Measuring Bank Funding Liquidity Risk

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Abstract

The standard framework to measure funding liquidity risk compares expected cumulative cash shortfalls over a particular time horizon against stock of available funding sources. This requires assigning cash-flows to future periods for financial products with uncertain cash-flow timing. There is lack of consensus on how to assign such cash-flows. A concern about models employed in the literature, so far, is that they give little credence to the distribution of time that financial products remain on a bank's book. Moreover, little is known about the run-off profile distribution of most bank financial products. Against this background, this study formulates an approach to measure bank funding liquidity risk at a business unit level on a run-off basis. A survival model is employed to assign cash-flows to future time horizons. The resulting model is applied to a case study of an individual Southern Africa retail bank.

JEL Classification: C14; C41; G21; G32.

Keywords: Funding liquidity risk; survival analysis; Cash-flow modelling; Non-maturing assets and liabilities.

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1. Introduction

The standard framework to measure bank funding liquidity risk (FLR) compares expected cumulative cash shortfalls over a particular period against the stock of available funding sources (Neu, 2007). A fundamental problem with this framework is how to assign cash-flows to future periods, especially on financial products with uncertain cash-flow timing (Vento and La Ganga, 2009). Such financial products are referred to as having indeterminate maturity, or loosely called non-maturing assets and liabilities (NoMALs). Equally, the term indeterminate maturity can refer to a financial product whose cash-flow timing pattern differs significantly from that specified on the product's contract. Examples of bank assets with uncertain cash-flow timing include credit card accounts and overdraft accounts. For bank liabilities, examples include demand deposits and savings accounts. Since products with uncertain cash-flow timing constitute a significant portion of a typical depository institution, it follows that an accurate understanding of these product's liquidity risk characteristics is of significant practical importance to FLR measurement (Kalkbrener and Willing, 2004).

Existing literature on NoMALs has largely focused on their valuation and interest rate sensitivity; little attention has been drawn to their impact on FLR profile of a bank. Examples of studies on NoMALs include: Jarrow and van Deventer (1998), Jonasi et al. (1999), Kalkbrener and Willing (2004), Maes and Timmermans (2005), Bardenhewer (2007), Frauendorfer and Schürle (2007) and Nyström (2008). One method to value NoMALs is arbitrage-free pricing methodology (Jarrow and van Deventer (1998); Nyström (2008)). This valuation approach uses risk-neutral probabilities, which have limited application to liquidity risk management. Instead, what matters for liquidity risk are real world probabilities to determine expected real world cash-flows (Bardenhewer, 2007). Another method values NoMALs by constructing a replicating portfolio that closely resembles the NoMAL's cash-flow features (Maes and Timmermans (2005); Frauendorfer and Schürle (2007)). Such models assume a particular maturity profile for the NoMALs. Thus, liquidity is exogenous to replicating portfolio models. In both replicating portfolio and arbitrage-free valuation approaches, scenarios are mostly specified in terms of market interest rates. While this is appropriate in terms of a valuation and/or interest rate sensitivity objective, it differs from how liquidity risk scenarios are specified in practice. In sum, while NoMALs have mostly

been modelled in the context of pricing and interest rate risk a gap exists in the literature where NoMALs are modelled mainly in the context of funding-liquidity-risk.

There is lack of consensus on how to assign cash-flows to future periods where cash-flow timing is stochastic. Many banks simply divide the balance position into core (stable) and floating (less stable) portion (Kalkbrener and Willing (2004); Neu (2007); Vento and La Ganga (2009); Basel Committee on Bank Supervision (2011)). The less stable portion is subsequently assigned to earlier time horizons while the core balance position is assigned to later time horizons. Management judgement is reportedly the most common approach to assign the balance position (Neu, 2007). Other less subjective approaches are based on a time series regression of NoMAL volume (Kalkbrener and Willing (2004); Neu (2007)). But, a volume time series shows how the balance position stock evolved over time, not the time those balance positions stayed on a bank's book. Thus, given the objective of allocating cash-flows to future periods, it seems reasonable to assume that a method that considers the period that balance positions stay on a bank's book better meets the objective.

The objective of this study is to propose a new approach to assign cash-flows with uncertain timing to future periods in the context of measuring bank FLR. This is done on a run-off basis – i.e., no future new business is assumed when calibrating the model. The contribution of the study is twofold. Firstly, the study develops a unique method to handle timing of stochastic cash-flows by utilising 'survival' models. The method is formulated using financial product positions as the subject of study. Of interest for these subjects is the time they spend with a bank. Secondly, application of the method is demonstrated. To date, empirical knowledge on run-off profiles for NoMALs is limited to a few studies calibrated without considering duration of balance positions. This limitation implies that many important questions remain unanswered. What form is the distribution of run-off profiles? How do run-off profiles vary across scenarios (stressed or 'normal' operating environment)? What is the average time positions stay on a bank's book? Are run-off rates proposed under Basel III reasonable? To what extent does stability of different funding sources differ from each other? This study attempts to demonstrate how these and related questions can be answered. The demonstration is by a case study of an individual Southern Africa retail bank.

The scope of this study excludes future new business. It is questionable whether additional assumptions needed when incorporating future new business, which results in a more complex design and uncertainty, leads to a better understanding of the risk being

measured. This perhaps explains why most practitioners prefer to measure risk on a run-off basis (Neu, 2007). To a large extent, bank supervision is on a run-off basis as supervisors are primarily concerned with existing business. By way of analogy, the value at risk framework, which is arguably the most commonly used market risk measurement standard, is also consistent with a run-off basis since the portfolio is assumed to be held constant over the period market risk is measured (Hull, 2011). Similarly, credit risk measurements largely focus on default exposures of existing portfolio. As pointed out by Bardenhewer (2007), it is ideal to model future new business separately from existing business given that they tend to have two distinct sources of information. Whereas run-off of existing business can be estimated statistically, future new business estimates are best extracted from expert knowledge supplied by the marketing and/sales department of a bank (Bardenhewer, 2007). Nonetheless, techniques utilised to analyse the existing portfolio can also be used to assess the impact of future new business on the overall projected position at a particular future time.

The paper proceeds as follows. Section 2 states the problem statement. Section 3 reviews the literature on bank FLR measurement. Section 4 develops the model to measure bank FLR. Section 5 presents a case study. Finally, section 6 concludes.

2. Problem Statement

The framework for (1) quantitative measurements of bank funding-liquidity-risk and (2) bank funding-liquidity indicators is fairly standard. These measurements include: balance sheet ratios, net cash capital position, maturity mismatches and funding ratios. Calculating these measurements is straightforward if cash-flow timing is certain. However, a significant portion of bank financial products have uncertain cash-flow timing (Jarrow and van Deventer, 1998). Consequently, banks face two fundamental problems in assessing FLR: that is;

(1) Assigning cash flows to future time horizons for financial products with uncertain timing; and/or
(2) Ascertaining the stable (core) and less stable (volatile) portion of financial products with uncertain cash-flow timing.

Bank regulators face similar problems on setting and monitoring quantitative prudential funding liquidity requirements (e.g.: liquidity coverage ratio and net stable funding requirement).

Concerns on the few quantitative models, in the literature, that handle the cash-flow timing problem of financial products are threefold:

(1) The models infer the run-off through studying a time series of the volume of a financial product. Nonetheless, a volume time series shows how stock of the balance position evolved over time, not the time those balance positions stayed with the bank. Thus, given the objective of allocating cash-flows to time horizons, it seems reasonable to assume that a method considering the time that balance positions stay on a bank's book better matches the objective.
(2) Quantitative models in the literature utilise parametric approaches, which restrict the evolution patterns of cash-flows. However, the literature is silent on determining non-parametric run-off patterns of financial products.
(3) Although liquidity risk assessment is scenario-specific, calibration of existing models lacks explicit recognition of liquidity scenario(s) that prevailed in the past.

This study is a step towards addressing these epistemological problems in FLR assessment. A time-to-event models is used to obtain actual past evolution of financial products. Liquidity risk scenarios that prevailed in past data are explicitly considered in deriving the funding liquidity scenario-specific run-off patterns.

3. Literature Review

This section reviews the literature on funding liquidity risk measurement. More specifically, Section 3.1 discusses issues on defining FLR. Section 3.2 proceeds to discuss the various quantitative funding liquidity indicators and funding liquidity risk measurements. The aim is to highlight how treatment of the evolution of financial products influences the outcome of funding liquidity measurement instruments. Section 3.3 discusses quantitative approaches to allocate cash-flows to future periods. Given shortcomings identified in the literature the section motivates the need of an alternative approach to allocate cash-flows. Section 3.3 also shows how a better understanding of the evolution of NoMAL cash-flows improves the

valuation of NoMALs, which is important for bank asset and liability management. Section 3.4 discusses FLR supervisory matters. Emphasis is particularly on how effectiveness of FLR prudential regulations depends on the treatment of cash-flows with uncertain timing.

3.1. Definition of Funding Liquidity Risk

In the literature, most definitions of funding-liquidity-risk share three elements. Firstly, they are defined from the perspective of an individual economic agent (e.g. firm, bank, individual) rather than a security or financial system. A common alternative reference to FLR is thus in terms of the individual economic agent. For example, Acharya et al. (2011) refer to FLR as 'bank liquidity'. Often, the literature refers to 'funding liquidity' simply as 'liquidity' (see, for example: Matz (2007); Committee of European Banking Supervisors (2009b); La Ganga and Trevisan (2010)). In these cases, it becomes apparent that the subject matter is funding liquidity given that it is described from an individual economic agent's point of view. Secondly, there is an element of offsetting cash-flows. For example, Bessis (2011) mentions of 'cash when needed' when defining FLR; Neu (2007) refers to 'net cumulative cash outflow'; and Vento and La Ganga (2009) mention of 'disequilibrium between cash inflows and outflows'. Thirdly, there is a time aspect over which liquidity is considered. For example, Basel Committee on Banking Supervision (2008) allude to funding liquidity as meeting obligations 'when due'; and IMF (2008) refer to FLR as settling obligations 'in a timely fashion'.

Differences on defining of funding-liquidity-risk exist in the literature. Variations mainly emanate from whether FLR considers (1) solvency, (2) cost of obtaining liquidity and (3) immediacy. These factors are discussed below.

Solvency: In the literature, FLR is occasionally considered as applicable only to solvent firms. For example, IMF (2008) defines FLR as "the ability of a solvent institution to make agreed-upon payments in a timely fashion". A problem with making solvency a necessary condition to discuss FLR is that insolvent banks can sometimes be liquid, particularly in the presence of information asymmetry, just like a solvent bank can be illiquid (Montes-Negret, 2009). Information asymmetry arises from the fact that although a firm might know its solvency status, the public might be unaware of this status. Often, however, FLR is defined without reference to solvency. This is perhaps more appropriate given that FLR and solvency tend to be difficult to distinguish, particularly in crisis periods where information asymmetry

tends to be more pronounced (Montes-Negret, 2009). Moreover, solvency is covered by capital, which is different from FLR that is covered by cash inflows (Neu, 2007).

Cost of obtaining liquidity: One school of thought is that funding can always be obtained to cover obligations (Montes-Negret, 2009). However, funding can sometimes be obtained only at a considerable cost. It is this additional cost that some FLR definitions attempt to account. For example, Vento and La Ganga (2009) define FLR as "the risk that a financial firm, though solvent, either does not have enough financial resources to allow it to meet its obligations as they fall due or can obtain such funding only at excessive cost". Similarly, Basel Committee on Banking Supervision (2008) define FLR as "the ability to fund increases in assets and meet obligations as they come due, without incurring unacceptable losses". In both definitions, there is a problem that the cost of liquidity is described in apparently subjective terms ('excessive cost' and 'unacceptable losses'), which vary across markets, individual firm, context, and time.

Immediacy: The speed with which an economic agent settles its obligations is a prominent aspect in a number of FLR Definitions. For example, Drehmann and Nikolaou (In press) define FLR as "the possibility that over a specific horizon the bank will become unable to settle obligations with immediacy".

Notwithstanding similarities in key elements of a FLR definition, consensus on a FLR definition remains elusive largely due to its ambiguity and vagueness. Ambiguity in the term FLR originates from multiple probable meanings in a given context. Vento and La Ganga (2009) discuss various possible interpretations of FLR in a banking context. Two examples they give are: (1) FLR is a measure of the "capability to turn an asset quickly without capital loss or interest penalty"; and (2) the risk of being unable to raise funds on the wholesale financial market. Ambiguity is however absent when the term FLR is used in other contexts. Vagueness in the term FLR arises from the possibility that the term is conveying multiple meanings. This is typically the case when FLR is discussed alongside market liquidity risk and/or systemic liquidity risk, particularly when authors only use the term 'liquidity' without specifying the dimension of liquidity risk. The context is thus of fundamental importance in deducing the meaning of FLR.

In this study, funding liquidity risk is defined as the risk of an individual economic agent failing to fund its net cumulative cash outflows over a specific period. Four features of this definition are worth noting. Firstly, it ignores solvency status of the individual economic

agent. The reason is that funding liquidity problems occur regardless of solvency status. To illustrate, it is conceivable that an insolvent firm can lack funding liquidity problems if the public unaware of its solvency status – information asymmetry. Moreover, it is questionable whether economic agents always first consider solvency before funding liquidity. Secondly, this study's FLR definition omits the cost of liquidity factor. Although unlikely, one can still face FLR problems even if the cost of liquidity remains unaltered. For instance, FLR problems can arise solely from increased onerousness of collateral requirements, e.g. more and/or higher quality collateral requirements – in terms of collateral being less risky and/or more easily convertible to cash. Thirdly, the definition disregards the element of obtaining funding with immediacy. This is because immediacy is deemed to be a market liquidity risk feature since it is more of a transactional property of markets where funding is sourced. In this sense, immediacy is viewed as one channel that market liquidity risk affects FLR. Fourthly, the definition is consistent with the approach by Drehmann and Nikolaou (In press) of distinguishing between funding liquidity and funding liquidity risk. As pointed out by Drehmann and Nikolaou (In press), funding liquidity is a binary concept: an individual agent can either fund or is unable to fund its obligations. In contrast, funding liquidity risk is defined on a continuum (that is, on a probability space) and is forward looking. By way of analogy, this is the same relationship between default and credit risk. In conclusion, while this study's FLR definition is somewhat general, it does provide a reasonable basis for measurement given the exclusion of subjective elements.

3.2. Funding Liquidity Regulatory Framework

This section discusses the bank regulatory framework for funding liquidity. Firstly, the section describes the role of the Basel Committee on Bank Supervision (BCBS) in globally shaping the banking regulatory framework. This is followed by an overview of the Basel III accord - a regulatory framework proposed by the BCBS to address bank capital, leverage and liquidity concerns. The aim of this section is to highlight how this study helps gain a better understanding of methods to estimate some key parameters that underpin Basel III's minimum liquidity standards.

3.2.1. The Role of the Basel Committee on Bank Supervision

One role of the Bank of International Settlement (BIS) is promoting financial sector stability through researching policy issues faced by supervisory authorities, thereby enabling it to set

international standards on banking supervision. To execute this role, the BIS set a committee called the Basel Committee on Banking Supervision (BCBS). Occasionally, the BCBS issues guidelines and standards in consultative papers described as 'Basel Accords'. Despite BCBS recommendations having no legal force, they have considerable influence on banking authorities around the world, including non-BCBS member countries (Shin, 2009). To date, the BCBS has issued three accords: namely, Basel I, Basel II and Basel III.

The Basel I and II accord ignored liquidity risk in their prudential regulatory framework. Nonetheless, the Basel Committee on Banking Supervision (2008) published a paper on best practices in liquidity risk management. This paper superseded two earlier publications[2] on the same best practices. In terms of funding liquidity risk measurement, the Basel Committee on Banking Supervision (1992) mainly recommended the use of a maturity ladder to determine the net funding requirements over different future times under alternative scenarios. The objective for this recommendation was to provide a bank's treasury with an indication of future funding requirements, thereby enabling the bank to attempt influencing its maturity profile in current treasury operations. Later, the Basel Committee on Banking Supervision (2000) extended its recommendations to include setting of limits by senior management on cumulative net funding requirements over specific time periods. Liquid assets formed part of the maturity profile and were recommended to be incorporated in earlier time periods in accordance to their expected time to liquidate rather than contractual maturity date.

Given the large extent to which banks faced liquidity challenges during the 2007-2009 global financial crisis, it became apparent that issuing only liquidity best practices was inadequate. This gave impetus for the BCBS to incorporate minimum standards to manage liquidity risk within the Basel III accord. Other objectives for Basel III include the strengthening of the capital adequacy ratio. The section below outlines key aspects of the Basel III accord in terms of liquidity risk regulatory standards.

3.2.2. Basel III and Funding Liquidity Risk

The minimum liquidity standards under Basel III are based on two complementary ratios: namely, the liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR). Whereas the LCR minimum requirement aims to strengthen banks' ability to withstand short-term liquidity shocks, the NSFR is set to "promote resilience over the long term" (BCBS, 2010a).

[2] Basel Committee On Banking Supervision (1992) and Basel Committee On Banking Supervision (2000).

The LCR is scheduled to become effective into law in the G20 countries on 1 January 2015, with the NSFR scheduled for 1 January 2018. These two minimum liquidity standards are discussed further below.

3.2.2.1. Liquidity Coverage Ratio

BCBS (2010a) defines the liquidity coverage as:

$$\text{LCR}=\frac{\text{Stock of high quality liquid assests}}{\text{Total NCO over the next 30 days}} \tag{1}$$

The LCR should be at least 100%.

The stock of liquid assets, i.e. the numerator of equation (1), is split into two categories: namely, level one and level two assets. Level one assets are considered to be highly liquid. Characteristics of level one assets include the following: 1) they are eligible for use as collateral when borrowing from the central bank; and 2) they are easy to convert to cash. Level two assets are relatively less liquid. To account for reduced liquidness, a discount of 15% to level two asset value is applied when calculating the liquidity coverage ratio. There are three key parameters in the practical implementation of the LCR requirement: 1) The discount to the value of liquid assets (haircut) that constitute the numerator of the LCR; 2) the run-off rates applied to assets and liability classes; and 3) the split of demand deposits into core and volatile portion.

Following Neu (2007), this study defines the core amount of a product class as the portion whose time on a bank's book exceeds one month at a given confidence level (for example, 95%). Such an approach is less subjective and takes account of the spread for run-off rates. The method of determining the stable amount portion is, in spirit, close to that of Neu (2007), Kalkbrener and Willing (2004) and Vento and La Ganga (2009). There is however one key difference. These studies derive run-off profiles from employing a time series analysis of volume increments governed by a Gaussian distribution. Instead, this study models run-off rates using a nonparametric 'survival' model, which does not restrict the functional form of run-off patterns.

3.2.2.2. Net Stable Funding Ratio

BCBS (2010a) defines the Net Stable Funding Ratio as:

$$\text{NSFR}=\frac{\text{Available amount of stable funding}}{\text{Required amount of stable funding}} \tag{2}$$

Basel III requires banks to have a NSFR of at least 100% (Basel Committee on Banking Supervision, 2010).

Available stable funding comprises of equity, preferred stock and demand deposits. The demand deposits are split between 'stable' and 'less stable' demand deposits. Stable deposits are defined as the portion of deposits expected to remain with the bank for at least one year (Basel Committee on Banking Supervision, 2010). The available amount of stable funding is calculated as the sum of the value of each funding source held, multiplied by a specific factor prescribed for each funding source. Whereas the available stable funds (i.e., the numerator of Eq. (5)) relates to liabilities, the required amount of stable funding (i.e., the denominator of Eq. (5)) considers assets.

The objective of the NSFR is "to promote more medium and long-term funding" for banks (Basel Committee on Banking Supervision, 2010). This is achieved through the effect of the NSFR in limiting the extent to which a bank can mismatch the duration of assets and liabilities (IMF, 2011). The same objective can be met by employing limits to the long-term funding ratio.

Practically implementing the NSFR raises the following question: How can the notional value of demand deposits be split between 'stable' and 'less stable' deposits? This study seeks to provide an answer to this question through empirical estimations of the run-off pattern for demand deposits. In this study, the stable portion of a product class, in the context of NSFR, is defined as the portion that stays with a bank for at least one year at a given confidence level (for example, 95%).

3.3. Models for Indeterminate Maturity Financial Products

In the literature, models that deal with cash-flow timing uncertainty relate to non-maturing assets and liabilities (NoMaLs). Hence, this section reviews literature on modelling NoMaLs in terms of (1) the evolution of cash-flows; and (2) valuation. The section starts by discussing how evolution of NoMAL cash-flows has been modelled in the literature. This is important since it provides an approach to handle NoMALs when calculating funding liquidity indicators and FLR measurements discussed in Section 3.4. Focus is on highlighting areas of

concern in the approaches that exist in the literature. Lastly, this section discusses models used to value NoMALs. Although FLR is concerned with cash-flows, not valuation, the aim of discussing valuation matters is to show how a better understanding of the evolution of cash-flows can lead to improvements in NoMAL valuation models: namely, (1) the replicating portfolio valuation approach; and (2) the option-adjusted spread valuation approach.

3.3.1. Models for the Evolution of NoMAL Cash-flows

Kalkbrener and Willing (2004) develop a model to value non-maturing liabilities that accounts for liquidity risk and interest rate risk. In their model, liquidity risk is future uncertainty in the NoMAL account balance. To project future NoMAL account balances, Kalkbrener and Willing (2004) assume that account balance increments are governed by a normal distribution. They also suggest a log-normal distribution as an alternative specification of the NoMAL volume increments. Several paths of the account balance process are then obtained by simulation. For each account balance trajectory, evolution of the existing portfolio (run-off) is obtained from the minimum account balance in earlier periods along that path. Thus, for each simulated account balance path, there is a corresponding simulated run-off path. At each projection time, t, the run-off account balance is then obtained, at a given level of confidence, by considering the distribution of simulated values as at time t. For example, say the simulation procedure generates 10,000 run-off paths. Further, say the objective is to estimate the run-off account balance, at time t, likely to be available with at least 99% level of confidence. This is the 99th (lower) percentile of the 10,000 run-off values projected as at time t.

Neu (2007) and Vento and La Ganga (2009) model the future NoMAL account balance by a log-linear time series regression where the dependent variable, a log-transformation of NoMAL account balance, is explained by the existing NoMAL account balance (the intercept), time (trend) and a normally distributed error term. Following this specification, the inference of the run-off, at a given level of confidence, is based on the estimated mean and volatility of the historical NoMAL account balance. Using a log-transformation on NoMAL account balance ensures that the model returns paths with non-negative NoMAL account balances.

Bardenhewer (2007), allocates NoMAL cash-flows using a three stage approach: (1) projecting the stock of NoMAL account balance; (2) Using an optimisation procedure to

determine weights of financial instruments in a replicating portfolio such that the volatility of the difference between the return from a replicating portfolio and the customer interest rate on the NoMAL is minimised; and (3) moderating the run-off profile. In stage one, after projecting the NoMAL volume, the series is split into a deterministic trend and a random component, indicating deviation from the trend. The deterministic trend of NoMAL account balance is modelled by a time series regression where the dependent variables are: the current NoMAL volume (the intercept); time; and deviations of customer interest rate on the NoMAL account from its historical average. This differs from Neu (2007) and Kalkbrener and Willing (2004) who exclude interest rates as an explanatory variable. At each projected future time, the trend volume presumed to be invested in a replicating portfolio. The remainder, i.e. the deviation of the projected volume from the trend is treated as the NoMAL's cash-flow realised within one month. Stage two involves constructing a replicating portfolio of the NoMAL. That is, selecting investments that closely resemble the NoMAL's term profile, currency and nature of cash flows. The allocation of cash-flows for the trend volume is then deduced from the cash-flow profile of the replicating portfolio after having derived the weights of constituent financial instruments of the replicating portfolio. The optimisation procedure used to derive replicating portfolio weights for a non-maturing liability is based on the premise that the replicating portfolio is a benchmark investment portfolio to hold against funds from the non-maturing liability. In stage three, moderation can be done through analysing past changes in the maturity profile of the NoMAL.

From the NoMAL models discussed above, it appears that quantitative models on the evolution of NoMAL cash-flows, in the literature, are grounded on an analysis of a stock of NoMAL volume. Yet a stock variable lacks information on the time NoMAL positions are retained by a firm. It is thus questionable whether analysing a stock variable is consistent with the objective of allocating the existing NoMAL account balance to future periods. Instead, it seems more appropriate to utilise a model that analyses the time NoMAL positions stay with a bank. To the best of the author's knowledge, use of duration for NoMAL positions on a bank's book, in the context of funding liquidity risk, has been unexplored so far.

The models by Kalkbrener and Willing (2004), Bardenhewer (2007), Neu (2007) and Vento and La Ganga (2009) have parameter values influenced by addition of NoMAL positions on a bank's book. Yet, adding NoMAL positions is uninformative about the evolution of cash-flows of those positions. The impact of 'new positions' is to increase

volatility of the stock variable, which is a key statistic influencing the out-come of models discussed in the literature. Thus, a better model, in terms of the objective of allocating future cash-flows, is one that is calibrated excluding 'new positions' but focuses on the run-off of NoMAL positions over time.

In Kalkbrener and Willing (2004), Neu (2007) and Vento and La Ganga (2009), volume increments are modelled by either a normal or lognormal distribution. The resulting run-off profiles from these models are thus from a parametric model. There is a gap in literature on non-parametric model specification. We thus lack a good understanding on the empirical distribution of unrestricted run-off profiles.

3.3.2. Replicating Portfolio Valuation Methods

As mentioned earlier, the replicating portfolio approach to valuing NoMAL is based on constructing a portfolio that closely resembles the NoMAL in terms of cash-flow properties such as the currency, and timing of cash-flows. The replicating portfolio is constructed from standard traded financial instruments; such as fixed income securities, money market instruments and standard swaps. Once the set of financial instruments for the replicating portfolio has been selected, what remains is to determine portfolio weights for constituent financial instruments in the replicating portfolio. This is typically done within an optimisation framework. Most models specify what is optimal in terms of the difference between the interest rate on the replicating portfolio and on the NoMAL. Frauendorfer and Schürle (2007) determine portfolio weights by minimising the expected downside deviation of the spread between the yield on the replicating portfolio and on the NoMAL. In contrast, Maes and Timmermans (2005) use the standard deviation of the same spread. Other constraints placed on the optimisation problem include the following: 1) the weights of the individual financial instruments in the replicating portfolio sum to one (e.g. Maes and Timmermans (2005); Bardenhewer (2007)); 2) Individual weights of constituent financial instruments are non-negative. That is, short position holdings are inacceptable. While most studies apply static weights, Frauendorfer and Schürle (2007) use dynamic weights, meaning that the replicating portfolio weights change over time with changes in NoMAL volume, NoMAL interest rate and market interest rates.

The method to optimise weights of assets in the replicating portfolio can result in cash-flow patterns that differ from the NoMAL's run-off. This explains why Bardenhewer (2007) moderates cash-flow patterns derived from the optimisation procedure. Such

moderation indicates potential limitations of using replicating portfolio approaches in measuring funding-liquidity-risk.

In the context of measuring funding liquidity risk, the most important factor is to determine the scenario-specific expected cash-flows. As pointed out by Bardenhewer (2007), it is principal payments, not interest payments, which are of major concern in funding-liquidity-risk. This is so because interest payments typically constitute a small fraction of the total cash-flows of a financial product. This is particularly true if market interest rates are relatively low. It therefore seems more appropriate, in the context of measuring FLR, to primarily model cash-flows based on their amortisation schedule and then attempt to understand how other factors, such as interest rates, influence this distribution of principal cash-flow timing.

The timing of cash-flows is exogenous to the replicating portfolio approach. That is, the model assumes that the replicating portfolio closely resembles the NoMAL with respect to cash-flow timing. What seems missing in the literature is a methodology to ascertain the extent to which the replicating portfolio of assets matches the cash-flow timing of the NoMAL. In this proposed study, this gap is filled by producing a distribution of the NoMAL run-off. From this distribution, one obtains a better understanding of the term profile of the NoMAL. Furthermore, other statistics such as the average term of the NoMAL can be obtained from the distribution and then compared against the duration of the prospective replicating portfolio. As a result, an additional constraint to the optimisation procedure for determining the replicating portfolio weights is that the replicating portfolio's duration equals the NoMaL's average term.

4. Survival Model Formulation of Funding Liquidity Risk

This section shows how bank data can be processed to enable inference on allocating cash-flows to future periods using survival methods.

4.1. Estimating the Run-off Profile

The subject of this study is one-hundredth of a monetary unit held in an individual account balance of a NoMAL of interest. Denote V_i as the total account balance for individual i. To illustrate, consider the following example: Say an individual account, i, has an account

balance of USD 100.00, then V_i = 10,000. In this example, the subject is a cent in individual i's account balance.

For each subject, the observation of interest is the time that the subject is on a bank's position: denote this variable by T. In terms of cash-flow, specifying T depends on whether one is considering an asset or a liability. From the bank's point of view, if a subject is a liability, e.g. a savings account, then exit of a subject from a bank's position corresponds to a cash outflow. In contrast, if the subject is an asset, e.g. a credit card account, then exit from a bank's position corresponds to a cash inflow.

It is assumed that the distribution of T for each subject is independent of other subjects. Admittedly, the subjects for this study are grouped into individual accounts and the survival times within an individual account tend to be correlated because the subjects share a common account holder. The effect of such intra-account correlations, however, is not to alter the 'survival' function estimates, but its variance (Williams, 1995). Although validity of the independence assumption is questionable for subjects belonging to a single account holder, it is reasonable when considering subjects derived from several account holders. In principle, the more diversified the number of account holders the more reasonable the assumption of independence.

Since the objective is to allocate the volume of cash-flows from a NoMAL account, it follows that one needs to make inference from the total account balance for the NoMAL account. To this end, denote V as the account balance for an individual business class. Adding a time index t to NoMAL account balance, i.e. V_t, and another index i for individual account balances, i.e. $V_{i,t}$, the relationship between the two variables, is shown in Eq. (3).

$$V_t = \sum_i V_{i,t} \tag{3}$$

In words, the account balance of an individual business class, at a particular time, is the sum of individual account balances in that business class, at that particular time. Notably, for the purpose of allocating non-maturing assets and liability cash-flows to future time horizons, the models in literature base their model calibration largely on a time series study of V_t (see for example, Neu (2007); Vento and La Ganga (2009)). As mentioned earlier, such an approach to model calibration has the problem that V_t does not necessarily inform us about

the time that a particular position remains on a bank's book, which is the most important aspect to consider if the objective is solely to allocate cash-flows to future time horizons.

To remedy the above problem, this study uses an experimental design where one observes only a fixed number of subjects. Here, focus is solely on understanding the time those subjects take on a bank's book. Following this reason, increments in an individual account are ignored as these are treated as subjects outside the study. To illustrate, consider the following example: Say an individual savings account, i, at inception, i.e. $t = 0$, has an account balance of USD 100.00, then this individual account contributes 10,000 subjects to the study. If the account holder were to deposit another USD 50.00 a week later within the observation period it is clear that this deposit event does not contain any information on the time on a bank's book for the 10,000 subjects under study. Therefore, the additional USD 50.00 needs to be ignored when considering the run-off as at $t = 0$. In sum, what matters for this study are only decrements (withdrawals for liabilities and repayments for assets), not increments (deposits for liabilities and drawdowns for assets) since increments are non-informative with regards to run-off patterns.

As a result of being interested only in decrements, there is need to define a monotonically decreasing function of the account balance, thereby enabling the study to employ survival methods as a tool in allocating cash-flows to future time horizons. This is done for an individual account, i, in Eq. (4) below.

$$\tilde{V}_{i,t} := \min_{0 \leq s \leq t} V_{i,s} \tag{4}$$

In Eq. (2), $\tilde{V}_{i,t}$ is a function that tracks only decrements of subjects that existed at inception (t = 0). Notably, the specification of Eq. (2) is structurally the same as that by Kalkbrener and Willing (2004). However, Kalkbrener and Willing (2004) use Eq. (2) to derive run-off patterns from a Monte Carlo simulation of NoMAL volume time series, which is different from its use in this study, which is a direct estimation of a survival function.

For simplicity, this study treats business classes independent of each other. Such an approach is similar to that taken by other studies in literature (see, for example: Kalkbrener and Willing (2004); Neu (2007); Vento and La Ganga (2009)). A consequence of treating business classes independently is that when studying the distribution of time on a bank's position for an individual business class, internal transfer of funds to other business class accounts is treated as a form of censoring.

The objective of the study is to employ survival analysis as a tool to estimate the proportion, $P(t)$, of the volume of an individual business class whose time on a bank's position would exceed t without imposing any distribution on $P(t)$. More formally, $P(t)$ is defined in Eq. (8).

$$P(t) = Pr(T \geq t) \tag{5}$$

To obtain $P(t)$, use is made of $\tilde{V}_{i,t}$. Consider a discrete time framework where we move one time step, from time t = 0 to time t = 1. The corresponding run-off for the account balance is $\tilde{V}_{i,0}$ to $\tilde{V}_{i,1}$. The difference ($\tilde{V}_{i,0}$ - $\tilde{V}_{i,1}$) can be explained as resulting from either the subjects exiting a bank's book or censorship (e.g.: account suspension). Thus, after each time step, we observe subjects exiting from a bank's book, censored observations and the subjects remaining on the banks book. By way of analogy, the subjects exiting a bank's book are treated as 'deaths' in the traditional survival analysis of lifetime experiments. The data from the account balance would thus be in a form required to perform traditional survival analysis (discrete and continuous time), thereby providing run-off patterns of financial products.

4.2. Implications of Using Survival Models in Funding Liquidity Risk Measurement

While this study is grounded on applying survival models, such models were originally developed to model lifetime data, not cash-flow timing. As such, it is critical to question the suitability of survival model assumptions when applied to cash-flow modelling. This section evaluates the applicability of survival model assumptions in the context of cash-flow modelling when measuring funding liquidity risk.

What are the similarities in modelling cash-flows and lifetime? Key similarities are mentioned below. Firstly, both cash-flow modelling and lifetime modelling model the length of time period that a subject remains in a particular state. Typically, in lifetime modelling, the subject can either be 'alive' or 'dead': in this case, the variable of interest is the time to 'death'. Similarly, a cash-flow position can either be on a bank's balance sheet or not on a bank's position: the variable of interest is thus the time on a bank's balance sheet. Secondly, both cash-flow and lifetime modelling are censored: that is, only partial information on the 'survival' time is known for some of the subjects under observation. Generally, censorship is

endemic in an experimental design where 'survival' times are observed over a limited experiment period.

What are the key differences between modelling cash-flows and lives? In the context of measuring funding liquidity risk, there are four key differences. Firstly, cash-flow modelling is scenario dependent whereas standard lifetime modelling is typically not modelled as state-dependent. In fact, the objective under cash-flow modelling is to obtain 'survival' distributions under different scenarios. This is irrespective of whether the survival analysis is extended to incorporate a regression model to estimate the relationship between covariates and 'survival' times. In contrast, lifetime modelling is concerned with the estimation of a single 'survival' time distribution, which is not state-dependent. This remains the case even when the 'survival' analysis is extended to incorporate covariates. Secondly, the time origin to base 'survival' analysis of financial products is unclear unlike the case with traditional fields where 'survival' models have been applied. The section below proceeds to discuss the approach taken to handle the time origin problem in the context of modelling cash-flow timing.

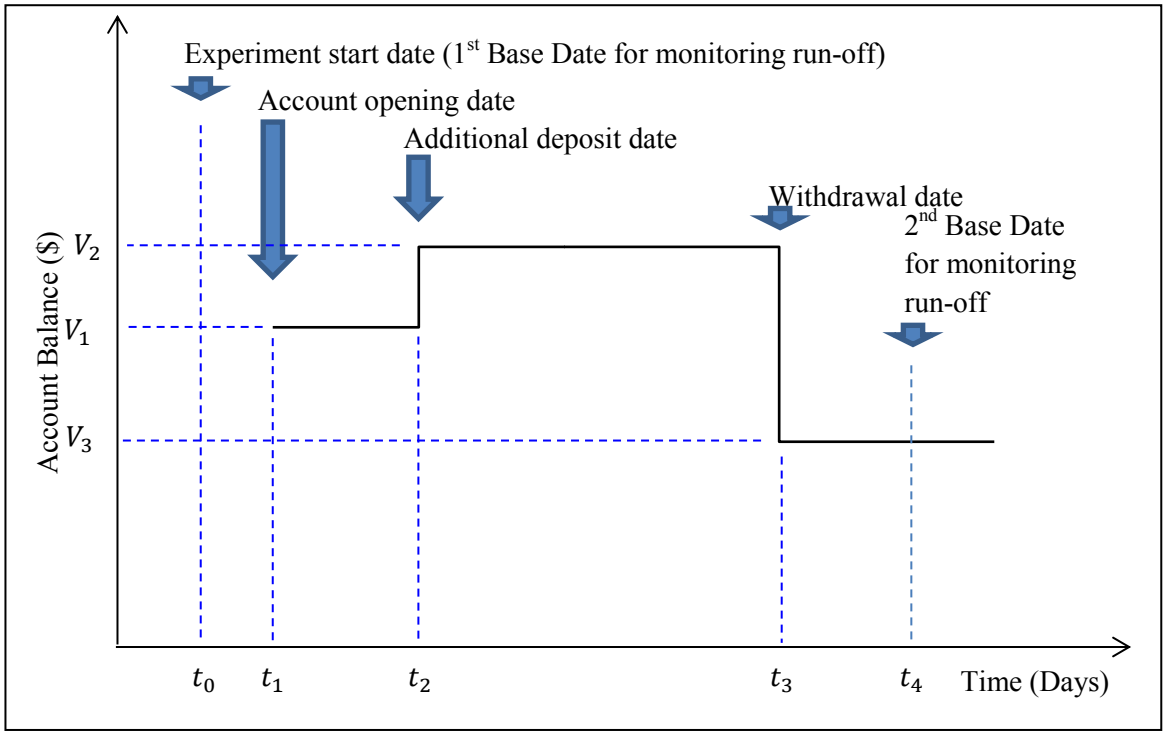
4.3. Determining the Time Origin for Run-off Profiles

This section extends the survival analysis described above by considering multiple run-off base dates, meaning the date from whence observation of run-off commences. The first run-off base date is the experiment start date. Subsequently, one, or more base dates can be established from whence run-off is observed. For these later base dates, it is possible to infer the run-off profile of individual accounts by considering past account balance information, under certain conditions. Thus, the time origin ($t = 0$), for an individual account, can occur earlier than the run-off base date. Since, different specifications of the time origin result in different survival curves, it is important to be explicit on how time origin is determined. This raises the following question: What is the time origin when studying the evolution of cash-flows for financial products with uncertain cash-flow timing? To answer this question, this section starts by illustrating with hypothetical, but practically possible, examples. Thereafter, the study's time origin is formally defined.

Consider a savings product as representing financial products with uncertain cash-flow timing. Suppose this savings product only has one account (hereinafter referred to as account 1). Further, say the firm decides that it will observe the run-off profile of the savings

product from various commencement dates starting at regular defined intervals. The motivation for doing so could be to consider potential run-off information from possible additional deposits into the savings product. Assume that the liquidity scenario remains constant. In this context, it is possible to make inference about the duration that a particular account balance has stayed with the firm under certain conditions. One such case is shown in Fig. 1.

Figure 1: Example Scenario Where Inference on Run-off Is Possible from Past Data



As shown in Fig. 1, t_0 is the experiment start date. At time t_0, only account 1 is under observation. So, the run-off profile commencing at time t_0 is fully informed by evolution of account 1. Note, however, that account 1's time series of the account balance is excluded from figure 4 because it is assumed to remain static over the period considered.

Next, say an additional savings account (hereinafter referred to as account 2) is opened later at time t_1. The account opening balance for account 2 is V_1. Notably, account 2 is excluded in the run-off profile monitored from time t_0 since account 2 was non-existent at time t_0. This highlights the point that the model developed in this proposed study is not influenced by the addition of 'new money'. This is ideal since 'new money' is uninformative about the duration that funds would be retained by the firm. In contrast, models in the

literature, for example, Kalkbrener and Willing (2004), Neu (2007), Bardenhewer (2007), and Vento and La Ganga (2009) are calibrated on the time series of the NoMAL volume. Such an approach has the limitation that 'new money', though uninformative of duration, influences the value of parameters such as volatility, which have a significant influence on the run-off profiles resulting from their models.

At time t_2, account 2 deposits an additional amount of $(V_2 - V_1)$. Similar to the event at time t_1, the deposit at time t_2 is uninformative with regards to run-off. At time t_3, there is a withdrawal from account 2 of $(V_2 - V_3)$. While this is informative with regards to the run-off profile of the savings product, the run-off monitored on a base date of t_0 misses this observation. This shows that for a particular financial product more information on its run-off profile can be obtained through observing run-offs from multiple base dates. Furthermore, the use of multiple run-off base dates makes the model dynamic as changes in the run-off profiles obtained from subsequent base dates could reflect behavioural changes. For example, it could be that 'new business' clients on the savings product have a higher propensity to withdraw relative to existing business, all else held constant.

Suppose that at time t_4 it is decided to track another run-off profile of the same savings product. For this other run-off profile, there are two savings accounts monitored; account 1 and account 2. The question then arises: what is the appropriate time origin for account 2: t_1, t_2, t_3 or t_4? Although t_4 is the run-off base date, it is still possible to learn about account 2's run-off profile by considering previous account balance information. Therefore, using t_4 as the time origin can potentially result in biased results. Time t_3 is inappropriate as a time origin because there is additional history relating to run-off that can be extracted by going further back in time. Time t_2 is one potential time origin since this is the furthest time that provides the experiment with the highest amount of subjects to study. Time t_1 is arguably an alternative time origin in the sense that a run-off from time t_1 can be determined. This is, however, at the expense of excluding the additional deposit of $(V_2 - V_1)$ made at time t_2 from the study. To summarise, t_2 and t_1 are potential equally sensible time origins. This proposed study will, however, only use t_2 as the time origin under the scenario depicted in figure 4, on the grounds of it optimising the number of studied subjects.

For consistency with the specification of time origin outlined on the Fig. 4 scenario, it follows that a different scenario where the account balance decreases as we move backwards

from the run-off base date would require us to set the run-off base date as the time origin: it is at the run-off base date where we maximise the number of studied subjects.

In light of the above examples, one can deduce that assuming a constant liquidity scenario, information on the run-off for an individual account can be inferred from observing the past trajectory if, and only if, backward movement from a run-off base date is non-decreasing for a least one time unit. Based on the criteria of maximising the number of studied subjects, the time origin ($t = 0$) is therefore the time corresponding to the furthest local maxima as we move backwards in time if, and only if, backward movement from a run-off base date is non-decreasing for a least one time unit. Otherwise, the time origin is the run-off base date.

3.2. Incorporating Liquidity Scenarios when Calibrating FLR Models

Funding-liquidity-risk assessment is scenario specific (Basel Committee on Banking Supervision, 2010). Emerging cash-flows from existing assets and liabilities considerably depend on the underlying FLR scenario since it is a major driver in the behaviour of the firm and its stakeholders (Neu (2007); Matz (2007)). While the influence of scenarios on a firm's funding-liquidity risk profile is widely acknowledged, the same explicit recognition seems to be lacking in the literature when calibrating models employed to project cash-flows. Intuitively, a FLR measurement model that employs data obtained under a specific scenario to correspondingly make inferences from that particular scenario is expected to better reflect the dynamics associated with various scenarios. By way of analogy, this is the motivation behind regime-switching models: to establish how relationships between variables differs between states. This study develops a model that explicitly incorporates FLR scenarios when calibrating FLR models.

In principle, it is difficult to envisage all FLR scenarios that can possibly be considered when analysing FLR. Moreover, the importance of analysing particular liquidity scenarios varies by firm and time. To thus make the specification of scenarios more applicable to a broader spectrum of firms, this study follows Matz (2007), Basel Committee on Banking Supervision (2010), and Committee of European Banking Supervisors (2009) by grouping liquidity scenarios as stemming from either bank-specific factors or market-specific factors. Examples of bank-specific factors include: credit rating downgrade; significant operational loss or credit risk event; and negative market rumours about the firm (Basel

Committee on Banking Supervision, 2010). Market-specific factors include: disorder in capital markets, economic recession, and payment system disruption (Matz, 2007). Fig. 2 shows the resulting four FLR scenarios arising from the grouping of FLR factors.

Figure 2: Funding-liquidity Scenario Matrix

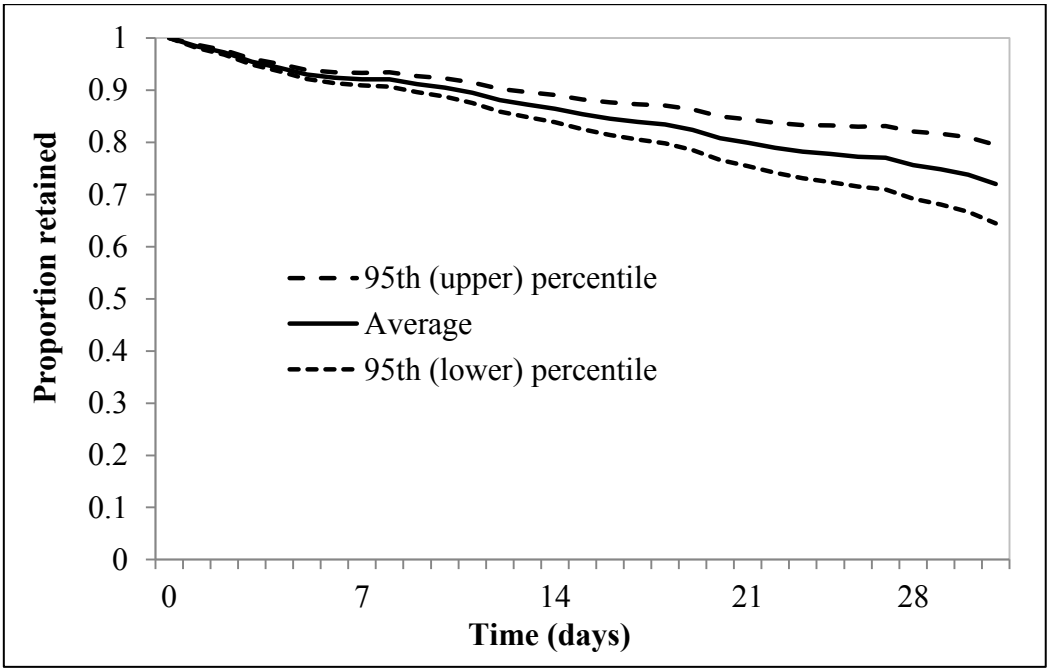
		Bank-specific Stress Scenario	
		No	Yes
Market stress scenario	No	**Scenario 1**	**Scenario 2**
	Yes	**Scenario 3**	**Scenario 4**

Examples of terms to describe scenario 1, in Fig. 2, include: 'business as usual' and 'normal operating environment'. Scenario 2 can be described as an 'idiosyncratic-stress scenario'. Scenario 3 can be termed a 'market stress scenario' while scenario 4 describes a state where funding liquidity stress is arising from both market-specific and bank-specific factors.

The time spent in a particular scenario is random. Thus, it is impossible to ex-ante establish which scenarios will be observed over an experiment period. For example, it could be that over the experiment period, 90% of the time was in only one scenario, with less credible time observed for other scenarios. In this case, only meaningful inference can be made to one scenario. Nevertheless, ex-post, one can make reasonable judgement of the potential evolution in other states by making inference relative to the state where credible time spent was experienced.

5.0 Case Study

Figure 3: Run-off Profile for Individual Savings Product under a 'Business as Usual' Liquidity Scenario



6.0 Conclusion

This paper developed a unique method to handle cash-flow timing uncertainty when measuring bank funding-liquidity-risk. More specifically, the study utilised a survival model to allocate cash-flows to future periods for bank products. In addition, the survival model was employed to split a notional amount of a financial product into a stable and unstable portion. Moreover, the proposed approach leads to a better understanding of run-off rate distribution for financial products, which is important for simulating the trajectory of cash-flows for existing business, rollovers and future new business.

While the proposed approach gives a better understanding to measuring scenario-specific funding liquidity risk, the study has its limitations. For instance, the proposed approach ignores potential correlations between 'survival' times of different assets and liabilities. Such correlations can be accounted for in future research through, for example, using copula functions: these functions would link together 'survival' distributions of various assets and liabilities with a dependence (correlation) structure. Another limitation is that the study ignores interactions between funding liquidity risk and other dimensions of liquidity risk (systemic liquidity risk and market liquidity risk). Therefore, the contributions proposed

by this study are only suggestive of further theoretical and empirical work in this relatively unstudied field of funding liquidity risk measurement.

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