwhile conservative administrators are less affected. However, those events are present and results in lower pensions. Modeling volatility allows contemplating all possible outcomes.

Modeling investment returns has been extensively investigated in the financial literature. There are different approaches to capture characteristics of asset returns. One possibility is to assume outcomes are independent and to model distribution of returns using a probability density function or mixed distributions. To model non normality behavior an alternative for heavy-tailed distributions can be stable Paretian as proposed by (Mandelbrot, 1963), logistic, Scaled t-student or Exponential Power Distributions are other options tested by (Gray & French, 1990) over log returns of the S&P 500 Composite Index. In (Gurrola, 2006) it is improved mixed normal distribution for measure market risk for Mexican Pension System. Other approach is to use conditional variance models, such as different variants of the GARCH (*Generalized Autoregressive Heteroskedasticity*) model. For correlated data another approach is to use VARIMA (*Vector Autoregressive Integrated Moving Average*) models, or simulating via Bootstrap methods as improved in (Ibarra, 2006) where also estimates replacement rates and probabilities of exercising the MPG.

In this study, it is adopt the first approach and assume returns are independent. But it can be used any method depending the characteristics of asset returns. Due correlation structure between administrators it is fitted a Multivariate Normal Distribution for each fund.

Multivariate Normal Distribution for real investment returns

Let $R = (r_1, \dots, r_n)$ the real investment returns for AFORE $i, i \in \{1, \dots, n\}$. R is normaly distributed if its probability density function is:

$$\phi(R; \mu, \Sigma) = |2\pi\Sigma|^{-1/2} exp\left(-\frac{1}{2}(R-\mu)'\Sigma^{-1}(R-\mu)\right)$$

The parameter $\mu = (\mu_1, \dots, \mu_n)$ is the monthly expected return for AFORE i and y Σ is the covariate matrix which represents the volatility and correlation structure between administrators. The estimation for μ, Σ was with the maximum likelihood method. The normality tests for each administrator are presented in Appendix C).

2.2.3. Methodology

Let a worker with age, salary contribution density and fund at time zero x_0, S_0, δ y $F_0 = 0$ respectively, the individual account is simulated with the following steps.

- At time t the SIEFORE is determined

$$SIEFORE\ j\ = \begin{cases} 1, & x_t \leq 26 \\ 2, & 26 < x_t \leq 37 \\ 3, & 37 < x_t \leq 45 \\ 4, & 45 < x_t \leq 59 \\ 5, & 59 < x_t \end{cases}$$

- Given SIEFORE j, it is generated a normal vector of monthly investment returns $R_j = (r_1, \dots, r_n)$ for all the administrators.

- The stochastic process of contributions X_t is the success of a random variable $Bernoulli(\delta)$ as described previously.

- The generated interest plus the contribution are accumulated into the fund for each administrator.

$$F_{i,t} = F_{i,t-1}(1 + r_i)(1 - C_i) + X_t(6.5\%S_t + SQ_t)$$

- After a year, salary is upgrade by one percent and the process continuing until retirement age.

- For administrator i the replacement rate for one scenario is calculated as

$$\Delta_i = max\left(F_{i,n} / \left(\frac{13}{12}\right)\left(\ddot{\boldsymbol{a}}_x^{(12)} + \ddot{\boldsymbol{a}}_y^{(12)} - \ddot{\boldsymbol{a}}_{xy}^{(12)}\right), MPG\right) / S_n$$

- It is constructed N replacement rates scenarios investments returns.

- The probability of exercising MPG is estimated as follows

$$\widehat{p_i} = \frac{\sum_{i=1}^{N} I\{Pension = MPG\}}{N}$$

The methodology described uses a multivariate normal distribution for real investment returns of administrators for comparing them, in (Rodríguez & Zuñiga, 2008) the simulated returns were over an optimized portfolio as proposed by (Markowitz, 1959) for all permitted assets by CONSAR. It is calculated the probability of exercising the MPG for distinct salary levels, but there is no difference between the administrators nor take into account contribution density and there is no replacement rates estimation. In this case the aim is not simulate an optimized portfolio over the administrators because workers just can choose one of them. In the next section is shown estimated probabilities of exercising the MPG and replacement rates for several workers profile.

2.3 Simulation Results

Thousand simulations were made for workers with initial age of twenty and thirty years old until retirement age for each administrator. Those simulations were elaborated with the statistical package R. Data was obtained from the regulator institution CONSAR and the Mexican Central Bank webpage at February 2012. At first, it will presented the probabilities of exercising the MPG and therefore the replacements rates distributions, mean and standard deviation

2.3.1. Probabilities of exercising the MPG

As can been seen in the next table the average workers -4.2 minimum wages and contributions density less than 100%- have more probabilities of exercising the MPG in comparison with the rest profiles. The differences between contribution densities are between .1 and .3 with same salary and age.[5]

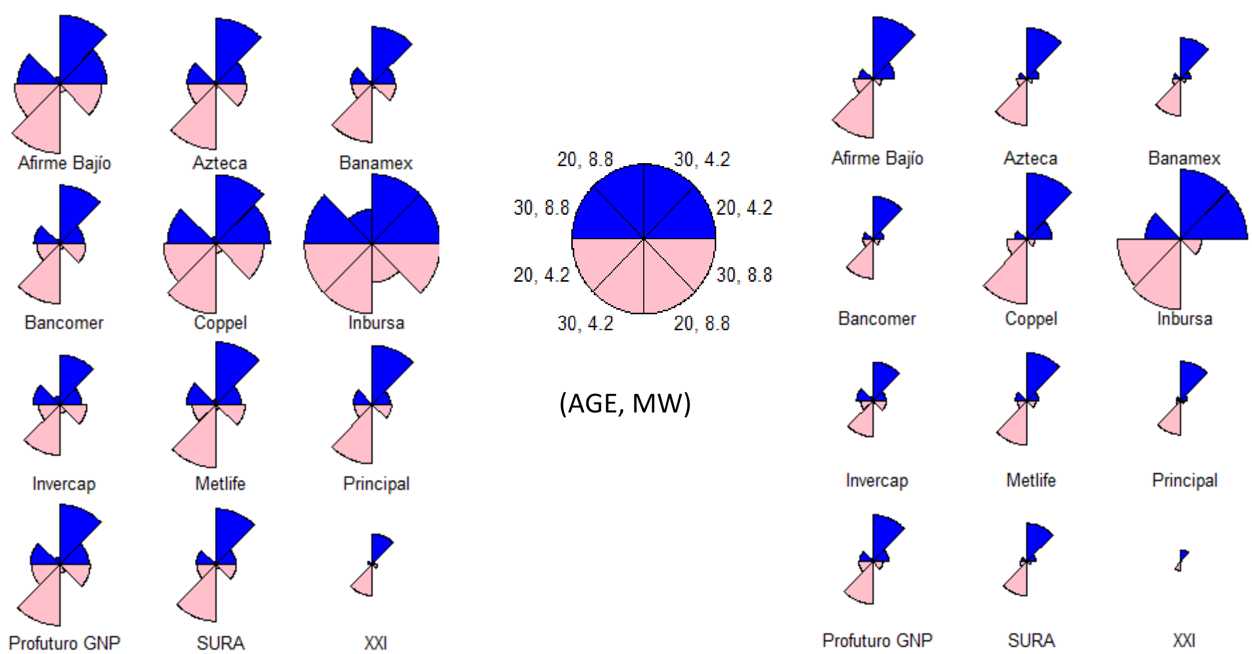
[5] Consult Appendix B) for numerical results for each administrator.

Table 2.1 Probabilities of exercising MPG (System)

Probability of exercising MPG		System			
		Salary			
		4.2 MW		8.8 MW	
		AGES			
Sex	**Density**	**20**	**30**	**20**	**30**
M	**70%**	0.446	0.844	0.103	0.446
	100%	0.253	0.661	0.014	0.172
F	**66%**	0.427	0.853	0.117	0.454
	100%	0.222	0.613	0.010	0.119

The previous table is the mean of the administrators, but there are differences among them. Through the next charts is easy observe the distinct probabilities between administrators, contribution density, salary and sex. The blue semicircle represents males, meanwhile pink females. The left charts presents average contribution density and the right contribution density of 100%.

Probabilities of exercising MPG



Average contribution density

Contribution density of 100%

The administrators that stand out for its shape are Inbursa and XXI, whom have the highest and smallest probabilities respectively. The actual low risk investment strategies makes almost sure greeting the MPG, meanwhile XXI has more chances to give pensions higher than the PMG. It is worth noting that persons who are twenty years old and average salary have similar probabilities than persons who are thirty years old and double salary, the double salary compensate ten years without contributions.

Previous results are similar to (Ibarra, 2006), for average workers the system is susceptible to exercise the MPG over 1250 contribution weeks. This gives an idea of possible scenarios where resources could be not enough to finance a minimum life level. Checking changes in probabilities due new investment strategies, reforms, commissions, etc is a good tool for measure the impact in pension benefits.

2.3.2. Replacement rates

As mentioned in previous sections, the replacement rate is the ratio between the pension and the last salary and it serve for comparing different pensions systems. Mexico as member of ILO is obligated to bring replacements rates at least of forty per cent. Such goal is reached in mean for twenty years old persons with average contribution density with the possibility to obtain pensions near eighty per cent; in the other hand older ages do not accomplish the ILO recommendation.

Table 2.2 System replacement rates[6]

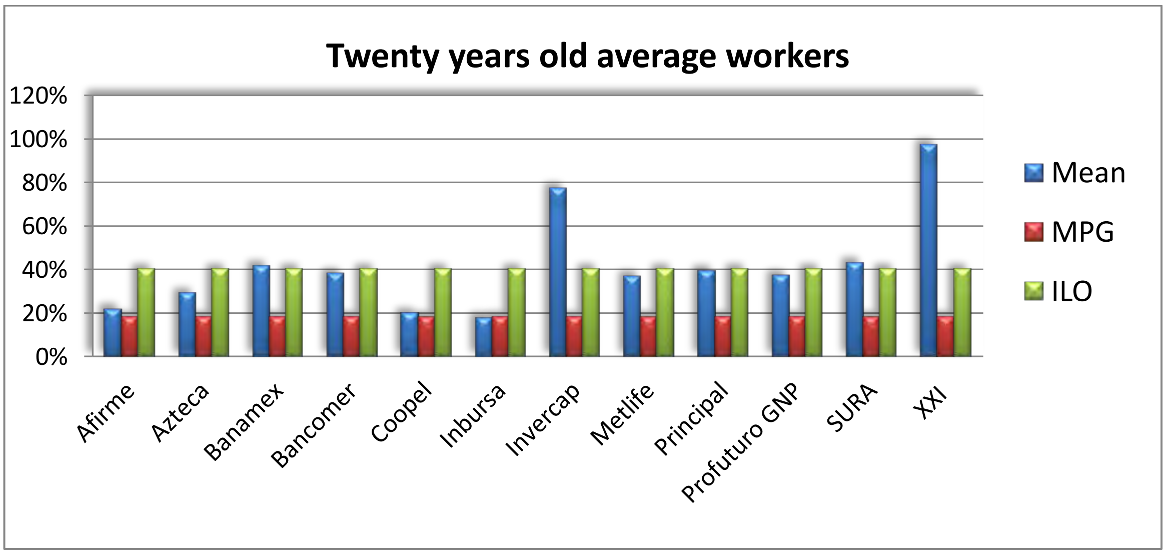
Replacements rates (μ, σ)		System							
		Salary							
		4.2 MW				8.8 MW			
		Ages							
		20		30		20		30	
Sex	**Density**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**
M	**70%**	42%	38%	23%	8%	39%	39%	16%	7%
	100%	59%	60%	27%	13%	54%	51%	22%	11%
F	**66%**	43%	39%	22%	6%	36%	31%	16%	9%
	100%	63%	64%	28%	14%	57%	54%	23%	12%

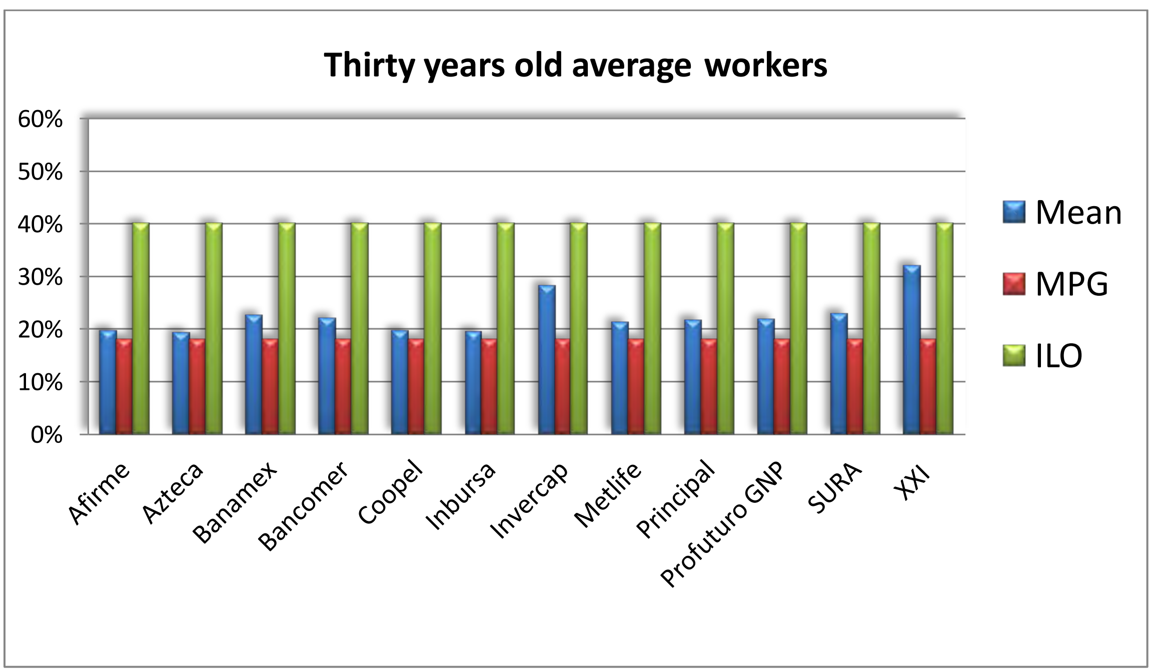
[6] For numerical resultsof each administrator consult Appendix A)

The social quota helps more to workers with fewer incomes and is reflected into replacement rates with differences up to seven per cent. As expected, low density cases presents fewer replacements rates than obtained with a fully-time working, differences can be up to twenty per cent in young females; for older, differences up to seven per cent.

In the next charts it is shown for each administrator their expected replacement rates in contrast with ILO recommendation and the MPG for average workers for two different ages.

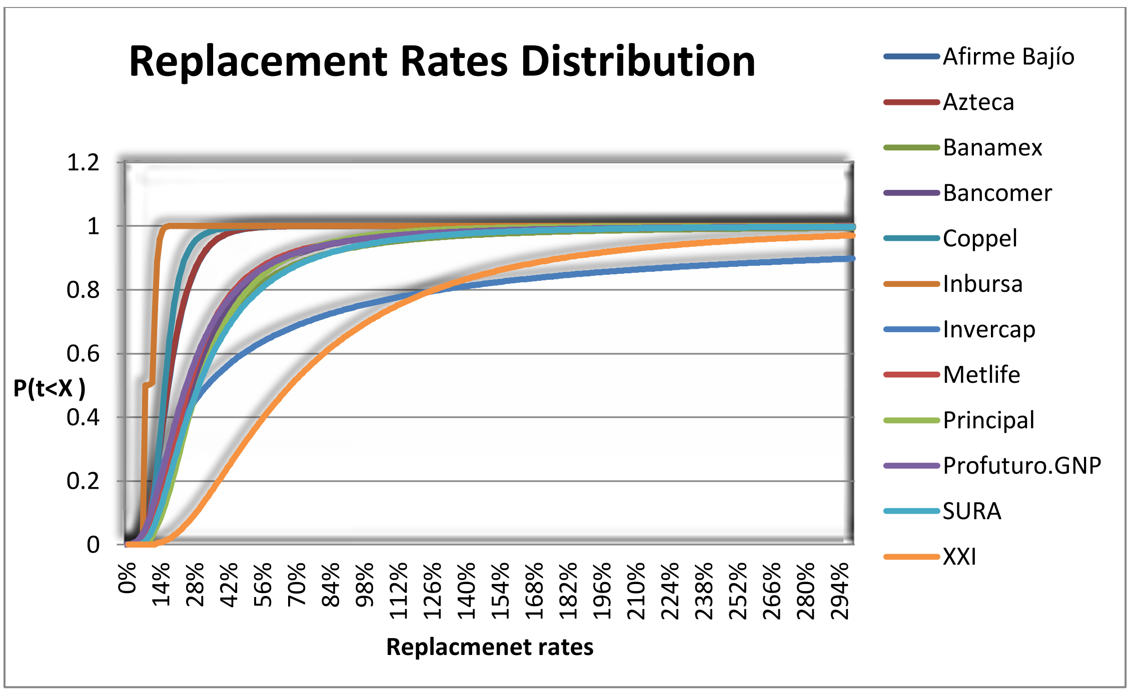
Chart 6 Replacement rates





In previous charts it is worth notice that the administrators with higher probabilities of exercising de MPG (Inbursa, Coppel y Afirme Bajío) have replacements rates that almost are equally to the pension subsidy by the government; the other group (Azteca, Banamex, Bancomer, Metlife, Principal, Profuturo GNP y SURA) is more homogeneous and near to ILO recommendation for youngest; Invercap and XXI have expected replacements rate above eighty and hundred per cent. In the other hand thirty years old average workers have replacements rates lower than ILO recommendation, similar to (Ibarra, 2006) where average workers in 1250 contributions weeks are under forty percent in their replacement rates. Other approach for comparing total outcomes is through their probability distribution function, for graphical purposes the distribution obtained by simulations were smoothed. It is appreciate which level could reach the administrators given a probability. For example almost all the administrators excluding Invercap and XXI with a probability of .80 will have replacements rates lower than sixty per cent.

Chart 7 Replacement rates distribution



Corresponding for twenty years old worker, 8.8 MW and average contribution density.

Distributions with heavier tails offer higher replacements rates with more probability. The distribution with the heaviest tail is from Invercap, there are scenarios where its risk taking reach higher pensions above the rest of administrators. However, for several scenarios Invercap has fewer pensions than others. For analyzing dynamism in replacement rates behavior it is presented correlations between them and estimated probabilities of pensions from administrator i is lower than administrator j. The high positive correlation in investment returns is reflected in benefits at retirement age. This could be an advantage, because is pensions probably increases or decreases together, thus AFORE election might continuing correct with elevate confidence. Based in probability of giving lower benefits than the other AFORE, Inbursa and XXI are the two extremes, in almost all cases they have the worst and the best expected pensions respectively. The table gives a better certainty to worker election. For any pair of AFOREs the result can be interpreted as the probability of correct decision.

Table 2.3 Correlation Matrix between replacement rates

P_{xy}	Afirme Bajío	Azteca	Banamex	Bancomer	Coppel	Inbursa	Invercap	Metlife	Principal	Profuturo GNP	SURA	XXI
Afirme Bajío	1	0.92	0.84	0.88	0.95	0.51	0.63	0.89	0.91	0.88	0.85	0.81
Azteca		1	0.89	0.93	0.90	0.47	0.74	0.91	0.94	0.91	0.91	0.84
Banamex			1	0.98	0.82	0.46	0.88	0.92	0.94	0.97	0.98	0.91
Bancomer				1	0.86	0.48	0.87	0.94	0.96	0.98	0.98	0.91
Coppel					1	0.53	0.63	0.87	0.90	0.84	0.84	0.78
Inbursa						1	0.28	0.47	0.55	0.47	0.47	0.46
Invercap							1	0.80	0.77	0.83	0.87	0.79
Metlife								1	0.96	0.95	0.94	0.90
Principal									1	0.96	0.97	0.90
GNP										1	0.97	0.91
SURA											1	0.93
XXI												1

Table 2.4 Probabilities that benefits from AFORE i will be lower than AFORE j

$P[\Delta_i < \Delta_j]$	Afirme Bajío	Azteca	Banamex	Bancomer	Coppel	Inbursa	Invercap	Metlife	Principal	Profuturo GNP	SURA	XXI
Afirme Bajío	0	0.90	0.91	0.90	0.22	0.02	0.83	0.89	0.97	0.81	0.94	1
Azteca		0	0.80	0.75	0.03	0	0.75	0.69	0.93	0.59	0.87	1
Banamex			0	0.26	0.02	0	0.64	0.31	0.54	0.17	0.63	1
Bancomer				0	0.02	0	0.71	0.41	0.74	0.26	0.83	1
Coppel					0	0.02	0.84	0.91	0.97	0.83	0.96	1
Inbursa						0	0.88	0.93	0.97	0.89	0.96	1
Invercap							0	0.23	0.36	0.18	0.35	0.84
Metlife								0	0.73	0.40	0.75	1
Principal									0	0.19	0.53	1
GNP										0	0.86	1
SURA											0	1
XXI												0

Corresponding for twenty years old worker, 8.8 MW and average contribution density

Conclusions

Simulating replacement rates also can be used for comparing, portfolios or investment strategies, pension plans, systems or market competitors based in risk measures of final benefits like expected value, standard deviation, interquantile ranges or confident regions, Value-at-Risk, among others measures through their probability density function; it can be analyzed correlation structure as the analysis made in this paper where replacement rates are high positive correlated due investment returns; also can be measure success over competitors estimating the probability of being a better system than other giving higher benefits, in this study XXI and Invercap are more probable to give higher pensions.

Globally, the Mexican individual account system has significant probabilities of exercising MPG for average workers, which is traduced in high fiscal cost due the enormous percentage of workers with low salary profiles. In addition, benefits at retirement are high probably to be lower than ILO recommendation and near to MPG excluding young people with XXI and Invercap as administrators. Final benefits for average contribution density workers differ from whom fully time contribute up to twenty per cent in replacement rates. Such difference gives an idea of how the system can be over estimated. It is worth of notice that results are obtained from probabilistic models based in historical data and it is not necessary that the same patterns will present again, however, is an approximation to reality with actual information and it is useful for taking with more confidence better decisions for population.

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Appendix

A) Replacement rates for each administrator

It is presented the replacement rate mean **μ** and its standard deviation **σ** for each administrator.

Replacement rates %		Afirme Bajío								Azteca							
		Salary								Salary							
		4.2 MW				8.8 MW				4.2 MW				8.8 MW			
		Age								Age							
		20		30		20		30		20		30		20		30	
Sex	**Density**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**
M	**70**	21.8	7.7	19.6	1.4	19.2	8.9	11.5	3.3	28.9	15.3	20.3	3.8	26.7	15.5	13.7	5.0
	100	29.7	13.2	20.7	4.0	26.9	12.3	16.0	5.3	41.1	23.0	23.2	7.4	37.1	20.9	19.2	7.4
F	**66**	22.3	8.5	19.6	1.1	18.7	8.2	11.6	3.5	30.0	16.0	20.2	2.9	25.8	14.2	13.9	5.8
	100	31.6	14.2	21.3	4.8	28.7	13.1	17.1	5.6	43.8	24.6	24.3	8.3	39.5	22.3	20.6	7.8

Replacement rates %		Banamex								Bancomer							
		Salary								Salary							
		4.2 MW				8.8 MW				4.2 MW				8.8 MW			
		Age								Age							
		20		30		20		30		20		30		20		30	
Sex	**Density**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**
M	**70**	41.6	34.9	22.4	8.8	39.9	37.5	16.6	8.1	38.1	30.4	22.0	8.3	36.1	31.7	15.9	7.7
	100	60.0	54.3	27.7	14.3	54.6	47.9	23.1	12.1	54.6	46.3	26.8	13.2	49.3	40.5	22.0	11.5
F	**66**	43.7	38.9	22.0	7.0	36.6	30.6	17.2	10.1	39.8	33.3	21.6	6.5	33.4	26.2	16.4	9.2
	100	63.9	57.9	29.2	15.6	58.2	51.0	24.7	12.9	58.2	49.4	28.2	14.4	52.6	43.2	23.6	12.2

Replacement rates %		Coppel								Inbursa							
		Salary								Salary							
		4.2 MW				8.8 MW				4.2 MW				8.8 MW			
		Age								Age							
		20		30		20		30		20		30		20		30	
Sex	**Density**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**	**μ**	**σ**
M	**70**	19.9	5.1	19.5	0.5	17.1	6.2	10.7	2.4	17.6	0.0	19.5	0.0	10.4	2.1	9.3	0.1
	100	26.5	9.7	20.0	2.3	24.0	8.5	15.0	4.0	17.7	0.8	19.5	0.0	15.6	2.0	11.1	2.0
F	**66**	20.1	5.5	19.5	0.3	16.7	6.0	10.9	2.7	17.6	0.0	19.5	0.0	10.1	2.0	9.3	0.1
	100	28.3	10.4	20.3	2.9	25.6	9.1	16.1	4.2	18.0	1.5	19.5	0.0	16.6	2.2	12.2	2.1

Replacement rates %		Invercap								Metlife							
		Salary								Salary							
		4.2 MW				8.8 MW				4.2 MW				8.8 MW			
		Age								Age							
		20		30		20		30		20		30		20		30	
Sex	**Density**	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
M	**70**	77.2	149.9	28.2	27.2	80.3	162.7	20.7	17.3	36.7	28.2	21.1	6.9	34.0	27.0	14.6	6.6
	100	122.1	260.8	35.7	35.7	104.8	197.9	27.1	23.9	52.2	42.6	25.1	11.1	47.3	38.3	20.2	9.9
F	**66**	82.8	144.7	26.6	18.4	68.0	109.2	22.4	23.4	37.7	29.9	20.7	4.5	32.3	24.3	15.2	8.2
	100	130.0	278.1	37.7	38.3	111.7	210.9	29.0	25.6	55.6	45.5	26.2	12.2	50.4	40.8	21.6	10.5

Replacement rates %		Principal								Profuturo GNP							
		Salary								Salary							
		4.2 MW				8.8 MW				4.2 MW				8.8 MW			
		Age								Age							
		20		30		20		30		20		30		20		30	
Sex	**Density**	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
M	**70**	39.2	25.9	21.7	6.7	36.8	25.0	16.4	6.8	37.1	32.8	21.8	8.8	34.5	32.5	15.2	7.6
	100	56.4	38.9	27.0	11.3	50.9	34.0	23.1	10.0	51.8	48.7	26.0	13.2	47.3	43.6	20.9	11.8
F	**66**	40.4	27.6	21.4	5.5	35.1	22.5	16.8	8.0	38.4	35.3	21.4	6.1	31.8	27.9	15.6	9.4
	100	60.2	41.5	28.5	12.4	54.3	36.2	24.7	10.7	55.2	51.9	27.3	14.4	50.4	46.5	22.3	12.6

Replacement rates %		SURA								XXI							
		Salary								Salary							
		4.2 MW				8.8 MW				4.2 MW				8.8 MW			
		Age								Age							
		20		30		20		30		20		30		20		30	
Sex	**Density**	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
M	**70**	43.0	35.2	22.8	9.4	41.0	37.3	17.3	8.4	97.3	84.7	32.0	18.2	93.1	86.4	28.1	14.5
	100	61.7	54.6	28.5	14.4	56.0	47.2	24.1	12.4	140.1	121.3	45.1	24.8	128.8	116.2	39.0	21.0
F	**66**	44.8	39.1	22.3	7.4	37.9	30.0	17.9	10.3	100.9	93.7	30.9	15.3	86.4	71.4	28.9	16.4
	100	65.8	58.2	30.1	15.7	59.7	50.4	25.7	13.2	149.4	129.3	48.2	26.5	137.3	123.9	41.7	22.4

B) Probabilities of exercising MPG

Probability of exercising the MPG		Afirme Bajío				Azteca				Banamex			
		Salary				Salary				Salary			
		4.2 MW		8.8 MW		4.2		8.8 MW		4.2 MW		8.8 MW	
		AGE				AGE				AGE			
Sex	**Density**	**20**	**30**	**20**	**30**	**20**	**30**	**20**	**30**	**20**	**30**	**20**	**30**
M	**70%**	0.698	0.986	0.106	0.615	0.440	0.936	0.050	0.427	0.327	0.816	0.047	0.321
	100%	0.330	0.887	0.021	0.203	0.183	0.728	0.005	0.124	0.146	0.582	0.011	0.109
F	**66%**	0.663	0.992	0.121	0.623	0.408	0.936	0.057	0.451	0.303	0.835	0.064	0.344
	100%	0.279	0.839	0.010	0.137	0.150	0.661	0.003	0.085	0.128	0.528	0.005	0.076

Probability of exercising the MPG		Bancomer				Coppel				Inbursa			
		Salary				Salary				Salary			
		4.2 MW		8.8 MW		4.2 MW		8.8 MW		4.2 MW		8.8 MW	
		AGE				AGE				AGE			
Sexo	**Densidad**	**20**	**30**	**20**	**30**	**20**	**30**	**20**	**30**	**20**	**30**	**20**	**30**
M	**70%**	0.362	0.845	0.067	0.376	0.796	0.996	0.101	0.716	1.000	1.000	0.500	0.999
	100%	0.166	0.619	0.013	0.143	0.377	0.949	0.006	0.186	0.986	1.000	0.003	0.536
F	**66%**	0.335	0.854	0.080	0.387	0.769	0.998	0.125	0.700	1.000	1.000	0.548	0.998
	100%	0.143	0.553	0.009	0.103	0.302	0.919	0.002	0.121	0.949	1.000	0.001	0.305

Probability of exercising the MPG		Invercap				Metlife				Principal			
		Salary				Salary				Salary			
		4.2 MW		8.8 MW		4.2 MW		8.8 MW		4.2 MW		8.8 MW	
		AGE				AGE				AGE			
Sexo	**Densidad**	**20**	**30**	**20**	**30**	**20**	**30**	**20**	**30**	**20**	**30**	**20**	**30**
M	**70%**	0.325	0.708	0.120	0.389	0.371	0.893	0.069	0.431	0.263	0.845	0.027	0.286
	100%	0.196	0.554	0.052	0.239	0.176	0.684	0.018	0.172	0.098	0.557	0.002	0.067
F	**66%**	0.306	0.728	0.126	0.404	0.347	0.909	0.078	0.430	0.267	0.853	0.028	0.281
	100%	0.181	0.515	0.043	0.203	0.145	0.633	0.012	0.133	0.074	0.487	0.001	0.048

Probability of exercising the MPG		Profuturo GNP				SURA				XXI			
		Salary				Salary				Salary			
		4.2 MW		8.8 MW		4.2 MW		8.8 MW		4.2 MW		8.8 MW	
		AGE				AGE				AGE			
Sexo	**Densidad**	**20**	**30**	**20**	**30**	**20**	**30**	**20**	**30**	**20**	**30**	**20**	**30**
M	**70%**	0.432	0.861	0.106	0.432	0.296	0.799	0.038	0.291	0.042	0.439	0.000	0.068
	100%	0.241	0.665	0.035	0.193	0.132	0.544	0.007	0.084	0.006	0.167	0.000	0.008
F	**66%**	0.407	0.872	0.122	0.451	0.282	0.812	0.056	0.299	0.037	0.447	0.003	0.074
	100%	0.204	0.605	0.024	0.155	0.105	0.488	0.005	0.062	0.004	0.132	0.000	0.005

C) Normality tests for real investment returns

Normality Test for AFOREs

Pearson Normality Test (P-values)					
AFORE	**SIEFORE 5**	**SIEFORE 4**	**SIEFORE 3**	**SIEFORE 2**	**SIEFORE 1**
Afirme Bajío	0.58112	0.79277	0.79277	0.13013	0.02512
Azteca	0.68658	0.68658	0.58112	0.02845	0.65825
Banamex	0.48280	0.25503	0.48280	0.01336	0.93332
Bancomer	0.58112	0.48280	0.58112	0.00117	0.06076
Coppel	0.96257	0.88945	0.48280	0.12244	0.03229
Inbursa	0.68658	0.79277	0.58112	0.61364	0.42709
Invercap	0.48280	0.79277	0.31912	0.10088	0.04383
Metlife	0.03304	0.12377	0.48280	0.03416	0.93010
Principal	0.25503	0.25503	0.31912	0.00467	0.30125
Profuturo GNP	0.07401	0.58112	0.48280	0.00094	0.04238
SURA	0.15865	0.15865	0.58112	0.00205	0.77666
XXI	0.68658	0.68658	0.58112	0.02499	0.22602

Source: Own elaboration with data published by regulator institution CONSAR and Mexican Central Bank